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INTERNATIONAL A LEVEL MATHEMATICS • DECISION MATHEMATICS 1

Decision 1

Chapter 2: Graphs and Networks

Exam Questions by Topic

Pearson Edexcel International Advanced Level (WDM11/01) • Calculator allowed • Answer book pages included

Multi-step exam questions at the difficulty of the real WDM11 paper, with a full mark scheme

Chapters (tap a chapter to jump to it)		Qs	Marks	Page	My score
1	**Algorithms** Flow charts, bubble sort, quick sort, bin packing, binary search	10	80	separate file	
2	**Graphs and Networks** Vocabulary, types of graph, degrees, adjacency and distance matrices	10	80	2	
3	**Algorithms on Graphs** Kruskal, Prim (networks and matrices), Dijkstra	10	80	separate file	
4	**Route Inspection** Eulerian graphs, pairing odd vertices, start and finish points	10	80	separate file	
5	**The Travelling Salesman Problem** Classical and practical problems, upper and lower bounds	10	80	separate file	
6	**Critical Path Analysis** Activity networks, event times, floats, Gantt charts, scheduling	10	80	separate file	
7	**Linear Programming** Formulation, feasible regions, objective lines, integer solutions	10	80	separate file	

Full mark scheme Free courses More worksheets

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CHAPTER 2 • KEY SKILLS

2 Graphs and Networks

Specification D1: Graphs and networks • 10 questions • 80 marks • about 96 minutes

1 Terminology

Walk: any route. Trail: no arc repeated. Path: no vertex repeated. Cycle: a closed path. Hamiltonian cycle: visits every vertex once.

2 Degree

The degree (valency) of a vertex is the number of arcs meeting it. A loop adds 2.

3 Handshaking lemma

Sum of degrees = 2× number of arcs, so there is always an even number of odd vertices.

4 Simple and connected

Simple: no loops, at most one arc per pair. Connected: there is a path between every pair of vertices.

5 Trees

A tree is connected with no cycles; with n vertices it has $n-1$ arcs. A spanning tree includes every vertex.

6 Special graphs

K_n has $\frac{1}{2}n(n-1)$ arcs. Bipartite: two sets, arcs only between sets. $K_{r,s}$ has rs arcs.

7 Matrices

Adjacency: number of arcs between each pair (2 for a double arc). Distance: weights, with – for no direct arc. A digraph's matrix need not be symmetric.

8 Isomorphic graphs

Same structure drawn differently: match degrees, then give a full vertex correspondence that maps every arc.

9 Give reasons

When asked "explain" or "give a reason", write a sentence using the precise term.

Skills map every skill above is tested at least once; interleaved skills come from GCSE or other chapters

Skill	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Marks	6	6	6	6	6	10	10	10	10	10
1 Terminology	•			•				•	•	
2 Degree	•		•		•	•	•	•		•
3 Handshaking lemma	•		•			•	•	•		•
4 Simple and connected		•				•	•			
5 Trees			•				•		•	•
6 Special graphs			•			•			•	
7 Matrices		•		•				•		•
8 Isomorphic graphs					•					
9 Give reasons	•	•	•		•	•	•	•	•	•
Interleaved: combinations and factorials (P1, S1)			•			•			•	
Interleaved: proof by contradiction (P4)			•				•			
Interleaved: odd vertices and route inspection (Chapter 4)	•									•

Exam difficulty. Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WDM11 paper, where one part leads into the next.

Answer book. After each question is a copy of the page the real D1 answer book prints for it (reprinted matrices, vertex layouts to draw on, tables), so you can practise using the answer book exactly as in the exam.

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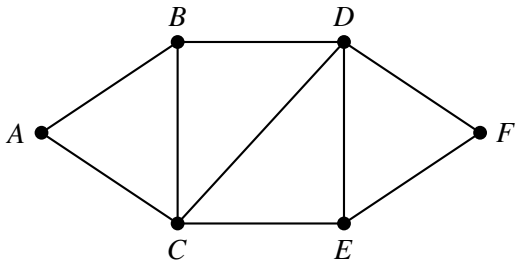


Figure 1

Figure 1 shows a graph G.

(a) Write down the degree of each vertex of G.

(2)

(b) Use your answer to part (a) to find the number of arcs in G, stating the result you use.

(2)

(c) Write down a path from A to F that uses exactly three arcs, and a Hamiltonian cycle starting at A.

(2)

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Question 1 continued D1 ANSWER BOOK

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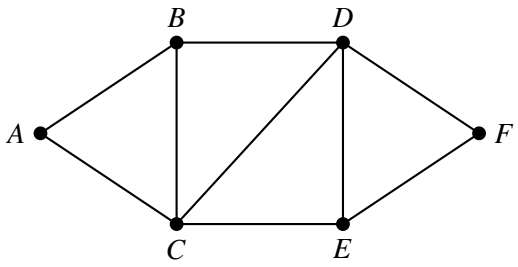


Diagram 1

Vertex	A	B	C	D	E	F
Degree						

(Total for Question 1 is 6 marks)

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2. The adjacency matrix below represents a graph H.

	P	Q	R	S	T
P	0	1	0	2	1
Q	1	0	1	0	0
R	0	1	0	1	1
S	2	0	1	0	0
T	1	0	1	0	0

(a) Draw H on the vertices given in the answer book.

(3)

(b) State, with a reason, whether H is simple, and whether H is connected.

(2)

(c) Write down the number of arcs in H.

(1)

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Question 2 continued **D1 ANSWER BOOK**

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	P	Q	R	S	T
P	0	1	0	2	1
Q	1	0	1	0	0
R	0	1	0	1	1
S	2	0	1	0	0
T	1	0	1	0	0

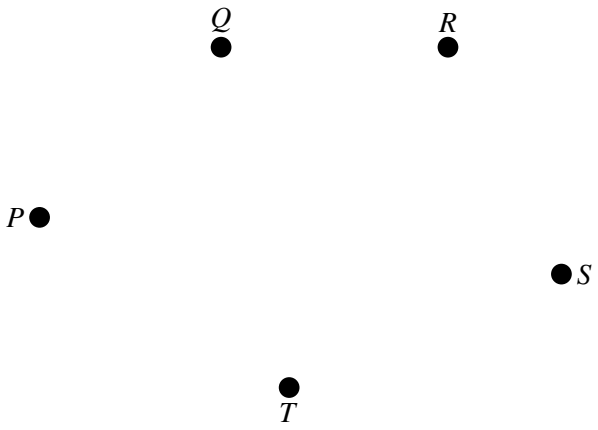


Diagram 2

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3. **(a)** Write down the number of arcs in the complete graph K_7

(1)

(b) Draw the complete bipartite graph $K_{3,4}$, and state its number of arcs.

(2)

(c) Explain why there is no graph whose vertices have degrees 3, 3, 2, 2 and 1

(2)

(d) Write down the minimum number of arcs in a connected graph with 9 vertices.

(1)

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Question 3 continued **D1 ANSWER BOOK**

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Diagram 3

(Total for Question 3 is 6 marks)

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4. The table shows the lengths, in km, of the roads directly joining five villages. A dash means there is no direct road.

	A	B	C	D	E
A	–	7	–	5	9
B	7	–	4	–	6
C	–	4	–	8	–
D	5	–	8	–	3
E	9	6	–	3	–

(a) Draw the network on the vertices given in the answer book, labelling each arc with its length.

(3)

(b) Find the total length of the roads.

(1)

(c) List the two paths from A to C that use exactly two arcs, and state which is shorter.

(2)

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Question 4 continued D1 ANSWER BOOK

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	A	B	C	D	E
A	–	7	–	5	9
B	7	–	4	–	6
C	–	4	–	8	–
D	5	–	8	–	3
E	9	6	–	3	–

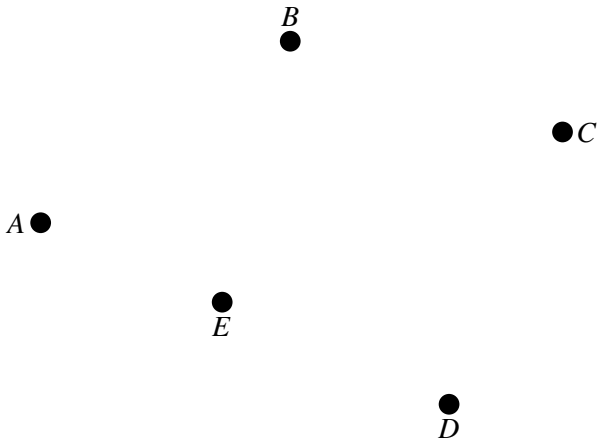


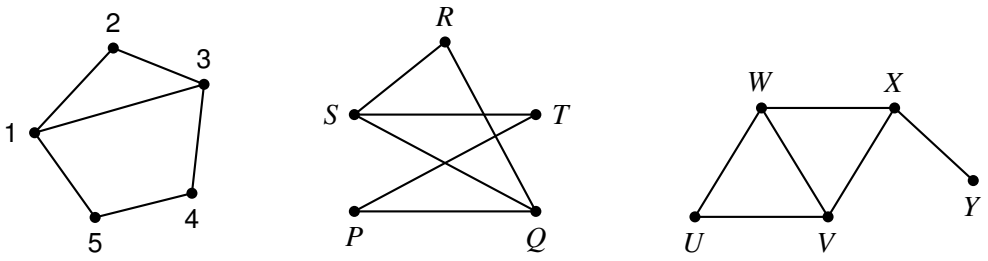
Diagram 4

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5.



Graph X Graph Y Graph Z

Figure 2

Figure 2 shows three graphs, X, Y and Z.

(a) Show that X and Y are isomorphic, by giving a correspondence between their vertices.

(4)

(b) Explain why Z is not isomorphic to X.

(2)

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Question 5 continued **D1 ANSWER BOOK**

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Vertex of $X \rightarrow$ vertex of Y				
1	2	3	4	5

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(Total for Question 5 is 6 marks)

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6. Four volunteers, A, B, C and D, are to be given jobs from the list 1, 2, 3, 4, 5.

A can do jobs 1 and 3; B can do jobs 2, 3 and 5; C can do job 1 only; D can do jobs 1 and 4

(a) Draw a bipartite graph to model this information.

(2)

(b) Write down the number of arcs, and the sum of the degrees of the vertices.

(2)

(c) Explain why job 1 must be given to C if every volunteer is to have a different job, and hence find such an allocation.

(3)

(d) State, with a reason, whether your bipartite graph is connected.

(1)

(e) Find how many more arcs would be needed to make the graph $K_{4,5}$

(2)

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Question 6 continued D1 ANSWER BOOK

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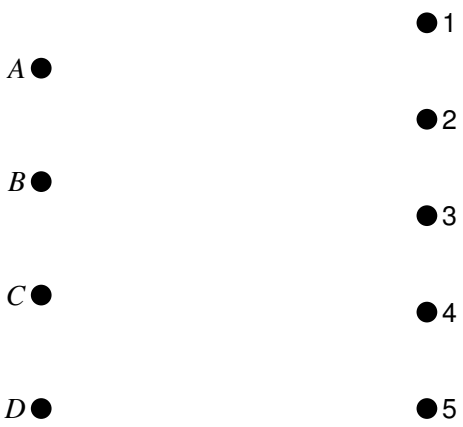


Diagram 5

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Question 6 continued

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(Total for Question 6 is 10 marks)

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7. **(a)** Prove that it is impossible for the five vertices of any graph to have degrees 1, 2, 3, 3 and 4

(2)

(b) Draw a simple graph with five vertices of degrees 2, 2, 3, 3 and 4, and state its number of arcs.

(3)

(c) Degrees 4, 4, 4, 1, 1 have an even total. Show that, even so, they cannot belong to a **simple** graph on five vertices.

(3)

(d) A tree has 6 vertices. Write down the number of arcs, and the sum of the degrees of its vertices.

(2)

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Question 7 continued D1 ANSWER BOOK

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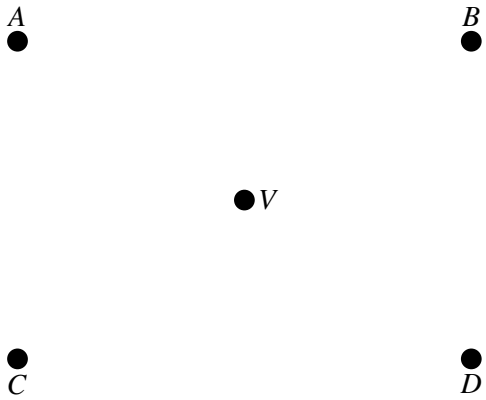


Diagram 6

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Question 7 continued

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(Total for Question 7 is 10 marks)

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8. Five junctions, A to E, are joined by one-way streets. An entry of 1 in the matrix means there is a street from the junction in that row to the junction in that column.

from \ to	A	B	C	D	E
A	0	1	1	0	0
B	0	0	1	1	0
C	0	0	0	1	1
D	1	0	0	0	1
E	0	1	0	0	0

(a) Draw the digraph on the vertices given in the answer book.

(3)

(b) Explain how the matrix shows that the streets are one-way.

(1)

(c) Complete the table of out-degrees and in-degrees, and use it to find the number of streets.

(3)

(d) List all the routes from A to E that use at most three streets and do not revisit a junction.

(3)

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Question 8 continued D1 ANSWER BOOK

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from \ to	A	B	C	D	E
A	0	1	1	0	0
B	0	0	1	1	0
C	0	0	0	1	1
D	1	0	0	0	1
E	0	1	0	0	0

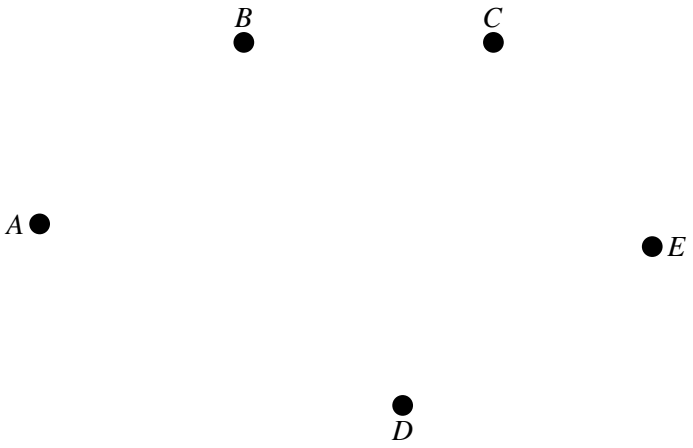


Diagram 7

Vertex	A	B	C	D	E
Out-degree					
In-degree					

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Question 8 continued

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(Total for Question 8 is 10 marks)

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9. **(a)** For the complete graph K_5, write down the number of arcs and the number of arcs in any spanning tree, and show that there are 12 different Hamiltonian cycles (a cycle and its reverse counted as the same).

(4)

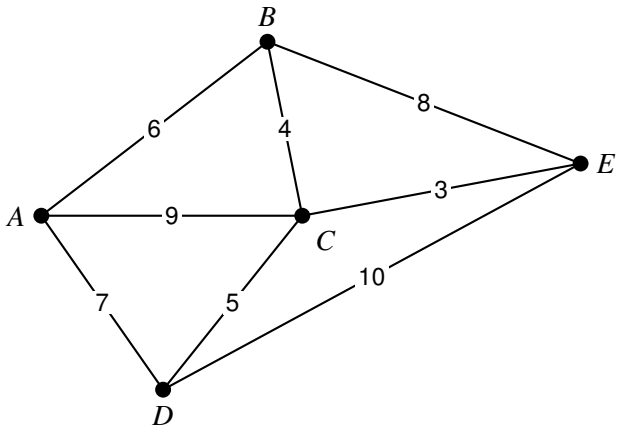


Figure 3

Figure 3 shows a network of paths between five huts. The number on each arc is the length of the path, in hundreds of metres.

(b) Find every Hamiltonian cycle starting and finishing at A, giving the length of each, and state the shortest.

(4)

(c) A new hut F is joined to the network by a single path, from E. Explain why the new network has no Hamiltonian cycle.

(2)

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Question 9 continued D1 ANSWER BOOK

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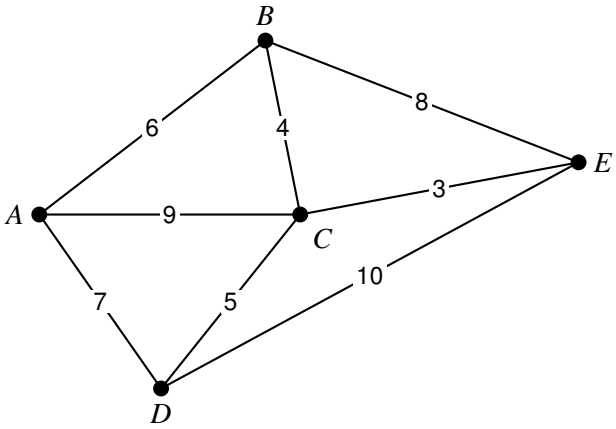


Diagram 8

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Question 9 continued

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10. The table gives the lengths, in metres, of the corridors in a building, joining six rooms A to F.

	A	B	C	D	E	F
A	–	5	8	–	–	6
B	5	–	3	7	–	–
C	8	3	–	4	9	–
D	–	7	4	–	2	5
E	–	–	9	2	–	6
F	6	–	–	5	6	–

(a) Draw the network on the vertices given in the answer book.

(3)

(b) Complete the table of degrees, and use the handshaking lemma to check the number of corridors.

(2)

(c) Explain why it is not possible to walk along every corridor exactly once and return to the start. State the least number of corridors that would need to be repeated, and suggest which.

(3)

(d) Find the total length of the corridors, and the number of corridors that must be removed to leave a spanning tree.

(2)

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Question 10 continued D1 ANSWER BOOK

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	A	B	C	D	E	F
A	–	5	8	–	–	6
B	5	–	3	7	–	–
C	8	3	–	4	9	–
D	–	7	4	–	2	5
E	–	–	9	2	–	6
F	6	–	–	5	6	–

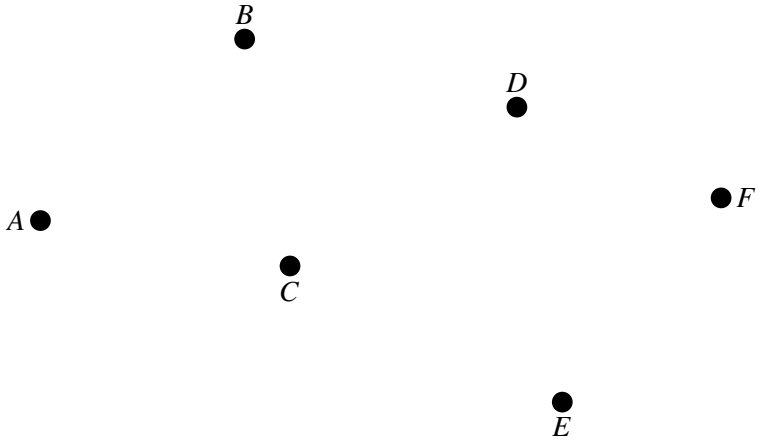


Diagram 9

Vertex	A	B	C	D	E	F
Degree						

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Question 10 continued

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(Total for Question 10 is 10 marks)

TOTAL FOR CHAPTER 2 IS 80 MARKS

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