



eClassroom

A-LEVEL MATHEMATICS • PURE 1

Algebraic Expressions

Worksheet

Pearson Edexcel International A Level (WMA11) & A Level (9MA0) • Calculator allowed: show every step

Free Algebraic Expressions worksheet for A-Level and International A-Level Maths, with full worked solutions



Tap a section to jump straight to it

By the end of this worksheet you can: use the laws of indices, including negative and fractional indices • expand and factorise polynomials completely • simplify surds and rationalise denominators • write expressions in the form $ax^p + bx^q$, ready for calculus • *Stretch:* solve equations in indices and prove results with surds.

Which questions are for me?

CORE The methods every A-level student must secure. Start here.

STRETCH Unfamiliar forms, equations in indices, surd algebra and proof.

Every section has both, clearly labelled. Questions get harder as you go, and every skill in Sections A to D is needed in Section E.

Track your progress

Section	B	C	D	E
Core marks	23	10	13	11
Stretch marks	16	14	17	8
My score				

Full worked solutions

Exam questions by topic

Algebra revision engine

Formula sheet builder

Name

Class

Date



Join students already acing their exams.

Book your FREE lesson at www.eclassroom.org.uk

A

Worked examples & Your turn

Read each example carefully. Cover it up, then answer the question beside it using the same method.

CORE

Secure the method

Worked example A1

Simplify $\frac{(2x^3y)^4}{8x^5y^2}$

Raise every factor to the power 4: $(2x^3y)^4 = 16x^{12}y^4$.
Then divide numbers and subtract powers.

$$2x^7y^2$$

Your turn A1

Simplify $\frac{(3a^2b)^3 \times 2a^{-1}b}{9a^4}$

Worked example A2

Expand and simplify $(x+3)(x-2)(2x-1)$

Two brackets first: $(x+3)(x-2) = x^2 + x - 6$. Then
multiply every term by $(2x-1)$

$$2x^3 - x^2 + 2x^2 - x - 12x + 6$$

$$2x^3 + x^2 - 13x + 6$$

Your turn A2

Expand and simplify $(2x-1)(x+4)(x-3)$

Worked example A3

Find the exact value of $\left(\frac{8}{27}\right)^{-\frac{2}{3}}$

The negative power flips the fraction. Then cube root,
then square.

$$\left(\frac{27}{8}\right)^{\frac{2}{3}} = \left(\frac{3}{2}\right)^2$$

$$\frac{9}{4}$$

Your turn A3

Find the exact value of $\left(\frac{25}{4}\right)^{-\frac{3}{2}}$



STRETCH

Worked example A4: factorising completelyFactorise completely $x^4 - 5x^2 + 4$ *A quadratic in x^2 : $(x^2 - 1)(x^2 - 4)$. Each bracket is a difference of two squares.*

$$(x - 1)(x + 1)(x - 2)(x + 2)$$

Your turn A4Factorise completely $x^4 - 13x^2 + 36$ **Worked example A5: the form $ax^p + bx^q$** Write $\frac{2x^3 - 5}{4\sqrt{x}}$ in the form $ax^p + bx^q$ *Divide each term by $4x^{\frac{1}{2}}$: $\frac{2x^3}{4x^{\frac{1}{2}}} - \frac{5}{4x^{\frac{1}{2}}}$*

$$\frac{1}{2}x^{\frac{5}{2}} - \frac{5}{4}x^{-\frac{1}{2}}$$

Your turn A5Write $\frac{x^2 - 6\sqrt{x}}{3x^3}$ in the form $ax^p + bx^q$ **Worked example A6: rationalising**Write $\frac{5 + \sqrt{3}}{2 - \sqrt{3}}$ in the form $a + b\sqrt{3}$ *Multiply top and bottom by the conjugate $2 + \sqrt{3}$. The bottom is $4 - 3 = 1$*

$$(5 + \sqrt{3})(2 + \sqrt{3}) = 10 + 5\sqrt{3} + 2\sqrt{3} + 3$$

$$13 + 7\sqrt{3}$$

Your turn A6Write $\frac{3 - \sqrt{2}}{4 + \sqrt{2}}$ in the form $a + b\sqrt{2}$ **Worked example A7: an equation in indices**Solve $9^{x+1} = 27^{2x}$ *Write both sides as powers of 3: $3^{2x+2} = 3^{6x}$, so the powers are equal.*

$$2x + 2 = 6x$$

$$x = \frac{1}{2}$$

Your turn A7Solve $4^x \times 8^{x-1} = 32$ 

B**Fluency**

Each question is a designed sequence. From one part to the next, ask yourself: what changed, and what did it do to the answer?

CORE*Secure the method***B1** Simplify

(a) $x^3 \times x^5$

(b) $(x^3)^5$

(c) $x^3 \div x^5$

(d) $\frac{(2x^3)^5}{4x^5}$

[4 marks]**B2** Find the exact value of

(a) $64^{\frac{1}{3}}$

(b) $64^{-\frac{1}{3}}$

(c) $64^{\frac{2}{3}}$

(d) $64^{-\frac{3}{2}}$

[4 marks]**B3** Expand and simplify

(a) $(x+5)(x-5)$

(b) $(2x-3)^2$

(c) $(x+1)(x^2-x+1)$

(d) $(x-2)^3$

[4 marks]**B4** Simplify

(a) $\sqrt{48}$

(b) $\sqrt{48} + \sqrt{12}$

(c) $\sqrt{48} \times \sqrt{12}$

(d) $\frac{\sqrt{48}}{\sqrt{12}}$

[4 marks]

CORE continued: rationalising and factorising**B5** Rationalise the denominator, simplifying fully.

(a) $\frac{6}{\sqrt{3}}$

(b) $\frac{4}{\sqrt{5}-1}$

(c) $\frac{\sqrt{7}+\sqrt{3}}{\sqrt{7}-\sqrt{3}}$

[3 marks]

B6 Factorise completely

(a) $6x^2 - 9x$

(b) $x^3 - 16x$

(c) $2x^3 + 5x^2 - 3x$

(d) $x^2y - 4y^3$

[4 marks]

STRETCH**B7** Write each in the form $ax^p + bx^q$, or $ax^p + bx^q + c$ in part (c).

(a) $\frac{x+4}{\sqrt{x}}$

(b) $\frac{3x^2-2}{x\sqrt{x}}$

(c) $\frac{(2+\sqrt{x})^2}{x}$

(d) $\frac{5x-2\sqrt{x}}{2x^2}$

[4 marks]

B8 (a) Expand $(3+\sqrt{2})(3-\sqrt{2})$ (b) Expand $(2\sqrt{5}-1)^2$ (c) Expand $(\sqrt{6}+\sqrt{2})^2$

(d) Solve $x\sqrt{3}+2=4x$, giving x in the form $\frac{a+b\sqrt{3}}{c}$

[4 marks]



STRETCH*continued: equations in indices***B9** Solve

(a) $2^x = \frac{1}{8}$

(b) $5^{2x-1} = 125$

(c) $9^x = 3\sqrt{3}$

(d) $4^x = 8^{1-x}$

[4 marks]

B10 Given that $y = 2^x$, write each expression in terms of y

(a) 2^{x+3}

(b) 4^x

(c) 8^{x-1}

(d) $\left(\frac{1}{2}\right)^{2x+1}$

[4 marks]



C**Reasoning**

Show your working and explain your thinking. A correct answer without a reason does not score full marks.

CORE

Secure the method

- C1** Tom writes $(x+3)^2 = x^2 + 9$
Explain Tom's mistake, and expand $(x+3)^2$ correctly. [2 marks]
- C2** Sam writes $\sqrt{16+9} = \sqrt{16} + \sqrt{9} = 7$
Explain why Sam is wrong. [2 marks]
- C3** Show that $(2\sqrt{3}+1)(\sqrt{3}-2) = 4 - 3\sqrt{3}$ [3 marks]
- C4** Explain why $8^{\frac{2}{3}}$ and $(8^2)^{\frac{1}{3}}$ are equal. Which is easier to work out without a calculator? Find the value. [3 marks]



STRETCH

C5 Jo rationalises $\frac{1}{3-\sqrt{5}}$ by multiplying the top and bottom by $3-\sqrt{5}$

Explain why this does not work, and rationalise the denominator correctly.

[3 marks]

C6 Show that $\frac{1}{\sqrt{5}+2} + \frac{1}{\sqrt{5}-2} = 2\sqrt{5}$

[3 marks]

C7 a and b are positive numbers. Prove that $(\sqrt{a} + \sqrt{b})^2 > a + b$

[3 marks]

C8 $x = 2 + \sqrt{3}$

(a) Show that $x + \frac{1}{x} = 4$

(b) Hence find the value of $x^2 + \frac{1}{x^2}$

(c) Hence find the value of $x^3 + \frac{1}{x^3}$

[5 marks]



D**Problem solving**

You decide the method. Read the whole question first, and show every step.

CORE

Secure the method

- D1** Figure 1 shows a solid cuboid. The edges of the cuboid have lengths $(2 + \sqrt{3})$ cm, $(2 - \sqrt{3})$ cm and $\sqrt{12}$ cm.

In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.

- (a) Show that the area of the face shaded in Figure 1 is exactly 1 cm^2

(1)

- (b) Find the exact volume of the cuboid, giving your answer in the form $k\sqrt{3} \text{ cm}^3$, where k is an integer.

(2)

[3 marks]

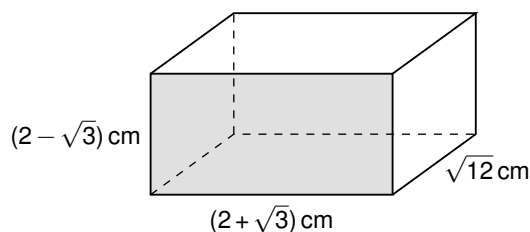


Figure 1

Diagram NOT accurately drawn

- D2** Figure 2 shows triangle ABC , in which angle $ABC = 90^\circ$, $AB = (\sqrt{5} + 1)$ cm and $BC = (\sqrt{5} - 1)$ cm.

Without using a calculator,

- (a) show that $AC = 2\sqrt{3}$ cm

(2)

- (b) find the exact area of triangle ABC

(1)

- (c) find the exact perimeter of triangle ABC , giving your answer in the form $(p\sqrt{5} + q\sqrt{3})$ cm, where p and q are integers to be found.

(1)

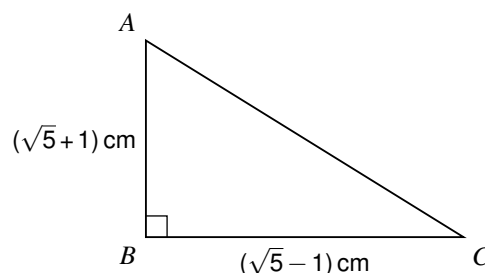


Figure 2

Diagram NOT accurately drawn

[4 marks]

- D3** Two colonies of bacteria, A and B , are being studied. The number of bacteria in colony A , t hours after the study begins, is modelled by the equation $N_A = 4^t$

The number of bacteria in colony B is modelled by the equation $N_B = 2 \times 8^{t-1}$

Using the laws of indices, and showing each step of your working, find the value of t for which the two colonies are predicted to contain the same number of bacteria.

[3 marks]



- D4** Figure 3 shows a sketch of the curve C with equation

$$y = x^4 - x^2 - 12$$

The curve C meets the x -axis at the points A and B , as shown in Figure 3.

- (a) Factorise $x^4 - x^2 - 12$ completely.

(2)

- (b) Hence write down the coordinates of A and the coordinates of B

(1)

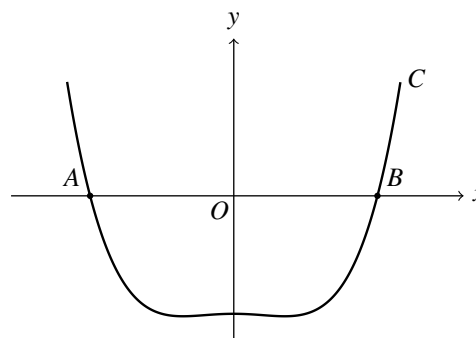


Figure 3

[3 marks]

STRETCH

- D5** Figure 4 shows an isosceles triangle PQR , in which $PQ = PR$. The point M is the midpoint of QR , and PM is perpendicular to QR

Given that

- $QR = (4 + 2\sqrt{3})$ cm
- the area of triangle PQR is $(10 + 6\sqrt{3})$ cm²

In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.

- (a) show that $PM = (2 + 2\sqrt{3})$ cm

(3)

- (b) Without using a calculator, show that PM is greater than 5 cm.

(1)

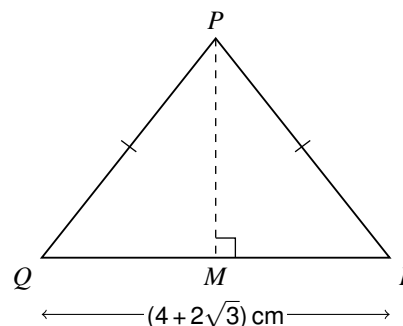


Figure 4

Diagram NOT accurately drawn

[4 marks]

- D6** Given that $\sqrt{37 + 20\sqrt{3}}$ can be written in the form $a + b\sqrt{3}$, where a and b are positive integers,

- (a) show that $a^2 + 3b^2 = 37$ and $ab = 10$

(2)

- (b) Hence find the value of a and the value of b

(2)

[4 marks]

- D7** Given that $(x+a)(x-2)(2x+1) \equiv 2x^3 - x^2 + bx + c$ where a , b and c are constants,

- (a) find the value of a

(2)

- (b) Hence find the value of b and the value of c

(2)

[4 marks]



D8 In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.

(a) Show that $\frac{\sqrt{3} + \sqrt{2}}{\sqrt{3} - \sqrt{2}} = 5 + 2\sqrt{6}$

(2)

(b) Use the result of part (a), or otherwise, to find the exact value of $\frac{\sqrt{3} + \sqrt{2}}{\sqrt{3} - \sqrt{2}} - \frac{\sqrt{3} - \sqrt{2}}{\sqrt{3} + \sqrt{2}}$

(3)

[5 marks]



E**Exam practice**

Exam-style questions in the style of WMA11. Write your answers in the spaces, and show every step.

CORE

Exam questions for everyone

E1 In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.

(a) Find the value of $125^{\frac{2}{3}}$

.....
(1)

(b) Simplify fully $\frac{(2x^{\frac{1}{2}})^3}{4x^{-1}}$ giving your answer in the form ax^n , where a and n are constants.

.....
(2)

Given that $y = 5^x$,

(c) express 25^{x+1} in terms of y

.....
(2)

(Total for Question E1 is 5 marks)



- E2** Figure 5 shows a rectangle $ABCD$, in which $AB = (3\sqrt{5} - 2)$ cm and $BC = (\sqrt{5} + 4)$ cm.

In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.

- (a) Show that the area of $ABCD$ is $(7 + 10\sqrt{5})$ cm²
(2)

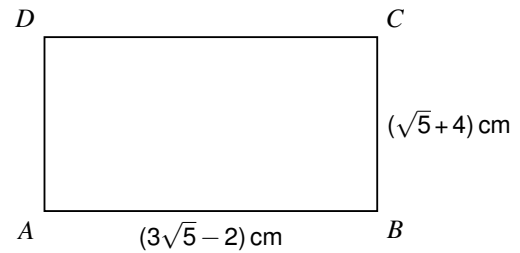


Figure 5

Diagram NOT accurately drawn

A second rectangle $PQRS$ has the same area as $ABCD$. The length of PQ is $(\sqrt{5} + 2)$ cm.

- (b) Find the length of QR , giving your answer in the form $(a + b\sqrt{5})$ cm, where a and b are integers to be found.

.....
(3)

- (c) Find the difference between the perimeters of the two rectangles, giving your answer in the form $p + q\sqrt{5}$, where p and q are integers.

.....
(1)

(Total for Question E2 is 6 marks)



STRETCH

E3 In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.

Given that $x > 0$,

(a) expand and simplify $\left(\sqrt{x} - \frac{3}{\sqrt{x}}\right)^2$

.....
(2)

(b) find the value of x for which this expression is zero.

.....
(1)

Given now that $x = 7 + 4\sqrt{3}$,

(c) show that $\sqrt{x} = 2 + \sqrt{3}$, and hence find $\sqrt{x} - \frac{3}{\sqrt{x}}$, giving your answer in the form $a + b\sqrt{3}$, where a and b are integers.

.....
(3)

(d) Hence find the exact value of $\left(\sqrt{x} - \frac{3}{\sqrt{x}}\right)^2$

.....
(2)

(Total for Question E3 is 8 marks)

Want more exam practice? The **Exam Questions by Topic** booklets have 10 more full exam questions on Pure 1 Chapter 1, Algebraic Expressions, with a full mark scheme.

[Open the booklet](#)

Finished? Mark your work with the **full worked solutions**, fill in your scores on the front page, then keep the skill sharp with unlimited questions on the **Algebra revision engine**.



Join students already acing their exams.

Book your FREE lesson at www.eclassroom.org.uk