# Graphs and Transformations

## Worked Solutions

Suitable for:

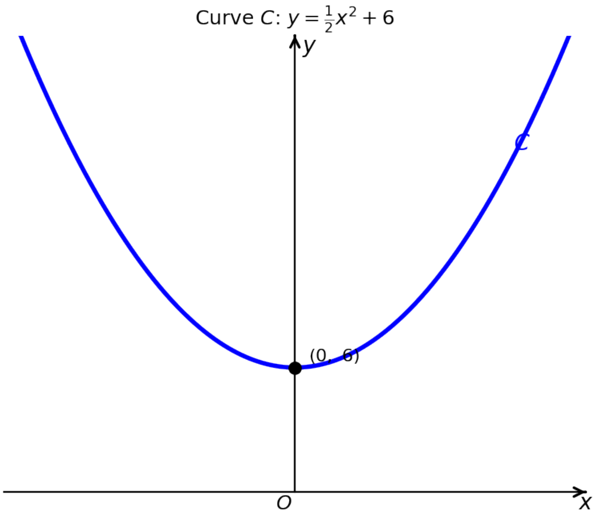
**Pearson Edexcel International A-Level (iAL) — Pure Mathematics 1**

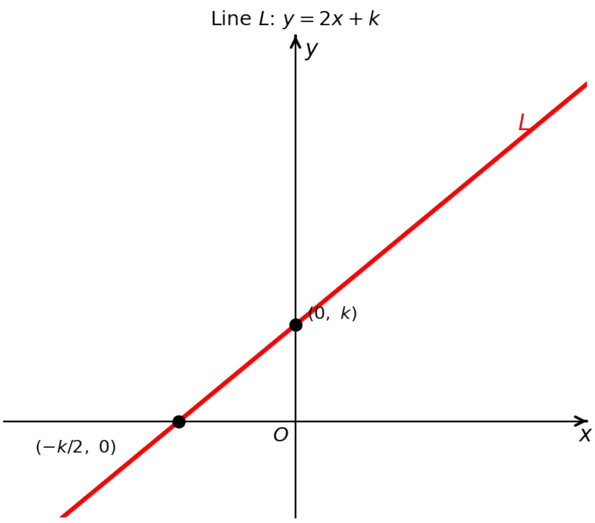
## Question 1 — Worked Solution

### (a) Sketches

*C*: upward parabola, minimum value 6 — does not cross *x*-axis. *y*-intercept (0, 6).

*L*: positive gradient 2. *y*-intercept (0, *k*), *x*-intercept (–*k*/2, 0).




### (b) Finding k

Set C equal to L:

$$\frac{1}{2}x^2 + 6 = 2x + k$$

$$\frac{1}{2}x^2 - 2x + (6 - k) = 0 \;\Rightarrow\; x^2 - 4x + 2(6 - k) = 0$$

For tangency, discriminant = 0:

$$\Delta = 16 - 4 \cdot 2(6 - k) = 0$$

$$16 - 8(6 - k) = 0 \Rightarrow 16 - 48 + 8k = 0 \Rightarrow 8k = 32$$

$$\therefore k = 4$$

## Question 2 — Worked Solution

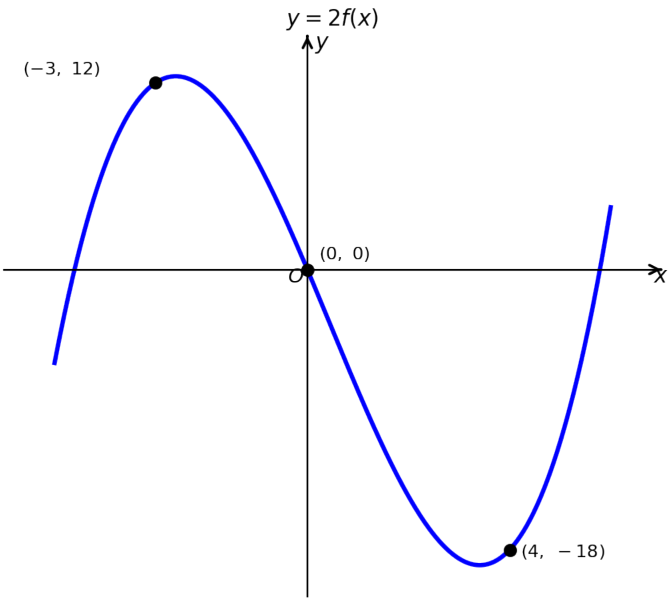
### (a) y = 2f(x) — vertical stretch, scale factor 2

Multiply all y-coordinates by 2. x-coordinates unchanged.

Max: A(–3, 6) → A'(–3, 12)

Min: B(4, –9) → B'(4, –18)

y-intercept: O(0, 0) → (0, 0) — unchanged



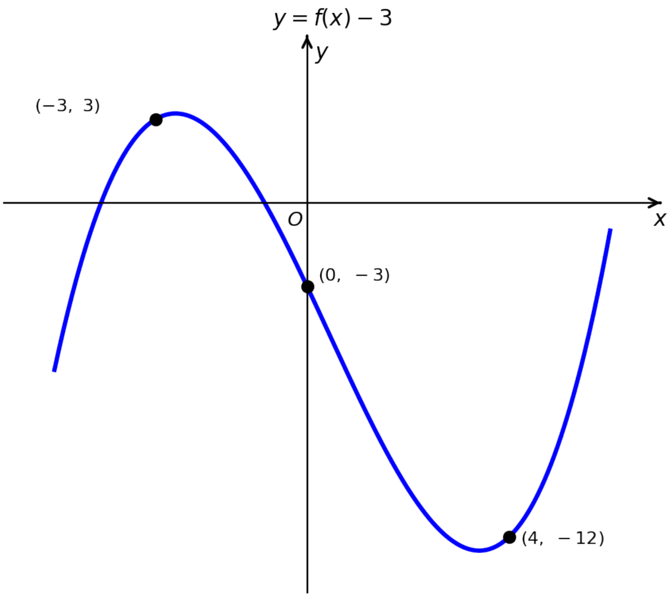

### (b) y = f(x) – 3 — vertical translation down 3

Subtract 3 from all y-coordinates. x-coordinates unchanged.

Max: A(–3, 6) → A'(–3, 3)

Min: B(4, –9) → B'(4, –12)

y-intercept: O(0, 0) → (0, –3)

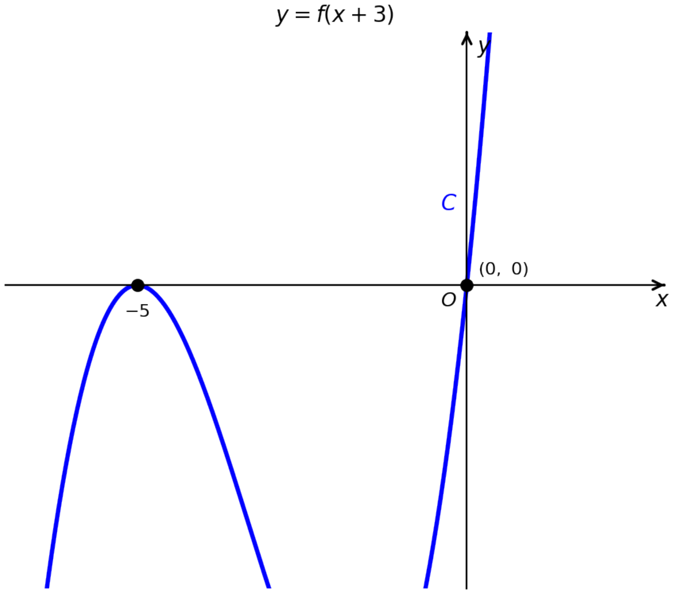

$y = f(x) - 3$
$y$
$(-3, 3)$
$O$
$x$
$(0, -3)$
$(4, -12)$

## Question 3 — Worked Solution

### (a) Sketch of y = f(x + 3)

f(x + 3): horizontal translation 3 units to the LEFT.

x-intercepts shift: (–2, 0) → (–5, 0) [touch]; (3, 0) → (0, 0) [cross]




$$\therefore \text{Meets } x\text{-axis at } (-5, 0) \text{ (touch) and } (0, 0) \text{ (cross)}$$

### (b) Equation of C

Substitute (x + 3) for x in f(x):

$$y = f(x+3) = (x+3+2)^2(x+3-3)$$

$$\therefore y = (x+5)^2 \cdot x$$

### (c) y-intercept of C

Set x = 0:

$$y = (0+5)^2 \times 0 = 0$$

$$\therefore \text{y-intercept: } (0,\ 0)$$

## Question 4 — Worked Solution

### (a) Finding a, b, c

C crosses (–2, 0) and touches (4, 0) — so (4, 0) is a repeated root:

$$y = (x + 2)(x - 4)^2$$

Expand:

$$(x - 4)^2 = x^2 - 8x + 16$$

$$y = (x + 2)(x^2 - 8x + 16) = x^3 - 8x^2 + 16x + 2x^2 - 16x + 32$$

$$y = x^3 - 6x^2 + 0x + 32$$

Verify max at (0, 32): y = 0 – 0 + 0 + 32 = 32 ✓

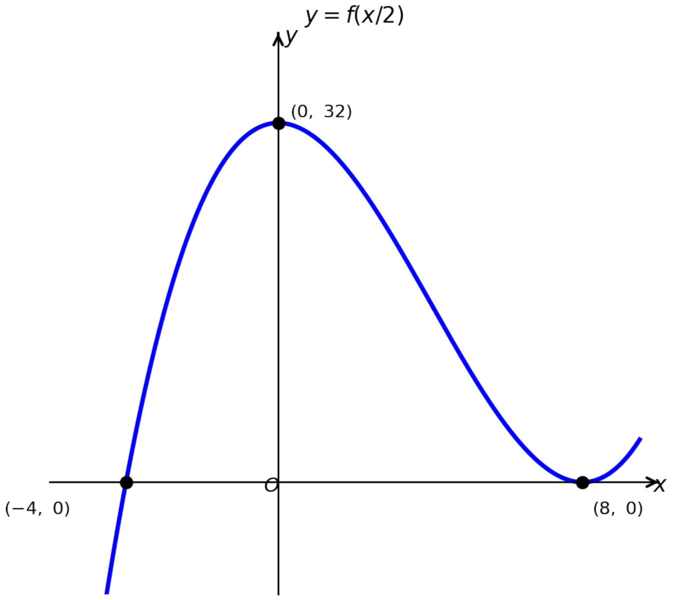
$$\therefore a = -6, \quad b = 0, \quad c = 32$$

### (b) Sketch of y = f(x/2)

Replace x with x/2: horizontal stretch, scale factor 2.

x-intercepts: (–2, 0) → (–4, 0); (4, 0) → (8, 0) [touch]

y-intercept: (0, 32) unchanged



## Question 5 — Worked Solution

### (a) x-coordinate of A

Set y = 0:

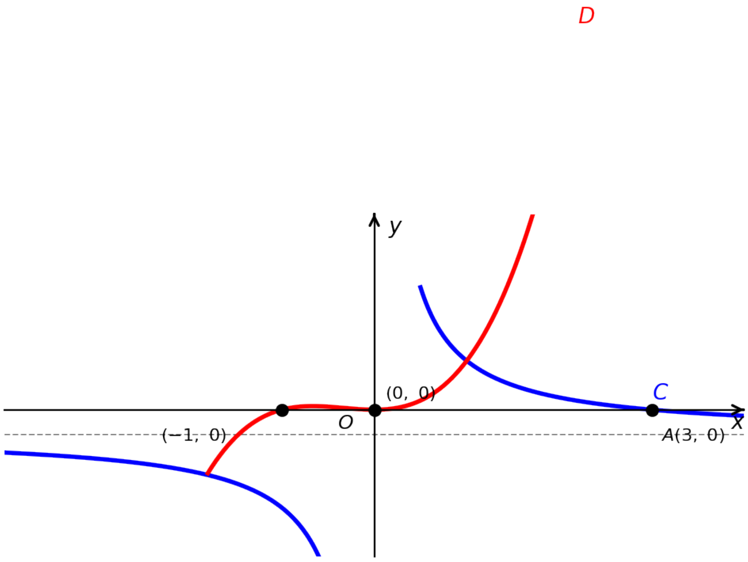
$$\frac{3}{x} - 1 = 0 \Rightarrow \frac{3}{x} = 1 \Rightarrow x = 3$$

$$\therefore x = 3$$

### (b) Sketch of D: y = x²(x + 1)

D touches x-axis at (0, 0) [double root] and crosses at (–1, 0).

Positive cubic — rises steeply for large positive x.




### (c) Number of real solutions

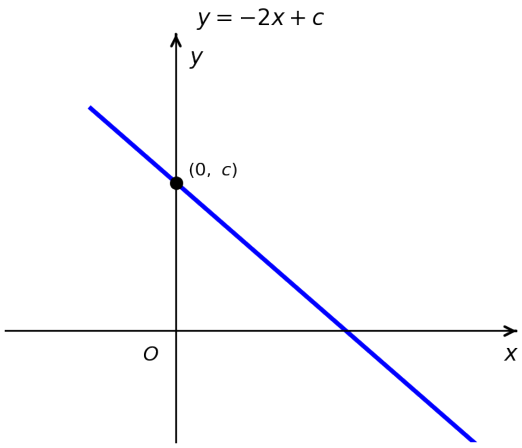
Solutions of x²(x+1) = 3/x – 1 occur where C and D intersect.

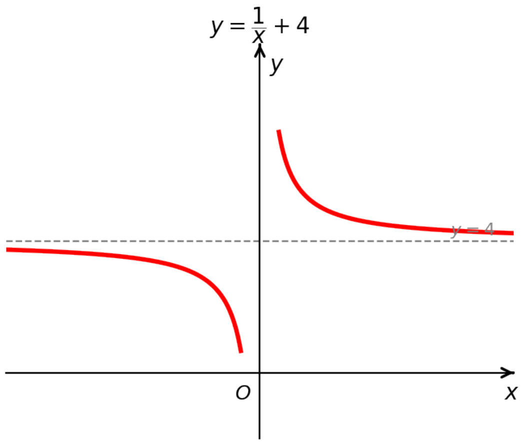
From the sketch, the curves intersect at 2 points.

$$\therefore \text{2 real solutions}$$

## Question 6 — Worked Solution

### (a) Sketches




### (b) Show $(4 – c)^2 > 8$

Set the line equal to the curve:

$$-2x + c = \frac{1}{x} + 4$$

$$x(-2x + c) = 1 + 4x \Rightarrow -2x^2 + cx = 1 + 4x$$

$$2x^2 + (4 - c)x + 1 = 0$$

For two distinct intersections, discriminant > 0:

$$\Delta = (4 - c)^2 - 4(2)(1) > 0$$

$$\therefore (4 - c)^2 > 8 \quad ✓$$

### (c) Range of values of c

$$|4 - c| > 2\sqrt{2}$$

$$c < 4 - 2\sqrt{2} \quad \text{or} \quad c > 4 + 2\sqrt{2}$$

Since c is a positive constant and $4 – 2\sqrt{2} \approx 1.17 > 0$:

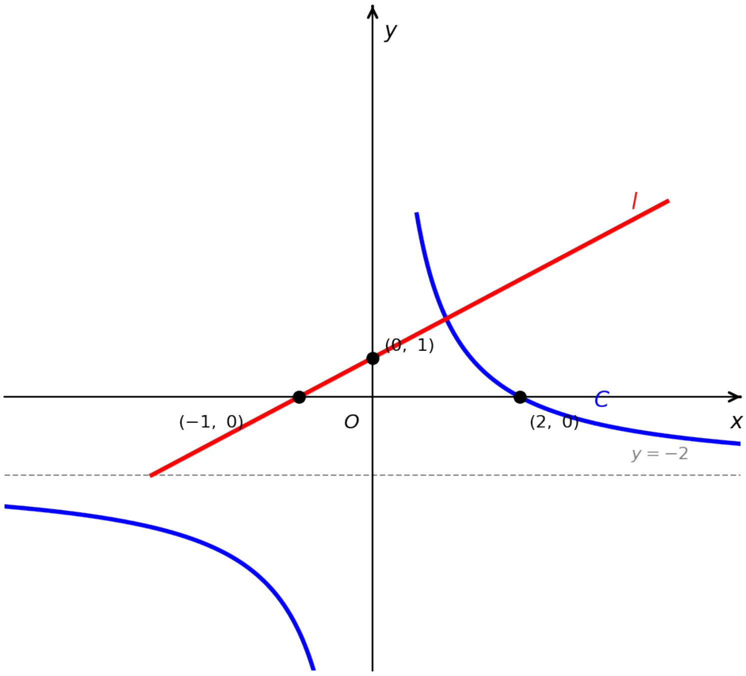
$$\therefore 0 < c < 4 - 2\sqrt{2} \quad \text{or} \quad c > 4 + 2\sqrt{2}$$

## Question 7 — Worked Solution

### (a) Sketch of C and l

C: $y = 4/x - 2$. Asymptotes $x = 0$, $y = -2$. x-intercept: $4/x = 2 \to x = 2$.

Line l: y-intercept (0, 1); x-intercept: $x + 1 = 0 \to x = -1$.




### (b) Asymptotes of C

$$\therefore x = 0 \quad \text{and} \quad y = -2$$

### (c) Points of intersection

Set equal:

$$\frac{4}{x} - 2 = x + 1 \Rightarrow \frac{4}{x} = x + 3$$

$$4 = x(x+3) = x^2 + 3x \Rightarrow x^2 + 3x - 4 = 0$$

$$(x+4)(x-1) = 0 \Rightarrow x = -4 \text{ or } x = 1$$

When $x = 1$: $y = 1 + 1 = 2$

When $x = -4$: $y = -4 + 1 = -3$

$$\therefore (1,\ 2) \quad \text{and} \quad (-4,\ -3)$$

## Question 8 — Worked Solution

### (a) Completing the square

$$2x^2 + 12x + 10 = 2(x^2 + 6x) + 10$$

$$= 2\left[(x+3)^2 - 9\right] + 10 = 2(x+3)^2 - 18 + 10$$

$$= 2(x+3)^2 - 8$$

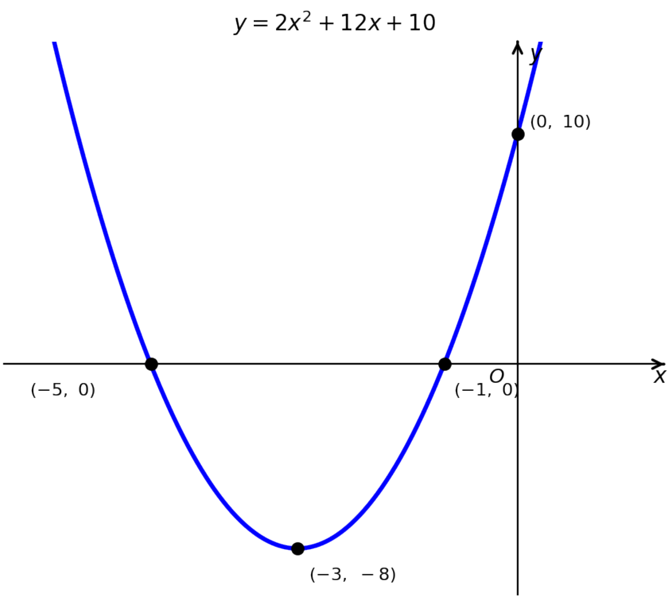
$$\therefore a = 2, \quad b = 3, \quad c = -8$$

### (b) Sketch

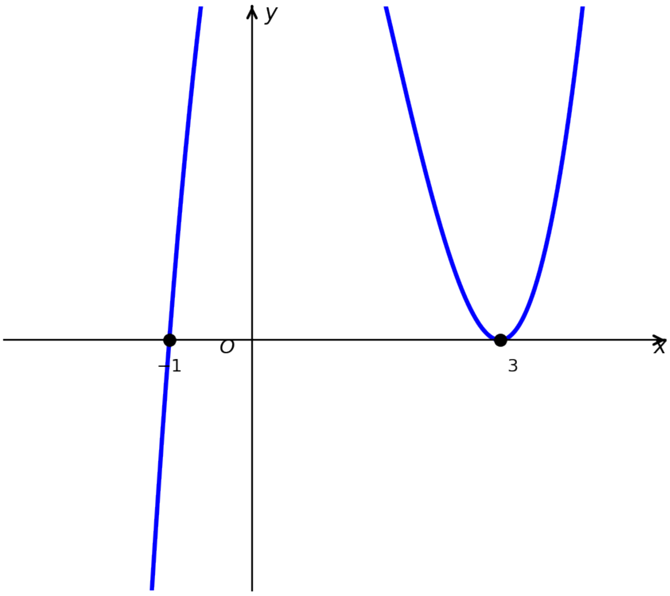
Factorise to find roots: $2x^2 + 12x + 10 = 2(x+1)(x+5) = 0$

Roots: $x = -1$ and $x = -5$ ✓ integers

y-intercept: (0, 10); vertex: (–3, –8)



## Question 9 — Worked Solution




**(a)(i) Find k**

For y = f(x) – k to pass through (0, 0):

$$f(0) - k = 0 \;\Rightarrow\; k = f(0)$$

$$f(0) = 4(0-3)^2(0+1) = 4 \times 9 \times 1 = 36$$

$$\therefore k = 36$$

**(a)(ii) Find c**

The minimum of f(x) occurs at the repeated root x = 3.

For f(x + c) to have minimum at the origin, we need x = 0 to map to x = 3:

$$0 + c = 3$$

Check: f(0 + 3) = 4(3–3)²(3+1) = 0 ✓ — minimum value is 0 at origin.

$$\therefore c = 3$$