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GCSE MATHEMATICS • ALGEBRA

Simultaneous Equations

Worked Solutions

Pearson Edexcel GCSE (1MA1) & International GCSE (4MA1) • No calculator

Full worked solutions for the free Simultaneous Equations worksheet, GCSE and iGCSE Maths



Tap a section to jump straight to it

Full worked solutions for every question on the Simultaneous Equations worksheet, in the same order and with the same question numbers.

How to mark your work. Tick each step you got right, not just the final answer. In the exam, method marks are given for correct working even when the final answer is wrong. If you lost marks, find the *first* line where your working is different from ours. That is the thing to fix.

In Section E: **M** = method mark, **P** = process mark, **A** = accuracy mark, **B** = independent mark, **C** = communication mark.

Question sheet

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A**Your turn answers**

Compare each line of your working with the solution.

FOUNDATION

- A1** Add: $6x = 18$, so $x = 3$ $12 + 3y = 18$, so $3y = 6$ $x = 3$, $y = 2$
- A2** Double the second: $6x - 2y = 22$ Add: $11x = 33$, so $x = 3$ $15 + 2y = 11$ $x = 3$, $y = -2$
- A3** In pence: $2c + 3t = 1080$ and $3c + t = 990$. Treble the second: $9c + 3t = 2970$
Subtract: $7c = 1890$, so $c = 270$ $810 + t = 990$, so $t = 180$ $c = 270$, $t = 180$: coffee £2.70, tea £1.80
- A4** The lines cross at $(1, 4)$, so $x = 1$, $y = 4$

HIGHER

- A5** $\times 7$ and $\times 3$: $35x + 21y = 147$ and $6x - 21y = -24$ Add: $41x = 123$, so $x = 3$ $15 + 3y = 21$ $x = 3$, $y = 2$
- A6** $x^2 = 3x - 2$ $x^2 - 3x + 2 = 0$ $(x - 1)(x - 2) = 0$ from $y = 3x - 2$: $(1, 1)$ and $(2, 4)$
- A7** $x^2 + (x + 1)^2 = 25$ $2x^2 + 2x - 24 = 0$ $x^2 + x - 12 = 0$ $(x + 4)(x - 3) = 0$ from $y = x + 1$: $(3, 4)$ and $(-4, -3)$

B**Fluency solutions**One mark for each correct part. The notes in *italics* explain why each sequence was chosen.**FOUNDATION**

- B1** (a) $2x = 14$: $(7, 3)$ (b) $2x = 6$: $(3, 7)$ (c) subtract, $x = 3$: $(3, 4)$ (d) subtract, $x = 3$: $(3, -4)$
Opposite signs: add. The same signs: subtract.
- B2** (a) $6x + 2y = 18$, $5x = 10$: $(2, 3)$ (b) $3x - 3y = 3$, $5x = 15$: $(3, 2)$ (c) $3x + 3y = 27$, $7x = 28$: $(4, 5)$
(d) $3x + 6y = 3$, $y = 1$: $(-1, 1)$



B3 (a) $x + x + 3 = 11$: $(4, 7)$ (b) $5x = 20$: $(4, 8)$ (c) $6y - 3 - y = 12$: $(5, 3)$ (d) $2x - 15 + 3x = 5$: $(4, 1)$

B4 (a) $(4, -3)$ (b) $3x = 6$: $(2, -4)$ (c) $y = x$, $4x = 5$: $(\frac{5}{4}, \frac{5}{4})$ (d) $2x = -2$: $(-1, 5)$

B5 (a) $x + y = 20$, $x - y = 6$: **13 and 7** (b) $3a + 2b = 90$, $a + 2b = 50$, $2a = 40$: apple **20p**, banana **15p**
 (c) $l - w = 3$, $2l + 2w = 26$: length **8 cm**, width **5 cm**

B6 (a) 1 and 5: **yes** (b) $6 - 6 = 0$ and $3 + 4 = 7$: **yes** (c) $12 - 2 = 10$ but $9 - 4 = 5 \neq 6$: **no** (d) $-2 = -2$ and $-2 = -2$: **yes**

HIGHER

B7 (a) $6x + 9y = 21$, $6x + 4y = 16$: $(2, 1)$ (b) $9x - 12y = 30$, $20x + 12y = 28$: $(2, -1)$
 (c) $8x + 10y = 46$, $15x + 10y = 60$: $(2, 3)$ (d) $18x + 15y = 24$, $20x - 15y = 90$: $(3, -2)$

B8 (a) $x^2 = 4$, and $y = x^2$: $(2, 4)$ and $(-2, 4)$ (b) $x^2 - x - 6 = 0$, $y = x + 6$: $(3, 9)$ and $(-2, 4)$
 (c) $x^2 - 2x + 1 = 0$, $(x - 1)^2 = 0$: one solution, $(1, 2)$ (d) $x^2 = -1$ has no solution: **none**
The same curve with different lines: two crossings, a touching line, and a line that misses.

B9 (a) $x^2 = 16$, and $x^2 + y^2 = 25$: $(4, 3)$ and $(-4, 3)$ (b) $x^2 - x - 12 = 0$, $y = x - 1$: $(4, 3)$ and $(-3, -4)$
 (c) $5x^2 = 20$, $y = 2x$: $(2, 4)$ and $(-2, -4)$ (d) $2x^2 - 8x + 8 = 0$, $(x - 2)^2 = 0$: $(2, 2)$, one point

B10 (a) $x(x + 1) = 12$, $x^2 + x - 12 = 0$, $y = x + 1$: $(3, 4)$ and $(-4, -3)$ (b) $x^2 - 3x = 0$, $y = x$: $(0, 0)$ and $(3, 3)$
 (c) $x^2 - 3x - 4 = 0$, $y = 3x$: $(4, 12)$ and $(-1, -3)$ (d) $x^2 = 4x$, $y = x$: $(0, 0)$ and $(4, 4)$

C

Reasoning solutions

Your wording can be different. Check that your reason makes the same point.

FOUNDATION

C1 Ben did not multiply the 5 by 2. It should be $2x + 2y = 10$ Subtract: $y = 2$, then $x + 2 = 5$ $x = 3$, $y = 2$

C2 Sam has not found y . Substitute $x = 3$: $9 + 2y = 13$, so $y = 2$



C3 Doubling $x + y = 5$ gives $2x + 2y = 10$, which cannot also equal 7. The lines are parallel, so they never meet.

C4 Elimination: add to get $3x = 9$, so $x = 3$ and $y = 1$ Substitution: $y = 7 - 2x$, so $x - (7 - 2x) = 2$, $3x = 9$, $x = 3$, $y = 1$

Both methods give $x = 3$, $y = 1$

HIGHER

C5 Each value of x needs its matching y . From $y = x + 2$: $(2, 4)$ and $(-1, 1)$

C6 Substituting a line into a circle gives a quadratic, which has 0, 1 or 2 solutions. Two different straight lines give a linear equation, which has one solution, or none if the lines are parallel.

C7 Kai used the wrong y for each x . The y values must come from $y = x - 1$: $(4, 3)$ and $(-3, -4)$

C8 $(10 - 2y)^2 + y^2 = 16$ $100 - 40y + 4y^2 + y^2 = 16$ $5y^2 - 40y + 84 = 0$
 $5y^2 - 40y + 84 = 5(y - 4)^2 + 4$, and $5(y - 4)^2 + 4 \geq 4 > 0$, so the equation has no solution: the line does not meet the circle.

D

Problem solving solutions

There is often more than one correct method. Any complete, correct method scores full marks.

FOUNDATION

D1 $x + y = 42$ and $x = 2y + 6$ $3y + 6 = 42$, so $y = 12$ $x = 30$, $y = 12$

D2 $x + y = 25$ and $10x + 20y = 360$ $10x + 10y = 250$, so $10y = 110$ $y = 11$, $x = 14$: 14 ten-pence and 11 twenty-pence coins

D3 Opposite sides are equal: $3x - y = 11$ and $x + 2y = 13$ $6x - 2y = 22$, so $7x = 35$ $x = 5$, $y = 4$

D4 $2m + c = 7$ and $5m + c = 16$ Subtract: $3m = 9$ $m = 3$, $c = 1$



HIGHER

D5 $25t - 5t^2 = 10t + 10$ $5t^2 - 15t + 10 = 0$ $t^2 - 3t + 2 = 0$ $(t - 1)(t - 2) = 0$

$t = 1$, height **20 m**, and $t = 2$, height **30 m**

D6 $2a + 3b = 13$ and $2b - 3a = -13$ $\times 3$ and $\times 2$: $6a + 9b = 39$ and $-6a + 4b = -26$ $13b = 13$ $b = 1$,
 $a = 5$

D7 $1 + a + b = 2$ and $9 + 3a + b = 12$, so $a + b = 1$ and $3a + b = 3$ Subtract: $2a = 2$ $a = 1$, $b = 0$
 Check with $x = 2$: $y = 4 + 2 = 6$, and $y = x^2 + x$ gives 6

D8 (a) $x^2 + (2x + k)^2 = 20$ $x^2 + 4x^2 + 4kx + k^2 = 20$ $5x^2 + 4kx + k^2 - 20 = 0$ **as required**
 (b) One solution means a repeated root: $(4k)^2 - 4 \times 5 \times (k^2 - 20) = 0$ $16k^2 - 20k^2 + 400 = 0$ $k^2 = 100$
 $k = 10$ or $k = -10$

E

Exam practice solutions

Marks are shown on the right of each line.

FOUNDATION

Foundation and Higher crossover

E1 Add: $4x = 12$ M1
 $x = 3$ A1
 $9 + y = 11$ $x = 3$, $y = 2$ A1

E2 $4x + 6y = 26$ and $15x - 6y = 12$ M1
 Add: $19x = 38$, so $x = 2$ A1
 $4 + 3y = 13$ $x = 2$, $y = 3$ A1

E3 $4a + 3c = 55$ and $2a + 5c = 45$ B1
 $4a + 10c = 90$ M1
 Subtract: $7c = 35$, so $c = 5$ A1
 $4a + 15 = 55$ $a = 10$, $c = 5$: adult £10, child £5 A1

E4 $3x + 2(2x - 3) = 22$ M1
 $7x - 6 = 22$, so $x = 4$ A1
 $y = 8 - 3$ $x = 4$, $y = 5$ A1



E5 Subtracting gives $y - (-y) = 2y$ and $8 - 2 = 6$, so $2y = 6$, not 10 C1
 $x = 5$, $y = 3$ B1

E6 $3(2) + 2(-1) = 6 - 2 = 4$ B1
 $2 - 4(-1) = 2 + 4 = 6$ Both equations are true B1

HIGHER

E7 $x^2 - 3x + 2 = 2x - 4$ M1
 $x^2 - 5x + 6 = 0$ M1
 $(x - 2)(x - 3) = 0$ A1
 From $y = 2x - 4$: $(2, 0)$ and $(3, 2)$ A1

E8 (a) The lines cross at about $(2.7, 1.7)$ B1
 (b) $2x + x - 1 = 7$ M1
 $3x = 8$ $\left(\frac{8}{3}, \frac{5}{3}\right)$ A1

E9 $2l + 2w = 26$ and $lw = 40$ B1
 $w = 13 - l$, so $l(13 - l) = 40$ M1
 $l^2 - 13l + 40 = 0$, $(l - 5)(l - 8) = 0$ M1
 $8 \text{ cm by } 5 \text{ cm}$ A1

E10 $x^2 + (3 - x)^2 = 17$ M1
 $2x^2 - 6x - 8 = 0$, so $x^2 - 3x - 4 = 0$ M1
 $(x - 4)(x + 1) = 0$: $A(4, -1)$ and $B(-1, 4)$ A1
 $AB = \sqrt{5^2 + 5^2} = \sqrt{50}$ M1
 $5\sqrt{2}$ A1

