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INTERNATIONAL A LEVEL MATHEMATICS • PURE 4 (WMA14)

Pure 4 Practice Paper 2

Edexcel IAL Pure Mathematics 4

AIMING FOR A

Written in the style of the Pearson Edexcel International Advanced Level paper (WMA14/01) • Calculator allowed

A full 75-mark paper, harder than a typical real series, with fewer steps given and topics combined in every question

Time	Total	You must have
1 hour 30 minutes	**75 marks**	**Mathematical Formulae and Statistical Tables, calculator**

Instructions

- Use black ink or ball-point pen. Pencil may be used for diagrams and sketches.
- Answer **all** questions, in the spaces provided.
- Show enough working to make your method clear. Answers without working may not gain full credit.
- Give inexact answers to **three significant figures** unless the question says otherwise.

Information

- There are 9 questions in this paper. The total mark is 75.
- The marks for each question are shown in brackets.

Advice

- Sit the paper in one go, in 1 hour 30 minutes, with the mark scheme closed.
- Read each question carefully and try every question. Check your answers if you have time.
- Then mark it line by line with the mark scheme and fill in your score.

Track your score

Q	Topic	Marks	Score
1	Improper partial fractions and a series	9	
2	A binomial approximation and a proof	8	
3	Area under a parametric curve	8	
4	A draining tank	8	
5	A minimum point and a volume	9	
6	Partial fractions and an exact area	8	
7	Implicit gradients and a proof	8	
8	The shortest distance to a line	9	
9	Series approximation of an integral	8	
	Total	**75**	

Rough grade guide (out of 75): A* **61** • A **54** • B **46** • C **38**

Based on recent WMA14 grade boundaries. Real boundaries change every series, so treat this as a guide, not a grade.

Mark scheme | Exam questions by topic | More practice papers

Name ..... Date .....

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1.

$$f(x) = \frac{4x^2 + x + 4}{(1+2x)(1-x)}$$

(a) Express $f(x)$ in the form

$$A + \frac{B}{1+2x} + \frac{C}{1-x}$$

where A, B and C are constants to be found.

(3)

(b) Find the series expansion of $f(x)$, in ascending powers of x, up to and including the term in x^2, and state the range of values of x for which it is valid.

(4)

(c) Given that the coefficient of x^2 in the expansion of $(1+kx)f(x)$ is zero, find the value of the constant k.

(2)

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Question 1 continued

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(Total for Question 1 is 9 marks)

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2. **(a)** Find the binomial expansion of

$$\sqrt{1-8x} \qquad |x| < \frac{1}{8}$$

in ascending powers of x, up to and including the term in x^3, simplifying each coefficient.

(3)

(b) By substituting $x = 0.01$ into your expansion, find a rational approximation to $\sqrt{23}$, giving your answer as a fraction in its simplest form.

(2)

(c) Prove by contradiction that $\sqrt{23}$ is irrational.

(3)

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Question 2 continued

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(Total for Question 2 is 8 marks)

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3. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

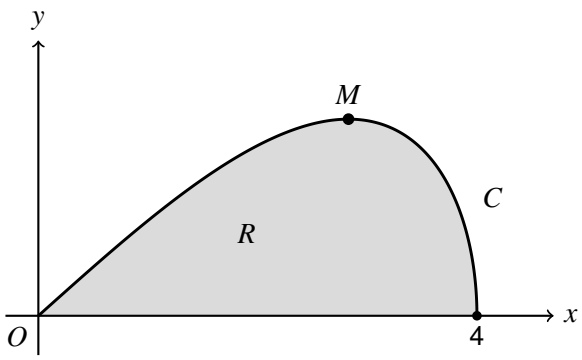

Figure 1

Figure 1 shows the curve C with parametric equations

$$x = 4\cos t, \qquad y = \sin 2t \qquad 0 \leqslant t \leqslant \frac{\pi}{2}$$

The point M is the maximum point on C.

(a) Find the exact coordinates of M.

(3)

The region R, shown shaded in Figure 1, is bounded by C and the x-axis.

(b) Find the exact area of R.

(5)

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Question 3 continued

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(Total for Question 3 is 8 marks)

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4.

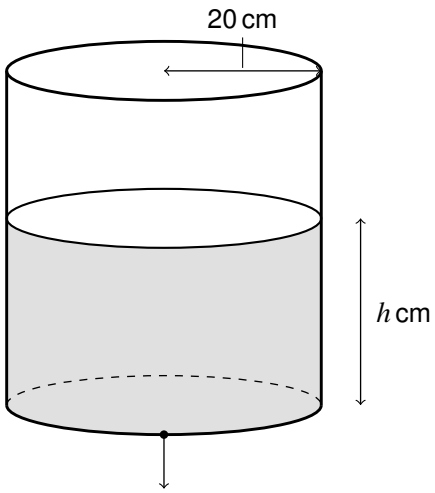

Figure 2

Figure 2 shows a tank in the shape of a vertical cylinder with radius 20 cm.

Water flows out of a small hole in the base of the tank. When the depth of the water is h cm, the water flows out at a rate of $80\pi\sqrt{h}$ cm^3 per minute.

At time $t = 0$ minutes the depth of the water is 100 cm, and no water flows into the tank.

(a) Form and solve a differential equation to find h in terms of t.

(5)

(b) Sketch the graph of h against t, for the time until the tank is empty.

(1)

(c) Find, to the nearest tenth of a minute, the time taken for the depth of the water to halve.

(2)

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Question 4 continued

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(Total for Question 4 is 8 marks)

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5. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

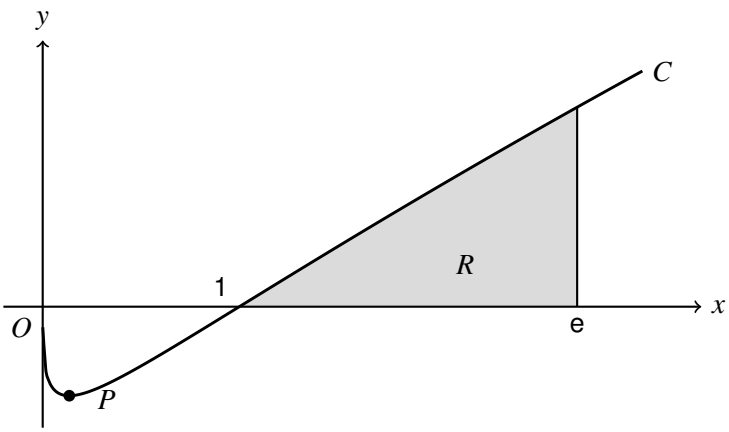

Figure 3

Figure 3 shows a sketch of the curve C with equation

$$y = \sqrt{x}\ln x \qquad x > 0$$

The point P is the minimum point on C.

(a) Find the exact coordinates of P.

(3)

The region R, shown shaded in Figure 3, is bounded by C, the x-axis and the line $x = \mathrm{e}$

The region R is rotated through 2π radians about the x-axis to form a solid of revolution.

(b) Find the exact volume of the solid.

(6)

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Question 5 continued

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(Total for Question 5 is 9 marks)

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6. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

$$f(x) = \frac{2x^2 + 1}{x^2(2x + 1)} \qquad x > 0$$

(a) Express $f(x)$ in partial fractions.

(3)

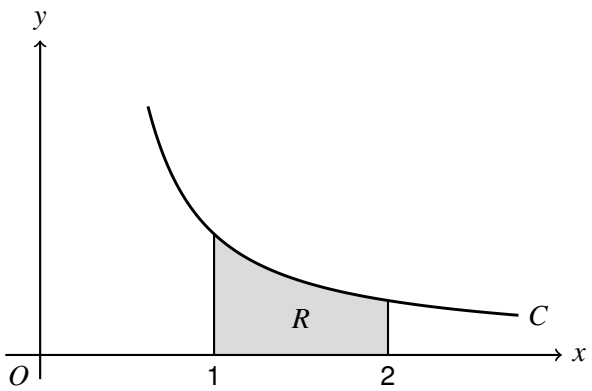

Figure 4

Figure 4 shows part of the curve with equation $y = f(x)$

The region R, shown shaded in Figure 4, is bounded by the curve, the x-axis and the lines $x = 1$ and $x = 2$

(b) Find the exact area of R, giving your answer in the form $p + \ln q$, where p and q are rational constants.

(5)

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Question 6 continued

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(Total for Question 6 is 8 marks)

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7. The curve C has equation

$$y^3 + y = x^3 - x$$

(a) Find $\frac{dy}{dx}$ in terms of x and y.

(3)

(b) Find an equation of the tangent to C at the point on C where $x = 1$

(2)

(c) Prove by contradiction that there is no point on C at which the gradient of C is less than -1

(3)

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Question 7 continued

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(Total for Question 7 is 8 marks)

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8. The line l has equation

$$\mathbf{r} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$$

The point A has coordinates (6, 4, 1), and P is the point on l with parameter λ.

(a) Find AP^2 in terms of λ, simplifying your answer.

(2)

(b) Use calculus to find the shortest distance from A to l, and the coordinates of the point F on l that is closest to A.

(4)

(c) Verify that AF is perpendicular to l.

(1)

(d) Find the coordinates of the reflection of A in the line l.

(2)

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Question 8 continued

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(Total for Question 8 is 9 marks)

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9. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

(a) Find the binomial expansion of $(1+x)^{-\frac{1}{2}}$, in ascending powers of x, up to and including the term in x^2.

(2)

(b) Use your expansion to find an approximate value of

$$\int_0^{0.2} \frac{x}{\sqrt{1+x}}\,\mathrm{d}x$$

giving your answer to 5 significant figures.

(2)

(c) Use the substitution $u = 1+x$ to find the exact value of this integral, giving your answer in the form $a+b\sqrt{30}$, where a and b are rational.

(3)

(d) Find the percentage error in your answer to part (b).

(1)

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Question 9 continued

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(Total for Question 9 is 8 marks)

TOTAL FOR PAPER IS 75 MARKS

END

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