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INTERNATIONAL A LEVEL MATHEMATICS • FURTHER PURE MATHEMATICS 2

Further Pure 2

Exam Questions by Topic: Mark Scheme

Pearson Edexcel International Advanced Level (WFM02/01) • Calculator allowed

Every question in the booklet, marked line by line the way an examiner marks it

Chapters (tap a chapter to jump to it)		Qs	Marks	Page	My score
1	**Inequalities** Rational and modulus inequalities, critical values, graphs	10	80	3	
2	**Series** Method of differences, partial fractions, telescoping sums	10	80	9	
3	**Complex Numbers** Exponential form, de Moivre's theorem, trigonometric identities, nth roots	10	80	16	
4	**Further Argand Diagrams** Loci, regions and transformations of the complex plane	10	80	22	
5	**First-Order Differential Equations** Integrating factors, substitutions, general and particular solutions	10	80	27	
6	**Second-Order Differential Equations** Auxiliary equations, complementary functions, particular integrals, substitutions	10	80	35	
7	**Maclaurin and Taylor Series** Higher derivatives, standard and compound series, Taylor series, series solutions	10	80	41	
8	**Polar Coordinates** Polar curves, areas, tangents parallel and perpendicular to the initial line	10	80	48	

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How to use this mark scheme

The same marking rules Pearson examiners use for WFM02.

Mark every line, not just the answer. Method marks are given for a correct method even when the final answer is wrong. If you lost marks, find the *first* line where your working differs from the scheme. That is the thing to fix.

Code	Meaning
M1	Method mark: for knowing a correct method and attempting to apply it. Small slips in the working do not lose this mark.
dM1	Dependent method mark: can only be given if the method mark before it has been given.
A1	Accuracy mark: for a correct answer or line of working. It can only be given if the method mark it depends on has been given.
B1	Independent mark: for a correct result or statement, whatever method was used.
*	The answer is given in the question. Every step must be shown, and the answer must follow from correct working (cso).
cao	Correct answer only.
cso	Correct solution only: no errors anywhere in the working.
ft	Follow through: the mark can be given for correct work using your own earlier (wrong) answer.
oe	Or equivalent: any correct equivalent form is accepted.

"Solutions relying entirely on calculator technology are not acceptable." When a WFM02 question says this, a correct final answer with no algebraic working scores **no marks**. Exact answers (surds, multiples of π) are required when asked for; otherwise give 3 significant figures, with arguments in radians.


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1 Chapter 1: Inequalities

Mark scheme. Marks are shown on the right of each line.

Question 1

6 marks

Part	Scheme	Marks
(a)	$\frac{x+1}{x-2} - 3 < 0$	M1
	$\frac{7-2x}{x-2} < 0$	A1
	Critical values 2 and $\frac{7}{2}$	B1
	Chooses the outside regions	M1
	$\boxed{x < 2 \text{ or } x > \frac{7}{2}}$	A1
	Multiplying by $x-2$ without considering its sign loses the method marks.	
		(5)
(b)	$\boxed{x < 2 \text{ or } x \geqslant \frac{7}{2}}$ (2 is still excluded)	B1
		(1)
		Total for Question 1: 6 marks

Question 2

6 marks

Part	Scheme	Marks
	Multiplies by $(x-1)^2$: $2(x-1) \geqslant x(x-1)^2$	M1
	$(x-1)(2-x(x-1)) \geqslant 0$	M1
	$-(x-1)(x-2)(x+1) \geqslant 0$	A1
	Critical values -1, 1, 2	A1
	Correct regions from a sketch of the cubic	M1
	$\boxed{x \leqslant -1 \text{ or } 1 < x \leqslant 2}$	A1
		(6)
		Total for Question 2: 6 marks

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Question 3

6 marks

Part	Scheme	Marks
(a)	V-shape with vertex $(3,0)$, y-intercept 3	B1
	Line through the origin crossing the left branch only	B1
		(2)
(b)	$-(x-3) = 2x$ gives $x = 1$	M1
	$x-3 = 2x$ gives $x = -3$, but then $\lvert x-3\rvert = 6 \neq -6$: rejected	A1
	Uses the sketch: $\lvert x-3\rvert$ is above the line to the left of the crossing	M1
	$\boxed{x < 1}$	A1
	Jan 2023 report: both cases of the modulus must be considered, and roots checked.	
		(4)
		Total for Question 3: 6 marks

Question 4

6 marks

Part	Scheme	Marks
(a)	Critical values -2, -1, 2	B1
	Sign of $\dfrac{(x-2)(x+2)}{x+1}$ in each interval	M1
	One correct region	A1
	$\boxed{-2 < x < -1 \text{ or } x > 2}$	A1
		(4)
(b)	The expression is undefined at $x = -1$ (the denominator is 0)	B1
		(1)
(c)	$\boxed{\{x : -2 < x < -1\} \cup \{x : x > 2\}}$	B1
		(1)
		Total for Question 4: 6 marks

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Question 5

6 marks

Part	Scheme	Marks
	$\frac{1}{x+2}-\frac{2}{x-1}>0$	M1
	$\frac{(x-1)-2(x+2)}{(x+2)(x-1)}=\frac{-x-5}{(x+2)(x-1)}>0$	A1
	Critical values -5, -2, 1	B1
	Sign analysis or sketch	M1
	One correct region	A1
	$\{x:x<-5\}\cup\{x:-2<x<1\}$	A1
		(6)
		Total for Question 5: 6 marks

Question 6

10 marks

Part	Scheme	Marks
(a)	$\lvert x^2-4\rvert$: W-shape touching the x-axis at $(\pm 2,0)$	B1
	Maximum of the reflected part at $(0,4)$	B1
	Line $y=3x$ through O	B1
		(3)
(b)	$x^2-4=3x$: $(x-4)(x+1)=0$	M1
	$x=4$; $x=-1$ rejected ($3x<0$ but the modulus is positive)	A1
	$4-x^2=3x$: $(x+4)(x-1)=0$	M1
	$x=1$; $x=-4$ rejected	A1
	Both rejections explained using the sign of $3x$	A1
		(5)
(c)	The line is above the curve between the crossings	M1
	$1<x<4$	A1
		(2)
		Total for Question 6: 10 marks

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Question 7

10 marks

Part	Scheme	Marks
(a)	$\frac{x+1}{x-3} - \frac{x}{x+2} \geqslant 0$	M1
	$\frac{(x+1)(x+2)-x(x-3)}{(x-3)(x+2)} = \frac{6x+2}{(x-3)(x+2)}$	A1
	Critical values -2, $-\frac{1}{3}$, 3	B1
	Sign analysis	M1
	One correct region	A1
	$\boxed{-2 < x \leqslant -\frac{1}{3} \text{ or } x > 3}$	A1
	Summer 2024 report: the values from the denominators must not be included.	(6)
(b)	Complement, excluding -2 and 3	M1
	$\boxed{x < -2 \text{ or } -\frac{1}{3} < x < 3}$	A1
		(2)
(c)	$(x-3)(x+2)$ is negative for $-2 < x < 3$	B1
	so multiplying by it would reverse the inequality in that interval	B1
		(2)
		Total for Question 7: 10 marks

Question 8

10 marks

Part	Scheme	Marks
(a)	V-shape with vertex $(2,0)$	B1
	Rectangular hyperbola shape with asymptotes $x = -3$ and $y = 0$	B1
	Correct relative positions (three crossings for $x > -3$)	B1
		(3)
(b)	$x \geqslant 2$: $(x-2)(x+3) = 6$, $x^2 + x - 12 = 0$	M1
	$x = 3$ ($x = -4$ rejected)	A1
	$x < 2$: $(2-x)(x+3) = 6$, $x^2 + x = 0$	M1
	$x = 0$ or $x = -1$	A1
	For $x < -3$ the right side is negative, so the inequality is true	B1
	Chooses regions from the sketch	M1
	$\boxed{x < -3,\ -1 \leqslant x \leqslant 0 \text{ or } x \geqslant 3}$	A1
		(7)
		Total for Question 8: 10 marks

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Question 9

10 marks

Part	Scheme	Marks
(a)	$3x \geqslant x^2+2$: $x^2-3x+2 \leqslant 0$ $(x-1)(x-2) \leqslant 0$ $\boxed{1 \leqslant x \leqslant 2}$	**M1** **A1** **A1** **(3)**
(b)	$x^2+2>0$ for all real x, so the inequality sign is unchanged	**B1** **(1)**
(c)	$kx^2-3x+2k \leqslant 0$ No real solutions when the quadratic is always positive: $9-8k^2<0$ $k^2>\frac{9}{8}$ $\boxed{k>\frac{3\sqrt{2}}{4}}$	**M1** **M1** **A1** **A1** **(4)**
(d)	Greatest value $\frac{3\sqrt{2}}{4}$; by symmetry (f is odd) least value $-\frac{3\sqrt{2}}{4}$ $\boxed{-\frac{3\sqrt{2}}{4} \leqslant f(x) \leqslant \frac{3\sqrt{2}}{4}}$	**M1** **A1** **(2)**
		Total for Question 9: 10 marks

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Question 10

10 marks

Part	Scheme	Marks
(a)	$x \geqslant 0$: $\dfrac{x}{x-1} - 2 < 0$, $\dfrac{2-x}{x-1} < 0$	M1
	$x < 1$ or $x > 2$, so $0 \leqslant x < 1$ or $x > 2$	A1
	$x < 0$: $\dfrac{-x}{x-1} - 2 < 0$, $\dfrac{2-3x}{x-1} < 0$	M1
	$x < \frac{2}{3}$ or $x > 1$, so all $x < 0$	A1
	Combines the cases	M1
	$\boxed{x < 1 \text{ or } x > 2}$	A1
		(6)
(b)	Complement, excluding $x = 1$	M1
	$\boxed{1 < x \leqslant 2}$	A1
		(2)
(c)	$x \geqslant 0$: $\dfrac{2x-1}{x-1} < 0$ gives $\frac{1}{2} < x < 1$; $x < 0$ gives no solutions	M1
	$\boxed{\frac{1}{2} < x < 1}$	A1
		(2)
	Total for Question 10: 10 marks	

2 Chapter 2: Series

Mark scheme. Marks are shown on the right of each line.

Question 1

6 marks

Part	Scheme	Marks
(a)	$\boxed{\frac{2}{r}-\frac{2}{r+1}}$	B1 (1)
(b)	Rows: $\left(2-\frac{2}{2}\right)+\left(\frac{2}{2}-\frac{2}{3}\right)+\cdots+\left(\frac{2}{n}-\frac{2}{n+1}\right)$ Survivors $2-\frac{2}{n+1}$ $=\frac{2(n+1)-2}{n+1}=\frac{2n}{n+1}$ *	M1 A1 A1* (3)
(c)	$\boxed{\frac{40}{21}}$	B1 (1)
(d)	$\boxed{2}$	B1 (1)
		Total for Question 1: 6 marks

Question 2

6 marks

Part	Scheme	Marks
(a)	$\frac{1}{r(r+2)}=\frac{A}{r}+\frac{B}{r+2}, A=\frac{1}{2}, B=-\frac{1}{2}$ $\boxed{\frac{1}{2r}-\frac{1}{2(r+2)}}$ *Jan 2023 report: give the fractions, not just the constants.*	M1 A1 (2)
(b)	$\frac{1}{2}\left[\left(1-\frac{1}{3}\right)+\left(\frac{1}{2}-\frac{1}{4}\right)+\cdots+\left(\frac{1}{n}-\frac{1}{n+2}\right)\right]$, with $\frac{1}{2}$ outside Survivors $\frac{1}{2}\left(1+\frac{1}{2}-\frac{1}{n+1}-\frac{1}{n+2}\right)$ Common denominator $4(n+1)(n+2)$ $\frac{3(n+1)(n+2)-2(2n+3)}{4(n+1)(n+2)}=\frac{n(3n+5)}{4(n+1)(n+2)}$ *	M1 A1 M1 A1* (4)
		Total for Question 2: 6 marks

Question 3

6 marks

Part	Scheme	Marks
(a)	$r^3-\left(r^3-3r^2+3r-1\right)$ $=3r^2-3r+1$ *	M1 A1* (2)
(b)	$\left(1^3-0^3\right)+\left(2^3-1^3\right)+\cdots+\left(n^3-(n-1)^3\right)$ $=n^3$ *	M1 A1* (2)
(c)	$3\sum r^2=n^3+3\cdot\frac{1}{2}n(n+1)-n$ $\sum r^2=\frac{1}{6}n\left(2n^2+3n+1\right)=\frac{1}{6}n(n+1)(2n+1)$ *	M1 A1* (2)
		Total for Question 3: 6 marks

Question 4

6 marks

Part	Scheme	Marks
(a)	$\frac{1}{r+1}-\frac{1}{r+2}$ Rows from $\frac{1}{2}-\frac{1}{3}$ to $\frac{1}{n+1}-\frac{1}{n+2}$ $\frac{1}{2}-\frac{1}{n+2}$ $=\frac{n}{2(n+2)}$ *	B1 M1 A1 A1* (4)
(b)	$\frac{2n}{2(2n+2)}-\frac{n}{2(n+2)}$ $\boxed{\frac{n}{2(n+1)(n+2)}}$	M1 A1 (2)
		Total for Question 4: 6 marks

Question 5

6 marks

Part	Scheme	Marks
(a)	$\frac{3}{r(r+3)} = \frac{1}{r} - \frac{1}{r+3}$	B1
	Rows written out to show a gap of three	M1
	Three positive survivors $1 + \frac{1}{2} + \frac{1}{3}$	A1
	Three negative survivors $\frac{1}{n+1} + \frac{1}{n+2} + \frac{1}{n+3}$	A1
	$\frac{11}{6} - \frac{1}{n+1} - \frac{1}{n+2} - \frac{1}{n+3}$ *	A1*
		(5)
(b)	$\boxed{\frac{11}{6}}$	B1
		(1)
		Total for Question 5: 6 marks

Question 6

10 marks

Part	Scheme	Marks
(a)	$A(4r+3) + B(4r-1) = 4$: $A = 1$, $B = -1$	M1
	$\boxed{\frac{1}{4r-1} - \frac{1}{4r+3}}$	A1
		(2)
(b)	Rows $\left(\frac{1}{3} - \frac{1}{7}\right) + \left(\frac{1}{7} - \frac{1}{11}\right) + \cdots$	M1
	Last row $\frac{1}{4n-1} - \frac{1}{4n+3}$	A1
	$\frac{1}{3} - \frac{1}{4n+3}$	M1
	$= \frac{4n}{3(4n+3)}$ *	A1*
	Jan 2023 report: extracting the wrong surviving terms, or not showing the rows, loses marks.	
		(4)
(c)	$S(3n) - S(n) = \frac{12n}{3(12n+3)} - \frac{4n}{3(4n+3)}$	M1
	$= \frac{4n}{3(4n+1)} - \frac{4n}{3(4n+3)}$	A1
	Common denominator	M1
	$= \frac{8n}{3(4n+1)(4n+3)}$ *	A1*
		(4)
		Total for Question 6: 10 marks

Question 7

10 marks

Part	Scheme	Marks
(a)	$\frac{1}{r(r+1)} - \frac{1}{(r+1)(r+2)} = \frac{(r+2)-r}{r(r+1)(r+2)}$	M1
	$= \frac{2}{r(r+1)(r+2)}$	A1
	Halving gives the result *	A1*
		(3)
(b)	$\frac{1}{2}\left[\left(\frac{1}{1\times2} - \frac{1}{2\times3}\right) + \cdots + \left(\frac{1}{n(n+1)} - \frac{1}{(n+1)(n+2)}\right)\right]$	M1
	$\frac{1}{2}\left(\frac{1}{2} - \frac{1}{(n+1)(n+2)}\right)$	M1
	$\boxed{\frac{1}{4} - \frac{1}{2(n+1)(n+2)}}$	A1
	$g(n) = \frac{1}{2(n+1)(n+2)}$	A1
		(4)
(c)	Sum to infinity $\frac{1}{4}$; $\frac{1}{2(n+1)(n+2)} < 0.001$	M1
	$(n+1)(n+2) > 500$: $21 \times 22 = 462$, $22 \times 23 = 506$	M1
	$\boxed{n = 21}$	A1
		(3)
	Total for Question 7: 10 marks	

Question 8

10 marks

Part	Scheme	Marks
(a)	$\frac{(r+1)^2 - r^2}{r^2(r+1)^2}$	M1
	$= \frac{2r+1}{r^2(r+1)^2}$ *	A1*
		(2)
(b)	Rows $\left(1 - \frac{1}{4}\right) + \left(\frac{1}{4} - \frac{1}{9}\right) + \cdots$	M1
	Last row $\frac{1}{n^2} - \frac{1}{(n+1)^2}$	A1
	$1 - \frac{1}{(n+1)^2}$	M1
	$= \frac{n^2+2n}{(n+1)^2} = \frac{n(n+2)}{(n+1)^2}$ *	A1*
		(4)
(c)	Sum to infinity 1 minus S(9) = $1 - \frac{1}{100}$	M1
	$\boxed{\frac{1}{100}}$	A1
		(2)
(d)	$S(n) = 1 - \frac{1}{(n+1)^2}$	M1
	and $\frac{1}{(n+1)^2} > 0$, so $S(n) < 1$	A1
		(2)
		Total for Question 8: 10 marks

Question 9

10 marks

Part	Scheme	Marks
(a)	$\dfrac{\sqrt{r+1}-\sqrt{r}}{(\sqrt{r+1}+\sqrt{r})(\sqrt{r+1}-\sqrt{r})}$	M1
	Denominator $(r+1)-r=1$ *	A1*
	Rows $\left(\sqrt{2}-1\right)+\left(\sqrt{3}-\sqrt{2}\right)+\cdots$	M1
	Last row $\sqrt{n+1}-\sqrt{n}$	A1
	$=\sqrt{n+1}-1$ *	A1*
		(5)
(b)	$\sqrt{n+1}=10$	M1
	$\boxed{n=99}$	A1
		(2)
(c)	$\sqrt{n+1}-1\to\infty$ as $n\to\infty$	B1
		(1)
(d)	$\left(\sqrt{25}-1\right)-\left(\sqrt{4}-1\right)$	M1
	$\boxed{3}$	A1
		(2)
		Total for Question 9: 10 marks

Question 10

10 marks

Part	Scheme	Marks
(a)	$r+3 = A(r+1)(r+2)+Br(r+2)+Cr(r+1)$	**M1**
	Two constants correct	**A1**
	$\boxed{\frac{3}{2r}-\frac{2}{r+1}+\frac{1}{2(r+2)}}$	**A1**
		(3)
(b)	Writes rows for $r = 1,2,3$ and $r = n-1,n$	**M1**
	Identifies cancelling diagonals (coefficients $\frac{3}{2}-2+\frac{1}{2}=0$)	**M1**
	Start survivors $\frac{3}{2}+\frac{3}{4}-1=\frac{5}{4}$	**A1**
	End survivors $-\frac{3}{2(n+1)}+\frac{1}{2(n+2)}$	**A1**
	$=\frac{5}{4}-\frac{2n+5}{2(n+1)(n+2)}$ *	**A1***
		(5)
(c)	Sum to infinity $\boxed{\frac{5}{4}}$	**B1**
	S(19) = 1.199, S(20) = 1.201: $\boxed{n=20}$	**B1**
		(2)
		Total for Question 10: 10 marks

3 Chapter 3: Complex Numbers

Mark scheme. Marks are shown on the right of each line.

Question 1

6 marks

Part	Scheme	Marks
(a)	$\boxed{6e^{\frac{\pi}{12}i}}$	B1 (1)
(b)	$\boxed{\frac{2}{3}e^{\frac{7\pi}{12}i}}$	B1 (1)
(c)	$\boxed{1+\sqrt{3}i}$	B1 (1)
(d)	$z^6 = 2^6e^{2\pi i}$ $= 64(\cos 2\pi + i\sin 2\pi) = \boxed{64}$	M1 A1 (2)
(e)	$\boxed{9e^{-\frac{\pi}{2}i}}$	B1 (1)
		Total for Question 1: 6 marks

Question 2

6 marks

Part	Scheme	Marks
(a)	$\sqrt{3}+i = 2e^{\frac{\pi}{6}i}$ $2^8e^{\frac{8\pi}{6}i} = 256e^{\frac{4\pi}{3}i}$ $256\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}i\right)$ $\boxed{-128-128\sqrt{3}i}$	B1 M1 M1 A1 (4)
(b)	$\frac{n\pi}{6} = 2k\pi$ $\boxed{n = 12}$	M1 A1 (2)
		Total for Question 2: 6 marks

Question 3

6 marks

Part	Scheme	Marks
(a)	$(c+\mathrm{i}s)^3 = c^3 + 3\mathrm{i}c^2s - 3cs^2 - \mathrm{i}s^3$	M1
	Real part: $\cos 3\theta = c^3 - 3cs^2$	A1
	$= c^3 - 3c\left(1 - c^2\right)$	M1
	$= 4c^3 - 3c$ *	A1*
		(4)
(b)	$x = \cos\theta$: $2\cos 3\theta = 1$, $3\theta = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}$	M1
	$\boxed{0.940, -0.174, -0.766}$	A1
		(2)
		Total for Question 3: 6 marks

Question 4

6 marks

Part	Scheme	Marks
(a)	$\left(z + z^{-1}\right)^4 = z^4 + 4z^2 + 6 + 4z^{-2} + z^{-4}$	M1
	$= \left(z^4 + z^{-4}\right) + 4\left(z^2 + z^{-2}\right) + 6$	M1
	$16\cos^4\theta = 2\cos 4\theta + 8\cos 2\theta + 6$	A1
	$\cos^4\theta = \frac{1}{8}(\cos 4\theta + 4\cos 2\theta + 3)$ *	A1*
		(4)
(b)	$\frac{1}{8}\left[\frac{1}{4}\sin 4\theta + 2\sin 2\theta + 3\theta\right]_0^{\frac{\pi}{2}}$	M1
	$\boxed{\frac{3\pi}{16}}$	A1
		(2)
		Total for Question 4: 6 marks

Question 5

6 marks

Part	Scheme	Marks
(a)	$8\mathrm{i} = 8\mathrm{e}^{\frac{\pi}{2}\mathrm{i}}$	B1
	$z = 2\mathrm{e}^{\left(\frac{\pi}{6}+\frac{2k\pi}{3}\right)\mathrm{i}}$	M1
	Two correct	A1
	$2\mathrm{e}^{\frac{\pi}{6}\mathrm{i}}, 2\mathrm{e}^{\frac{5\pi}{6}\mathrm{i}}, 2\mathrm{e}^{-\frac{\pi}{2}\mathrm{i}}$	A1
		(4)
(b)	Three points on a circle of radius 2, at angles 120° apart	B1
	an equilateral triangle	B1
		(2)
		Total for Question 5: 6 marks

Question 6

10 marks

Part	Scheme	Marks
(a)	$\cos 5\theta = \mathrm{Re}(c+\mathrm{i}s)^5 = c^5 - 10c^3s^2 + 5cs^4$	M1
	Binomial coefficients correct	A1
	Replaces s^2 by $1-c^2$	M1
	$c^5 - 10c^3 + 10c^5 + 5c - 10c^3 + 5c^5$	A1
	$= 16c^5 - 20c^3 + 5c$ *	A1*
	Summer 2024 report: show the full expansions; every step is needed for the final mark.	
		(5)
(b)	$x = \cos\theta$: $2\cos 5\theta - 1 = 0$, $\cos 5\theta = \frac{1}{2}$	M1
	$5\theta = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{11\pi}{3}, \frac{13\pi}{3}$	M1
	$x = \cos\frac{\pi}{15}, \cos\frac{\pi}{3}, \cos\frac{7\pi}{15}, \cos\frac{11\pi}{15}, \cos\frac{13\pi}{15}$	A1
	Smallest positive: $\cos\frac{7\pi}{15} =$ 0.105	A1
	Summer 2024 report: the first root found is not necessarily the smallest positive one.	
		(4)
(c)	The five values of θ in $(0, \pi)$ give five different values of $\cos\theta$, and a quintic has at most five roots	B1
		(1)
		Total for Question 6: 10 marks

Question 7

10 marks

Part	Scheme	Marks
(a)	$\sin 5\theta = \text{Im}(c+\text{i}s)^5 = 5c^4s - 10c^2s^3 + s^5$	**M1**
	Correct coefficients	**A1**
	Replaces c^2 by $1-s^2$	**M1**
	$5s\left(1-s^2\right)^2 - 10\left(1-s^2\right)s^3 + s^5$	**A1**
	$= 16s^5 - 20s^3 + 5s$ *	**A1***
	Jan 2023 report: binomial coefficients were sometimes omitted.	**(5)**
(b)	$16s^5 - 20s^3 + 4s = 0$	**M1**
	$4s\left(4s^4 - 5s^2 + 1\right) = 4s\left(4s^2-1\right)\left(s^2-1\right) = 0$	**A1**
	$s \neq 0$ in $(0,\pi)$; $s^2 = \frac{1}{4}$ or 1, so $s = \frac{1}{2}$ or 1 (square roots taken)	**M1**
	$\theta = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$	**A1**
	All three and no extras	**A1**
	Jan 2023 report: candidates forgot to square root to get $\sin\theta$ *from* $\sin^2\theta$.	**(5)**
		Total for Question 7: 10 marks

Question 8

10 marks

Part	Scheme	Marks
(a)	$16+16\sqrt{3}\mathrm{i} = 32\mathrm{e}^{\frac{\pi}{3}\mathrm{i}}$ $z = 2\mathrm{e}^{(\frac{\pi}{15}+\frac{2k\pi}{5})\mathrm{i}}$ At least three correct $\boxed{2\mathrm{e}^{\frac{\pi}{15}\mathrm{i}},\ 2\mathrm{e}^{\frac{7\pi}{15}\mathrm{i}},\ 2\mathrm{e}^{\frac{13\pi}{15}\mathrm{i}},\ 2\mathrm{e}^{-\frac{11\pi}{15}\mathrm{i}},\ 2\mathrm{e}^{-\frac{\pi}{3}\mathrm{i}}}$	**B1** **M1** **A1** **A1** **(4)**
(b)	Five triangles with sides 2, 2 and angle $\frac{2\pi}{5}$ $5\times\frac{1}{2}\times 2\times 2\sin\frac{2\pi}{5} = 10\sin\frac{2\pi}{5}$ $\boxed{9.51}$	**M1** **A1** **A1** **(3)**
(c)	The roots are $z_0,\ z_0\omega,\ldots,z_0\omega^4$ with $\omega = \mathrm{e}^{\frac{2\pi}{5}\mathrm{i}}$ Sum $= z_0\dfrac{\omega^5-1}{\omega-1} = 0$ since $\omega^5 = 1$ *	**M1** **A1*** **(2)**
(d)	$\boxed{16+16\sqrt{3}\mathrm{i}}$ (the product of the roots of $z^5 - w = 0$ is w)	**B1** **(1)**
		Total for Question 8: 10 marks

Question 9

10 marks

Part	Scheme	Marks
(a)	$\left(z+z^{-1}\right)^6 = z^6+6z^4+15z^2+20+15z^{-2}+6z^{-4}+z^{-6}$	M1
	Correct expansion	A1
	Pairs: $2\cos 6\theta + 12\cos 4\theta + 30\cos 2\theta + 20$	M1
	$64\cos^6\theta = \ldots$	A1
	$32\cos^6\theta = \cos 6\theta + 6\cos 4\theta + 15\cos 2\theta + 10$ *	A1*
		(5)
(b)	$\frac{1}{32}\left[\frac{1}{6}\sin 6\theta + \frac{3}{2}\sin 4\theta + \frac{15}{2}\sin 2\theta + 10\theta\right]_0^{\frac{\pi}{2}}$	M1
	All sine terms are 0 at both limits	M1
	$\boxed{\frac{5\pi}{32}}$	A1
		(3)
(c)	$\frac{1}{32}\left(-\frac{1}{6}+0+\frac{15}{2}+\frac{5\pi}{2}\right)$	M1
	$\boxed{\frac{11}{48}+\frac{5\pi}{64}}$	A1
		(2)
		Total for Question 9: 10 marks

Question 10

10 marks

Part	Scheme	Marks
(a)	$w = \frac{-1\pm\sqrt{3}\mathrm{i}}{2}$	M1
	$\boxed{w = \mathrm{e}^{\pm\frac{2\pi}{3}\mathrm{i}}}$	A1
		(2)
(b)	$z^3 = \mathrm{e}^{\frac{2\pi}{3}\mathrm{i}}$: $z = \mathrm{e}^{\left(\frac{2\pi}{9}+\frac{2k\pi}{3}\right)\mathrm{i}}$	M1
	$\mathrm{e}^{\frac{2\pi}{9}\mathrm{i}}$, $\mathrm{e}^{\frac{8\pi}{9}\mathrm{i}}$, $\mathrm{e}^{-\frac{4\pi}{9}\mathrm{i}}$	A1
	Conjugates from $z^3 = \mathrm{e}^{-\frac{2\pi}{3}\mathrm{i}}$	M1
	$\boxed{\mathrm{e}^{\pm\frac{2\pi}{9}\mathrm{i}}, \mathrm{e}^{\pm\frac{4\pi}{9}\mathrm{i}}, \mathrm{e}^{\pm\frac{8\pi}{9}\mathrm{i}}}$	A1
		(4)
(c)	$\left(z-\mathrm{e}^{\mathrm{i}\alpha}\right)\left(z-\mathrm{e}^{-\mathrm{i}\alpha}\right) = z^2-2z\cos\alpha+1$	M1
	Uses $\alpha = \frac{2\pi}{9}, \frac{4\pi}{9}, \frac{8\pi}{9}$	A1
	$\boxed{\left(z^2-2z\cos\frac{2\pi}{9}+1\right)\left(z^2-2z\cos\frac{4\pi}{9}+1\right)\left(z^2-2z\cos\frac{8\pi}{9}+1\right)}$	A1
		(3)
(d)	Coefficient of z^5: $-2\left(\cos\frac{2\pi}{9}+\cos\frac{4\pi}{9}+\cos\frac{8\pi}{9}\right)=0$ *	B1
		(1)
		Total for Question 10: 10 marks

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4 Chapter 4: Further Argand Diagrams

Mark scheme. Marks are shown on the right of each line.

Question 1

6 marks

Part	Scheme	Marks
(a)	Circle centre $(3, -2)$	**B1**
	radius 4	**B1**
		(2)
(b)	Half-line from $(-1, 0)$ (open circle there)	**B1**
	at $\frac{\pi}{4}$ to the positive real direction	**B1**
		(2)
(c)	Half-line $y = x + 1$, $x > -1$; substitutes: $(x-3)^2 + (x+3)^2 = 16$	**M1**
	$2x^2 + 18 = 16$ has no real roots: they do not meet	**A1**
		(2)
		Total for Question 1: 6 marks

Question 2

6 marks

Part	Scheme	Marks
(a)	$(x-3)^2 + (y-4)^2 = (x+1)^2 + (y+1)^2$	**M1**
	Expands correctly	**A1**
	$-8x - 10y + 23 = 0$	**M1**
	$8x + 10y = 23$ *	**A1***
	Summer 2024 report: in a geometric method, the midpoint and the negative reciprocal gradient are both needed.	
		(4)
(b)	The line $8x + 10y = 23$ drawn with negative gradient	**B1**
	Shades the side containing $3 + 4i$ (where $8x + 10y \geqslant 23$)	**B1**
	Summer 2024 report: the wrong side or partial shading was common.	
		(2)
		Total for Question 2: 6 marks

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Question 3

6 marks

Part	Scheme	Marks
(a)	$(x-6)^2+y^2=4\left(x^2+y^2\right)$	M1
	$3x^2+3y^2+12x-36=0$	A1
	$x^2+y^2+4x-12=0$	M1
	$(x+2)^2+y^2=16$	A1
	Centre -2, radius 4	A1
		(5)
(b)	6	B1
		(1)
		Total for Question 3: 6 marks

Question 4

6 marks

Part	Scheme	Marks
(a)	Distance from O to $(4,3)$ is 5	M1
	Greatest 7	A1
	Least 3	A1
		(3)
(b)	Tangent from O: angle between O-to-centre and the tangent is $\sin^{-1}\frac{2}{5}$	M1
	$\tan^{-1}\frac{3}{4}+\sin^{-1}\frac{2}{5}$	M1
	1.06	A1
		(3)
		Total for Question 4: 6 marks

Question 5

6 marks

Part	Scheme	Marks
(a)	An arc from -2 to 2 (endpoints excluded)	B1
	semicircle, since the angle is $\frac{\pi}{2}$	B1
	above the real axis (test $z=2$i: the quotient is i)	B1
		(3)
(b)	Circle with diameter from -2 to 2	M1
	$x^2+y^2=4$	A1
	$y>0$	A1
		(3)
		Total for Question 5: 6 marks

Question 6

10 marks

Part	Scheme	Marks
(a)	$w(z-1)=z+1,\ z(w-1)=w+1$ $z=\dfrac{w+1}{w-1}$ * *Summer 2024 report: rearranging for z incorrectly was surprisingly common.*	**M1** **A1*** **(2)**
(b)	$\left\lvert\dfrac{w+1}{w-1}\right\rvert=2$, so $\lvert w+1\rvert=2\lvert w-1\rvert$ $(u+1)^2+v^2=4\left((u-1)^2+v^2\right)$ $3u^2+3v^2-10u+3=0$ $\left(u-\frac{5}{3}\right)^2+v^2=\frac{16}{9}$ Centre $\frac{5}{3}$ Radius $\frac{4}{3}$ *Summer 2024 report: equal coefficients of u^2 and v^2 are needed for a circle; square root for the radius.*	**M1** **M1** **A1** **M1** **A1** **A1** **(6)**
(c)	z on the imaginary axis: $\lvert z+1\rvert=\lvert z-1\rvert$ $z+1=\dfrac{2w}{w-1},\ z-1=\dfrac{2}{w-1}$, so $\lvert w\rvert=1$ *	**M1** **A1*** **(2)**
		Total for Question 6: 10 marks

Question 7

10 marks

Part	Scheme	Marks
(a)	Perpendicular bisector of O and 4	B1
	Vertical line through 2	B1
	$\boxed{x = 2}$	B1
		(3)
(b)	$z = \frac{1}{w} = \frac{u - \mathrm{i}v}{u^2 + v^2}$	M1
	Real part: $\frac{u}{u^2 + v^2} = 2$	M1
	$u^2 + v^2 - \frac{1}{2}u = 0$	A1
	$\left(u - \frac{1}{4}\right)^2 + v^2 = \frac{1}{16}$	M1
	Centre $\frac{1}{4}$	A1
	Radius $\frac{1}{4}$ *	A1*
	Jan 2023 report: substitute the whole real part, not only its numerator.	
		(6)
(c)	$\boxed{w = 0}$: it would need $z = \frac{1}{0}$, i.e. z infinitely far along the line	B1
		(1)
		Total for Question 7: 10 marks

Question 8

10 marks

Part	Scheme	Marks
(a)	Circle centre 3, radius 3, through O	B1
	Half-line $y = x$ from O	B1
	Meets the circle at $3 + 3\mathrm{i}$	B1
	Region between the real axis and $y = x$, inside the circle, shaded	B1
		(4)
(b)	Triangle with vertices O, 3 and $3 + 3\mathrm{i}$: area $\frac{9}{2}$	M1
	Quarter of the circle from 6 to $3 + 3\mathrm{i}$: $\frac{1}{4}\pi(3)^2$	M1
	Adds	M1
	$\boxed{\frac{9}{2} + \frac{9\pi}{4}}$	A1
		(4)
(c)	The point 6 on the real axis	M1
	$\boxed{6}$	A1
		(2)
		Total for Question 8: 10 marks

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Question 9

10 marks

Part	Scheme	Marks
(a)	Arc through -4 and 4i (endpoints excluded)	B1
	Major arc (angle $\frac{\pi}{4} < \frac{\pi}{2}$)	B1
	on the side away from O (test $z = -8+4\text{i}$: quotient $1+\text{i}$)	B1
		(3)
(b)	Chord AB from -4 to 4i has length $4\sqrt{2}$	M1
	Angle at the centre $2 \times \frac{\pi}{4} = \frac{\pi}{2}$	M1
	Radius $= \dfrac{4\sqrt{2}}{\sqrt{2}} = 4$, centre at distance 4 from both points	A1
	Centre $-4+4\text{i}$, radius 4	A1
		(4)
(c)	$\lvert -4+4\text{i}\rvert = 4\sqrt{2}$	M1
	Greatest $\lvert z\rvert$ = distance to centre + radius, on the arc	M1
	$4\sqrt{2}+4$	A1
		(3)
		Total for Question 9: 10 marks

Question 10

10 marks

Part	Scheme	Marks
(a)	$z = \dfrac{3}{w} + 2 = \dfrac{3+2w}{w}$	M1
	$\lvert z\rvert = 1$: $\lvert 3+2w\rvert = \lvert w\rvert$	M1
	$(3+2u)^2 + 4v^2 = u^2 + v^2$	M1
	$3u^2 + 3v^2 + 12u + 9 = 0$	A1
	$(u+2)^2 + v^2 = 1$	M1
	so the image is $\lvert w+2\rvert = 1$ *	A1*
		(6)
(b)	$z = 0$ maps to $w = -\frac{3}{2}$, inside the circle	M1
	$\lvert w+2\rvert < 1$	A1
		(2)
(c)	Real z gives real w	M1
	The real axis, excluding $w = 0$	A1
		(2)
		Total for Question 10: 10 marks

5 Chapter 5: First-Order Differential Equations

Mark scheme. Marks are shown on the right of each line.

Question 1

6 marks

Part	Scheme	Marks
	IF $= e^{\int 2\,dx} = e^{2x}$	B1
	$\frac{d}{dx}\left(ye^{2x}\right) = e^{x}$	M1
	Correct	A1
	$ye^{2x} = e^{x} + c$	M1
	Divides every term by e^{2x}	M1
	$\boxed{y = e^{-x} + ce^{-2x}}$	A1
		(6)
		Total for Question 1: 6 marks

Question 2

6 marks

Part	Scheme	Marks
	$\frac{dy}{dx} + \frac{2}{x}y = x$	M1
	IF $= e^{2\ln x} = x^2$	A1
	$\frac{d}{dx}\left(x^2y\right) = x^3$	M1
	$x^2y = \frac{1}{4}x^4 + c$	A1
	Divides by x^2	M1
	$\boxed{y = \frac{1}{4}x^2 + \frac{c}{x^2}}$	A1
	Using IF $e^{\int 2\,dx}$ without dividing by x first is a common error.	
		(6)
		Total for Question 2: 6 marks

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Question 3

6 marks

Part	Scheme	Marks
(a)	IF $= e^{-\int \tan x \, dx} = e^{\ln \cos x} = \cos x$	B1
	$\frac{d}{dx}(y \cos x) = \cos^2 x = \frac{1}{2}(1 + \cos 2x)$	M1
	$y \cos x = \frac{1}{2}x + \frac{1}{4}\sin 2x + c$	A1
	$y = \frac{\frac{1}{2}x + \frac{1}{4}\sin 2x + c}{\cos x}$	A1
		(4)
(b)	$x = 0, y = 0$: $c = 0$	M1
	$y = \frac{2x + \sin 2x}{4 \cos x}$	A1
		(2)
		Total for Question 3: 6 marks

Question 4

6 marks

Part	Scheme	Marks
	IF $= x$	B1
	$\frac{d}{dx}(xy) = x \ln x$	M1
	By parts: $\frac{1}{2}x^2 \ln x - \int \frac{1}{2}x \, dx$	M1
	$xy = \frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + c$	A1
	Divides by x	M1
	$y = \frac{1}{2}x \ln x - \frac{1}{4}x + \frac{c}{x}$	A1
		(6)
		Total for Question 4: 6 marks

Question 5

6 marks

Part	Scheme	Marks
(a)	$\frac{dz}{dx} = -y^{-2}\frac{dy}{dx}$, so $\frac{dy}{dx} = -y^2\frac{dz}{dx}$	M1
	$-y^2\frac{dz}{dx} + y = xy^2$; divides by $-y^2$	M1
	$\frac{dz}{dx} - z = -x$ *	A1*
		(3)
(b)	IF e^{-x}: $ze^{-x} = \int -xe^{-x}\,dx = xe^{-x} + e^{-x} + c$	M1
	$z = x + 1 + ce^{x}$	A1
	$\boxed{y = \frac{1}{x + 1 + ce^{x}}}$	A1
		(3)
		Total for Question 5: 6 marks

Question 6

10 marks

Part	Scheme	Marks
(a)	$\frac{dz}{dx} = -2y^{-3}\frac{dy}{dx}$	M1
	$\frac{dy}{dx} = -\frac{1}{2}y^3\frac{dz}{dx}$	A1
	Substitutes: $-\frac{1}{2}y^3\frac{dz}{dx} + 2y = 2xy^3$	M1
	Multiplies by $-2y^{-3}$: $\frac{dz}{dx} - 4y^{-2} = -4x$	M1
	$\frac{dz}{dx} - 4z = -4x$ *	A1*
	Summer 2024 report: all steps must be shown in a "show that"; take care with indices.	(5)
(b)	IF e^{-4x}: $ze^{-4x} = \int -4xe^{-4x}\,dx$	M1
	$= xe^{-4x} + \frac{1}{4}e^{-4x} + c$ (by parts)	A1
	$z = x + \frac{1}{4} + ce^{4x}$ (constant multiplied by e^{4x})	A1
	$\boxed{y^2 = \frac{1}{x + \frac{1}{4} + ce^{4x}}}$	A1
	Summer 2024 report: reverse the substitution by reciprocating the whole expression.	(4)
(c)	$\frac{1}{4} = \frac{1}{4} + c$, $c = 0$: $\boxed{y^2 = \frac{4}{4x+1}}$	B1
		(1)
		Total for Question 6: 10 marks

Question 7

10 marks

Part	Scheme	Marks
(a)	$\frac{dy}{dx} = v + x\frac{dv}{dx}$	B1
	$\frac{x^2 + v^2x^2}{vx^2} = \frac{1+v^2}{v}$	M1
	$v + x\frac{dv}{dx} = \frac{1}{v} + v$	A1
	$x\frac{dv}{dx} = \frac{1}{v}$ *	A1*
		(4)
(b)	Separates: $\int v\,dv = \int \frac{1}{x}\,dx$	M1
	$\frac{1}{2}v^2 = \ln x + c$	A1
	$v = \frac{y}{x}$: $\frac{y^2}{2x^2} = \ln x + c$	M1
	$\boxed{y^2 = 2x^2 \ln x + Cx^2}$	A1
		(4)
(c)	$4 = 0 + C$	M1
	$\boxed{y^2 = 2x^2 \ln x + 4x^2}$	A1
		(2)
		Total for Question 7: 10 marks

Question 8

10 marks

Part	Scheme	Marks
(a)	IF $= e^{\ln(t+1)} = t+1$	B1
	$\frac{d}{dt}(v(t+1)) = 2(t+1)$	M1
	$v(t+1) = (t+1)^2 + c$	A1
	$v = t+1+\frac{c}{t+1}$	M1
	$t = 0$, $v = 5$: $c = 4$	M1
	$v = t+1+\frac{4}{t+1}$ *	A1*
		(6)
(b)	$\frac{dv}{dt} = 1 - \frac{4}{(t+1)^2} = 0$	M1
	$t = 1$	A1
	Least speed $4\,\text{m s}^{-1}$ at $t = 1$	A1
		(3)
(c)	$\frac{4}{t+1} \to 0$, so $v \approx t+1$: the speed increases steadily (by about $1\,\text{m s}^{-1}$ each second)	B1
		(1)
		Total for Question 8: 10 marks

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Question 9

10 marks

Part	Scheme	Marks
(a)	Standard form: IF = $e^{\int \frac{2x}{x^2+1}dx} = x^2+1$, or the left side is exact	M1
	$\frac{d}{dx}\left((x^2+1)y\right) = 4x$	A1
	$(x^2+1)y = 2x^2+c$	M1
	Divides by x^2+1	M1
	$\boxed{y = \frac{2x^2+c}{x^2+1}}$	A1
		(5)
(b)	$c = 3$	M1
	$\boxed{y = \frac{2x^2+3}{x^2+1}}$	A1
		(2)
(c)	$\frac{2(x^2+1)+1}{x^2+1} = 2+\frac{1}{x^2+1}$ *	B1
	$0 < \frac{1}{x^2+1} \leqslant 1$	M1
	$\boxed{2 < y \leqslant 3}$	A1
		(3)
	Total for Question 9: 10 marks	

Question 10

10 marks

Part	Scheme	Marks
(a)	$\frac{du}{dx} = \frac{dy}{dx} - 1$	M1
	$= u^2 + 1 - 1$	A1
	$= u^2$ *	A1*
		(3)
(b)	$\int u^{-2}\,du = \int 1\,dx$	M1
	$-\frac{1}{u} = x + c$	A1
	$u = -\frac{1}{x+c}$	M1
	$\boxed{y = x - \frac{1}{x+c}}$	A1
		(4)
(c)	$1 = -\frac{1}{c}$, $c = -1$	M1
	$\boxed{y = x - \frac{1}{x-1}}$	A1
	$\boxed{x = 1}$	B1
		(3)
		Total for Question 10: 10 marks

6 Chapter 6: Second-Order Differential Equations

Mark scheme. Marks are shown on the right of each line.

Question 1

6 marks

Part	Scheme	Marks
(a)	$m^2 - 5m + 6 = 0$, $m = 2, 3$	M1
	$\boxed{y = Ae^{2x} + Be^{3x}}$	A1
		(2)
(b)	$m = -2$ (repeated)	M1
	$\boxed{y = (A + Bx)e^{-2x}}$	A1
		(2)
(c)	$m = -1 \pm 2i$	M1
	$\boxed{y = e^{-x}(A\cos 2x + B\sin 2x)}$	A1
		(2)
	Total for Question 1: 6 marks	

Question 2

6 marks

Part	Scheme	Marks
	$m^2 - m - 2 = 0$, $m = 2, -1$; CF $Ae^{2x} + Be^{-x}$	B1
	PI $y = ax + b$	B1
	$0 - a - 2(ax + b) = 4x$	M1
	$a = -2$, $b = 1$	A1
	CF + PI	M1
	$\boxed{y = Ae^{2x} + Be^{-x} - 2x + 1}$	A1
		(6)
	Total for Question 2: 6 marks	

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Question 3

6 marks

Part	Scheme	Marks
(a)	CF $A\cos 3x + B\sin 3x$	B1
	PI λe^x: $\lambda + 9\lambda = 10$	M1
	$\lambda = 1$	A1
	$y = A\cos 3x + B\sin 3x + e^x$	A1
		(4)
(b)	$A + 1 = 2$, $3B + 1 = 4$	M1
	$y = \cos 3x + \sin 3x + e^x$	A1
		(2)
		Total for Question 3: 6 marks

Question 4

6 marks

Part	Scheme	Marks
(a)	e^{2x} is part of the CF $Ae^x + Be^{2x}$, so it gives 0 on substitution	B1
		(1)
(b)	PI $\lambda x e^{2x}$	B1
	$y' = \lambda(1 + 2x)e^{2x}$, $y'' = \lambda(4 + 4x)e^{2x}$	M1
	$\lambda[(4 + 4x) - 3(1 + 2x) + 2x] = 3$	M1
	$\lambda = 3$	A1
	$y = Ae^x + Be^{2x} + 3xe^{2x}$	A1
		(5)
		Total for Question 4: 6 marks

Question 5

6 marks

Part	Scheme	Marks
	$m = -1 \pm i$: CF $e^{-x}(A\cos x + B\sin x)$	B1
	PI $p\cos x + q\sin x$ (both terms)	B1
	Substitutes: $(p + 2q)\cos x + (q - 2p)\sin x = 10\sin x$	M1
	$p + 2q = 0$, $q - 2p = 10$	A1
	$p = -4$, $q = 2$	A1
	$y = e^{-x}(A\cos x + B\sin x) - 4\cos x + 2\sin x$	A1
		(6)
		Total for Question 5: 6 marks

Question 6

10 marks

Part	Scheme	Marks
(a)	$\frac{dy}{dt} = \frac{dy}{dx}\frac{dx}{dt} = x\frac{dy}{dx}$ *	B1
	$\frac{d^2y}{dt^2} = \frac{d}{dt}\left(x\frac{dy}{dx}\right) = \frac{dx}{dt}\frac{dy}{dx} + x\frac{d^2y}{dx^2}\frac{dx}{dt}$	M1
	$= x\frac{dy}{dx} + x^2\frac{d^2y}{dx^2}$ (chain rule, $\frac{dx}{dt} = x$)	A1
	$x^2\frac{d^2y}{dx^2} = \frac{d^2y}{dt^2} - \frac{dy}{dt}$ *	A1*
	Summer 2024 report: the chain rule must be used; inserting factors to reach the given answer earns no credit.	(4)
(b)	$\frac{d^2y}{dt^2} - \frac{dy}{dt} - 2\frac{dy}{dt} + 2y = e^{3t}$ *	B1
		(1)
(c)	CF $Ae^t + Be^{2t}$	B1
	PI λe^{3t}: $(9 - 9 + 2)\lambda = 1$	M1
	$\boxed{y = Ae^t + Be^{2t} + \frac{1}{2}e^{3t}}$	A1
	Summer 2024 report: the CF in (c) must be in t, not x.	(3)
(d)	$e^t = x$, $e^{2t} = x^2$, $e^{3t} = x^3$	M1
	$\boxed{y = Ax + Bx^2 + \frac{1}{2}x^3}$	A1
		(2)
	Total for Question 6: 10 marks	

Question 7

10 marks

Part	Scheme	Marks
(a)	$m^2+2m+5=0$, $m=-1\pm 2\mathrm{i}$	M1
	CF $\mathrm{e}^{-t}(A\cos 2t+B\sin 2t)$	A1
	PI constant k: $5k=10$	M1
	$k=2$	A1
	$\boxed{x=\mathrm{e}^{-t}(A\cos 2t+B\sin 2t)+2}$	A1
		(5)
(b)	$t=0$, $x=0$: $A=-2$	B1
	Differentiates: $\dot{x}=\mathrm{e}^{-t}[(2B-A)\cos 2t-(2A+B)\sin 2t]$	M1
	$\dot{x}(0)=0$: $2B-A=0$, $B=-1$	A1
	$\boxed{x=2-\mathrm{e}^{-t}(2\cos 2t+\sin 2t)}$	A1
		(4)
(c)	$\mathrm{e}^{-t}\to 0$: the oscillations die away and $x\to 2$	B1
		(1)
		Total for Question 7: 10 marks

Question 8

10 marks

Part	Scheme	Marks
(a)	$m=2$ is a repeated root, so the CF is $(A+Bx)\mathrm{e}^{2x}$	B1
	Both e^{2x} and $x\mathrm{e}^{2x}$ are in the CF, so they give 0	B1
		(2)
(b)	PI $\lambda x^2\mathrm{e}^{2x}$	B1
	Differentiates twice and substitutes	M1
	$2\lambda=6$, $\lambda=3$	A1
	$\boxed{y=\left(A+Bx+3x^2\right)\mathrm{e}^{2x}}$	A1
		(4)
(c)	$A=1$	B1
	$y'=(B+6x)\mathrm{e}^{2x}+2\left(A+Bx+3x^2\right)\mathrm{e}^{2x}$; $B+2A=0$	M1
	$\boxed{y=\left(3x^2-2x+1\right)\mathrm{e}^{2x}}$	A1
		(3)
(d)	$3x^2-2x+1$ has discriminant $4-12<0$ and is positive; $\mathrm{e}^{2x}>0$, so $y>0$	B1
		(1)
		Total for Question 8: 10 marks

Question 9

10 marks

Part	Scheme	Marks
(a)	$y' = \frac{z'}{x} - \frac{z}{x^2}$	B1
	$y'' = \frac{z''}{x} - \frac{2z'}{x^2} + \frac{2z}{x^3}$	M1
	Correct	A1
	Substitutes: $z'' - \frac{2z'}{x} + \frac{2z}{x^2} + \frac{2z'}{x} - \frac{2z}{x^2} + 4z = 0$	M1
	$\frac{d^2z}{dx^2} + 4z = 0$ *	A1*
		(5)
(b)	$m = \pm 2i$	M1
	$z = A\cos 2x + B\sin 2x$	A1
	$\boxed{y = \frac{A\cos 2x + B\sin 2x}{x}}$	A1
		(3)
(c)	$x = \frac{\pi}{4}: \frac{4B}{\pi} = \frac{4}{\pi}, B = 1; x = \frac{\pi}{2}: -\frac{2A}{\pi} = -\frac{2}{\pi}, A = 1$	M1
	$\boxed{y = \frac{\cos 2x + \sin 2x}{x}}$	A1
		(2)
		Total for Question 9: 10 marks

Question 10

10 marks

Part	Scheme	Marks
(a)	$m^2+m-2=0$, $m=1,-2$	**M1**
	$\boxed{Ae^x+Be^{-2x}}$	**A1**
		(2)
(b)	e^x is in the CF, so try $ax+b+\lambda xe^x$	**B1**
	Polynomial part: $a-2(ax+b)=4x-2$	**M1**
	$a=-2$, $b=0$	**A1**
	Exponential part: $\lambda[(2+x)+(1+x)-2x]e^x=3e^x$	**M1**
	$\lambda=1$: PI $\boxed{-2x+xe^x}$	**A1**
		(5)
(c)	$e^{-2x}\to\infty$ as $x\to-\infty$, so $B=0$	**M1**
	$y(0)=A=1$	**A1**
	$\boxed{y=e^x-2x+xe^x}$	**A1**
		(3)
		Total for Question 10: 10 marks

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7 Chapter 7: Maclaurin and Taylor Series

Mark scheme. Marks are shown on the right of each line.

Question 1

6 marks

Part	Scheme	Marks
(a)	$f'(x) = -\tan x$	B1
	$f''(x) = -\sec^2 x$	B1
	$f'''(x) = -2\sec^2 x \tan x$	B1
	$f''''(x) = -2\sec^4 x - 4\sec^2 x \tan^2 x$	B1
		(4)
(b)	$f(0) = 0, f'(0) = 0, f''(0) = -1, f'''(0) = 0, f''''(0) = -2$	M1
	$\ln(\cos x) \approx -\frac{1}{2}x^2 - \frac{1}{12}x^4$ *	A1*
		(2)
		Total for Question 1: 6 marks

Question 2

6 marks

Part	Scheme	Marks
(a)	$\ln(1+2x) - \ln(1-x)$	M1
	$2x - 2x^2 + \frac{8}{3}x^3$	A1
	$-\left(-x - \frac{1}{2}x^2 - \frac{1}{3}x^3\right)$	A1
	$\boxed{3x - \frac{3}{2}x^2 + 3x^3}$	A1
		(4)
(b)	Needs $-1 < 2x \leqslant 1$ and $-1 \leqslant x < 1$	M1
	$\boxed{-\frac{1}{2} < x \leqslant \frac{1}{2}}$	A1
		(2)
		Total for Question 2: 6 marks

Question 3

6 marks

Part	Scheme	Marks
(a)	$f'(x) = \sec^2 x$, $f''(x) = 2\sec^2 x \tan x$	M1
	$f\left(\frac{\pi}{4}\right) = 1$, $f'\left(\frac{\pi}{4}\right) = 2$, $f''\left(\frac{\pi}{4}\right) = 4$	A1
	Taylor form with $\frac{4}{2!}$	M1
	$\boxed{1+2\left(x-\frac{\pi}{4}\right)+2\left(x-\frac{\pi}{4}\right)^2}$	A1
		(4)
(b)	$x-\frac{\pi}{4} = 0.014602$	M1
	$\boxed{1.02963}$	A1
		(2)
		Total for Question 3: 6 marks

Question 4

6 marks

Part	Scheme	Marks
(a)	$\left(1+x+\frac{1}{2}x^2+\frac{1}{6}x^3\right)\left(x-\frac{1}{6}x^3\right)$	M1
	$x+x^2+\left(\frac{1}{2}-\frac{1}{6}\right)x^3$	A1
	x^4 terms: $\frac{1}{6}x^4-\frac{1}{6}x^4 = 0$	M1
	Correct conclusion *	A1*
		(4)
(b)	$\frac{x^2+\frac{1}{3}x^3+\cdots}{x^2} = 1+\frac{1}{3}x+\cdots$	M1
	$\boxed{1}$	A1
		(2)
		Total for Question 4: 6 marks

Question 5

6 marks

Part	Scheme	Marks
(a)	$y'(0) = 0 + 1 = 1$	B1
	$y'' = 2x + 2yy'$, so $y''(0) = 2$	M1
	$y''' = 2 + 2(y')^2 + 2yy''$	M1
	$y'''(0) = 2 + 2 + 4 = 8$	A1
	Summer 2024 report: $\left(\frac{dy}{dx}\right)^2$ *and* $\frac{d^2y}{dx^2}$ *are different.*	(4)
(b)	$1 + x + \frac{2}{2!}x^2 + \frac{8}{3!}x^3$	M1
	$\boxed{y = 1 + x + x^2 + \frac{4}{3}x^3}$	A1
		(2)
		Total for Question 5: 6 marks

Question 6

10 marks

Part	Scheme	Marks
(a)	$y' = e^x(\cos x - \sin x)$	M1
	$y'' = -2e^x \sin x$	A1
	$y''' = -2e^x(\sin x + \cos x)$	A1
	$y'''' = -4e^x \cos x = -4y$ *	A1*
	Summer 2024 report: simplify each derivative as you go; $-2\sin x - 2\sin x$ *is* $-4\sin x$.	(4)
(b)	$y^{(5)} = -4y'$, $y^{(6)} = -4y''$	M1
	$\boxed{y^{(5)}(0) = -4,\ y^{(6)}(0) = 0}$	A1
		(2)
(c)	$y(0) = 1$, $y'(0) = 1$, $y''(0) = 0$, $y'''(0) = -2$, $y''''(0) = -4$	B1
	Maclaurin form with factorials	M1
	$1 + x - \frac{2}{3!}x^3 - \frac{4}{4!}x^4 - \frac{4}{5!}x^5$	A1
	$\boxed{y = 1 + x - \frac{1}{3}x^3 - \frac{1}{6}x^4 - \frac{1}{30}x^5}$	A1
		(4)
		Total for Question 6: 10 marks

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Question 7

10 marks

Part	Scheme	Marks
(a)	$y''(1) = -(1)(0) - 1 = \boxed{-1}$	**B1** **(1)**
(b)	Differentiates: $y''' + y' + xy'' + y' = 0$ (product rule on xy') $y''' = -2y' - xy''$ *	**M1** **A1*** **(2)**
(c)	$y'''(1) = 0 + 1 = 1$; $y'''' = -3y'' - xy'''$ $y''''(1) = 3 - 1 = \boxed{2}$	**M1** **A1** **(2)**
(d)	Taylor form about 1 with factorials Correct coefficients $\boxed{y = 1 - \frac{1}{2}(x-1)^2 + \frac{1}{6}(x-1)^3 + \frac{1}{12}(x-1)^4}$	**M1** **A1** **A1** **(3)**
(e)	$1 - 0.02 + 0.001333 + 0.000133$ $\boxed{0.9815}$	**M1** **A1** **(2)**
		Total for Question 7: 10 marks

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Question 8

10 marks

Part	Scheme	Marks
(a)	$\left(1+2x+2x^2+\frac{4}{3}x^3\right)\left(1+x+x^2+x^3\right)$	M1
	x and x^2 terms: $3x$, $(1+2+2)x^2$	A1
	x^3: $1+2+2+\frac{4}{3}$	M1
	$=\frac{19}{3}$ *	A1*
		(4)
(b)	$\boxed{-1<x<1}$	B1
		(1)
(c)	Series: 1.35633	B1
	True value $\frac{e^{0.2}}{0.9}=1.35711$	M1
	$\boxed{0.058\%}$	A1
		(3)
(d)	$\left[x+\frac{3}{2}x^2+\frac{5}{3}x^3+\frac{19}{12}x^4\right]_0^{0.1}$	M1
	$\boxed{0.1168}$	A1
		(2)
		Total for Question 8: 10 marks

Question 9

10 marks

Part	Scheme	Marks
(a)	$\cos 2x = 1 - \frac{(2x)^2}{2} + \frac{(2x)^4}{24}$ $1 - \cos 2x = 2x^2 - \frac{2}{3}x^4$ *	M1 A1* (2)
(b)	$x \sin x = x^2 - \frac{1}{6}x^4$ $\frac{x^2 - \cdots}{2x^2 - \cdots}$ $\boxed{\frac{1}{2}}$	B1 M1 A1 (3)
(c)	Numerator $\frac{1}{2}x^2 + \frac{1}{6}x^3$ $\frac{\frac{1}{2}x^2 + \cdots}{x^2 + \cdots}$ $\boxed{\frac{1}{2}}$	B1 M1 A1 (3)
(d)	$\frac{-\frac{1}{2}x^2 + \frac{1}{3}x^3}{x^2}$ $\boxed{-\frac{1}{2}}$	M1 A1 (2)
		Total for Question 9: 10 marks

Question 10

10 marks

Part	Scheme	Marks
(a)	$y' = \dfrac{e^x}{1+e^x}$ *	**B1**
	$y'' = \dfrac{e^x}{(1+e^x)^2}$	**M1**
	$= \dfrac{e^x}{1+e^x} \cdot \dfrac{1}{1+e^x} = y'(1-y')$ *	**A1***
		(3)
(b)	$y''' = y''(1-2y')$	**M1**
	$y'''' = y'''(1-2y') - 2(y'')^2$	**M1**
	$y(0) = \ln 2$, $y'(0) = \frac{1}{2}$, $y''(0) = \frac{1}{4}$, $y'''(0) = 0$, $y''''(0) = -\frac{1}{8}$	**A1**
	Maclaurin form	**M1**
	$\boxed{y = \ln 2 + \frac{1}{2}x + \frac{1}{8}x^2 - \frac{1}{192}x^4}$	**A1**
	Summer 2024 report: $y(0) = \ln 2$, not 1; find the values first, then substitute.	
		(5)
(c)	$\ln(1+e^x) - \frac{x}{2} = \ln\left(e^{-\frac{x}{2}}(1+e^x)\right) = \ln\left(e^{-\frac{x}{2}} + e^{\frac{x}{2}}\right)$	**B1**
	This is unchanged when $x \to -x$ (an even function), so it has only even powers	**B1**
		(2)
		Total for Question 10: 10 marks

8 Chapter 8: Polar Coordinates

Mark scheme. Marks are shown on the right of each line.

Question 1

6 marks

Part	Scheme	Marks
(a)	$r^2 = 4r\cos\theta$	M1
	$x^2 + y^2 = 4x$, i.e. $(x-2)^2 + y^2 = 4$	A1
	Circle, centre $(2,0)$, radius 2	A1
		(3)
(b)	$r\sin\theta = r^2\cos^2\theta$	M1
	$r = \tan\theta\sec\theta$	A1
		(2)
(c)	$\left(2, \frac{2\pi}{3}\right)$	B1
		(1)
		Total for Question 1: 6 marks

Question 2

6 marks

Part	Scheme	Marks
(a)	$r^2 = 4 + 4\cos\theta + \cos^2\theta$	M1
	$= 4 + 4\cos\theta + \frac{1}{2}(1 + \cos 2\theta)$ *	A1*
		(2)
(b)	$\frac{1}{2}\int_0^{2\pi} \left(\frac{9}{2} + 4\cos\theta + \frac{1}{2}\cos 2\theta\right) d\theta$	M1
	$\frac{1}{2}\left[\frac{9}{2}\theta + 4\sin\theta + \frac{1}{4}\sin 2\theta\right]_0^{2\pi}$	A1
	$\frac{1}{2} \times 9\pi$	M1
	$\frac{9\pi}{2}$	A1
		(4)
		Total for Question 2: 6 marks

Question 3

6 marks

Part	Scheme	Marks
(a)	Loop in the first quadrant from O back to O	B1
	Greatest $r = 3$ at $\boxed{\theta = \frac{\pi}{4}}$	B1
		(2)
(b)	$\frac{1}{2}\int_0^{\frac{\pi}{2}} 9\sin^2 2\theta \, d\theta$	M1
	$\frac{9}{4}\int_0^{\frac{\pi}{2}} (1-\cos 4\theta)\, d\theta$	M1
	$\frac{9}{4}\left[\theta - \frac{1}{4}\sin 4\theta\right]_0^{\frac{\pi}{2}}$	A1
	$\boxed{\frac{9\pi}{8}}$	A1
		(4)
		Total for Question 3: 6 marks

Question 4

6 marks

Part	Scheme	Marks
(a)	$y = (1+\cos\theta)\sin\theta$	B1
	$\frac{dy}{d\theta} = \cos\theta + \cos^2\theta - \sin^2\theta$	M1
	$= \cos\theta + 2\cos^2\theta - 1$	A1
	$= 0$ gives $2\cos^2\theta + \cos\theta - 1 = 0$ *	A1*
		(4)
(b)	$(2\cos\theta - 1)(\cos\theta + 1) = 0$; $\cos\theta = -1$ gives the pole	M1
	$\boxed{\left(\frac{3}{2}, \frac{\pi}{3}\right)}$	A1
		(2)
		Total for Question 4: 6 marks

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Question 5

6 marks

Part	Scheme	Marks
(a)	$x = \cos\theta + 2\cos^2\theta$	B1
	$\frac{dx}{d\theta} = -\sin\theta - 4\cos\theta\sin\theta$	M1
	$= -\sin\theta(1 + 4\cos\theta)$ *	A1*
		(3)
(b)	$\sin\theta = 0$: $\theta = 0$, $r = 3$	M1
	$\cos\theta = -\frac{1}{4}$: $\theta = 1.82$, $r = \frac{1}{2}$	A1
	$(3, 0)$ and $\left(\frac{1}{2}, 1.82\right)$	A1
		(3)
		Total for Question 5: 6 marks

Question 6

10 marks

Part	Scheme	Marks
(a)	$OP\cos\theta = \frac{9}{4}$ on l; on C: $3(1 + \cos\theta)\cos\theta = \frac{9}{4}$	M1
	$4\cos^2\theta + 4\cos\theta - 3 = 0$, $(2\cos\theta - 1)(2\cos\theta + 3) = 0$	A1
	$\cos\theta = \frac{1}{2}$: $\theta = \frac{\pi}{3}$, $r = \frac{9}{2}$ *	A1*
		(3)
(b)	Area of C from 0 to $\frac{\pi}{3}$: $\frac{1}{2}\int_0^{\frac{\pi}{3}} 9(1 + \cos\theta)^2 d\theta$	M1
	$(1 + \cos\theta)^2 = \frac{3}{2} + 2\cos\theta + \frac{1}{2}\cos 2\theta$ (three terms)	M1
	$\frac{9}{2}\left[\frac{3}{2}\theta + 2\sin\theta + \frac{1}{4}\sin 2\theta\right]_0^{\frac{\pi}{3}}$	A1
	$= \frac{9\pi}{4} + \frac{81\sqrt{3}}{16}$	A1
	Triangle O, $\left(\frac{9}{4}, 0\right)$, P: $\frac{1}{2} \times \frac{9}{4} \times \frac{9\sqrt{3}}{4} = \frac{81\sqrt{3}}{32}$	M1
	Subtracts the triangle	M1
	$\frac{9\pi}{4} + \frac{81\sqrt{3}}{32}$	A1
	Summer 2024 report: limits of 0 to π or $\frac{\pi}{2}$, and errors in the triangle, were the common errors.	
		(7)
		Total for Question 6: 10 marks

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Question 7

10 marks

Part	Scheme	Marks
(a)	$3\cos\theta = 1 + \cos\theta$	**M1**
	$\cos\theta = \frac{1}{2}$	**A1**
	$\left(\frac{3}{2}, \frac{\pi}{3}\right)$ and $\left(\frac{3}{2}, -\frac{\pi}{3}\right)$	**A1**
		(3)
(b)	Region: C_2 for $\lvert\theta\rvert \leqslant \frac{\pi}{3}$, C_1 for $\frac{\pi}{3} \leqslant \lvert\theta\rvert \leqslant \frac{\pi}{2}$; use symmetry	**M1**
	$2 \times \frac{1}{2}\int_0^{\frac{\pi}{3}} (1+\cos\theta)^2 \, d\theta = \frac{\pi}{2} + \frac{9\sqrt{3}}{8}$	**M1**
	Correct	**A1**
	$2 \times \frac{1}{2}\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 9\cos^2\theta \, d\theta$	**M1**
	$= 9\left[\frac{\theta}{2} + \frac{1}{4}\sin 2\theta\right]_{\frac{\pi}{3}}^{\frac{\pi}{2}} = \frac{3\pi}{4} - \frac{9\sqrt{3}}{8}$	**A1**
	Adds	**M1**
	$\frac{5\pi}{4}$	**A1**
		(7)
		Total for Question 7: 10 marks

Question 8

10 marks

Part	Scheme	Marks
(a)	$\frac{1}{2}\int_0^{\pi} \theta^2 \, d\theta$	M1
	$\frac{1}{2}\left[\frac{\theta^3}{3}\right]_0^{\pi}$	A1
	$\boxed{\frac{\pi^3}{6}}$	A1
		(3)
(b)	$x = \theta\cos\theta$	B1
	$\frac{dx}{d\theta} = \cos\theta - \theta\sin\theta = 0$	M1
	Divides by $\cos\theta$: $\theta\tan\theta = 1$ *	A1*
		(3)
(c)	$f'(\theta) = -2\sin\theta - \theta\cos\theta$	B1
	$0.85 - \frac{f(0.85)}{f'(0.85)}$	M1
	$\boxed{\theta = 0.860}$	A1
	$x = 0.860\cos 0.860 = \boxed{0.561}$	A1
		(4)
		Total for Question 8: 10 marks

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Question 9

10 marks

Part	Scheme	Marks
(a)	For $\frac{\pi}{4} < \theta < \frac{3\pi}{4}$, $\cos 2\theta < 0$	**M1**
	so $r^2 < 0$, which is impossible	**A1**
		(2)
(b)	One loop: $\frac{1}{2}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} 4\cos 2\theta \, d\theta$	**M1**
	$= [\sin 2\theta]_{-\frac{\pi}{4}}^{\frac{\pi}{4}} = 2$	**A1**
	Two loops	**M1**
	$\boxed{4}$	**A1**
		(4)
(c)	$y^2 = r^2 \sin^2\theta = 4\cos 2\theta \sin^2\theta$; $\frac{d}{d\theta}\left(y^2\right) = 0$	**M1**
	$4\sin 2\theta(\cos 2\theta - 2\sin^2\theta) = 4\sin 2\theta(1 - 4\sin^2\theta) = 0$	**A1**
	$\sin\theta = \frac{1}{2}$, $\theta = \frac{\pi}{6}$	**M1**
	$\boxed{\left(\sqrt{2}, \frac{\pi}{6}\right)}$	**A1**
		(4)
		Total for Question 9: 10 marks

Question 10

10 marks

Part	Scheme	Marks
(a)	$\frac{1}{2}\int_0^{2\pi}\left(a^2+4a\cos\theta+4\cos^2\theta\right)\mathrm{d}\theta$	M1
	$=\frac{1}{2}\left(2\pi a^2+4\pi\right)=\pi\left(a^2+2\right)$	A1
	$a^2+2=18,\ a=4$ *	A1*
		(3)
(b)	$y=(4+2\cos\theta)\sin\theta$	B1
	$\frac{\mathrm{d}y}{\mathrm{d}\theta}=4\cos\theta+2\cos 2\theta=4\cos^2\theta+4\cos\theta-2$	M1
	$2\cos^2\theta+2\cos\theta-1=0,\ \cos\theta=\frac{-1+\sqrt{3}}{2}$ (the other root is less than -1) *	A1*
	$r=4+\left(\sqrt{3}-1\right)$	M1
	$r=3+\sqrt{3}$	A1
		(5)
(c)	Distance $=2r\sin\theta$, with $\sin\theta=0.9306$	M1
	8.81	A1
		(2)
		Total for Question 10: 10 marks