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INTERNATIONAL A LEVEL MATHEMATICS • PURE 2 (WMA12)

Pure 2 Practice Paper 3

Edexcel IAL Pure Mathematics 2

AIMING FOR A/A*

Written in the style of the Pearson Edexcel International Advanced Level paper (WMA12/01) • Calculator allowed

A full 75-mark paper set above the hardest recent series: long questions, unfamiliar settings and very few steps given

Time	Total	You must have
1 hour 30 minutes	**75 marks**	**Mathematical Formulae and Statistical Tables, calculator**

Instructions

- Use black ink or ball-point pen. Pencil may be used for diagrams and sketches.
- Answer **all** questions, in the spaces provided.
- Show enough working to make your method clear. Answers without working may not gain full credit.
- Give inexact answers to **three significant figures** unless the question says otherwise.

Information

- There are 8 questions in this paper. The total mark is 75.
- The marks for each question are shown in brackets.

Advice

- Sit the paper in one go, in 1 hour 30 minutes, with the mark scheme closed.
- Read each question carefully and try every question. Check your answers if you have time.
- Then mark it line by line with the mark scheme and fill in your score.

Track your score

Q	Topic	Marks	Score
1	A binomial coefficient condition	8	
2	Geometric series with a trigonometric ratio	9	
3	Areas forming a geometric series	10	
4	Two circles through two points	10	
5	The largest triangle in a circle	10	
6	Proof of divisibility	9	
7	Logarithms of $\sin x$ and $\cos x$	10	
8	Working back from the trapezium rule	9	
	Total	**75**	

Rough grade guide (out of 75): A **45** • B **38** • C **31**

Based on recent WMA12 grade boundaries. Real boundaries change every series, so treat this as a guide, not a grade.

Mark scheme | Exam questions by topic | More practice papers

Name ..... Date .....

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1. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

In the expansion of

$$(x+3)(1+2x)^n$$

where n is a positive integer, the coefficient of x^3 is 540

(a) Find the value of n, showing that there is only one possible value.

(6)

(b) Find the coefficient of x^4 in the expansion.

(2)

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Question 1 continued

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(Total for Question 1 is 8 marks)

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2. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

A geometric series has first term 5 and common ratio $2\cos\theta$, where $0 \leqslant \theta < 360^\circ$

The series has a sum to infinity, and the sum to infinity is $\frac{8}{3}$ times the sum of the second and third terms.

(a) Find the possible values of the common ratio.

(6)

(b) Hence find all the possible values of θ, giving your answers to one decimal place.

(3)

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Question 2 continued

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(Total for Question 2 is 9 marks)

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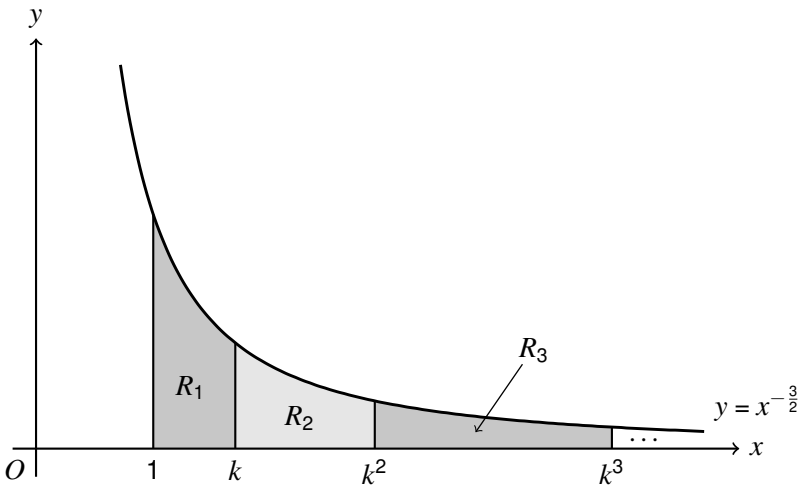

Figure 1

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

Figure 1 shows part of the curve with equation $y = x^{-\frac{3}{2}}$, $x > 0$

The constant k is greater than 1

For each positive integer n, the region R_n is bounded by the curve, the x-axis and the lines $x = k^{n-1}$ and $x = k^n$

The regions R_1, R_2 and R_3 are shown shaded in Figure 1.

(a) Show that the areas of $R_1, R_2, R_3, \ldots$ form a geometric sequence, and find its common ratio in terms of k.

(6)

(b) Show that the sum to infinity of the areas does not depend on the value of k.

(2)

(c) Given that $k = 4$, find the least value of n for which the total area of $R_1, R_2, \ldots, R_n$ exceeds 1.99

(2)

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Question 3 continued

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(Total for Question 3 is 10 marks)

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4. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

The points A and B have coordinates (1, 1) and (7, 1) respectively.

(a) Show that there are exactly two circles of radius 4 that pass through both A and B, and find an equation for each circle.

(4)

(b) Sketch both circles on the same axes, showing the coordinates of their centres and the positions of A and B.

(2)

(c) Find the area of the region that lies inside both circles, giving your answer to 3 significant figures.

(4)

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Question 4 continued

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Question 4 continued

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(Total for Question 4 is 10 marks)

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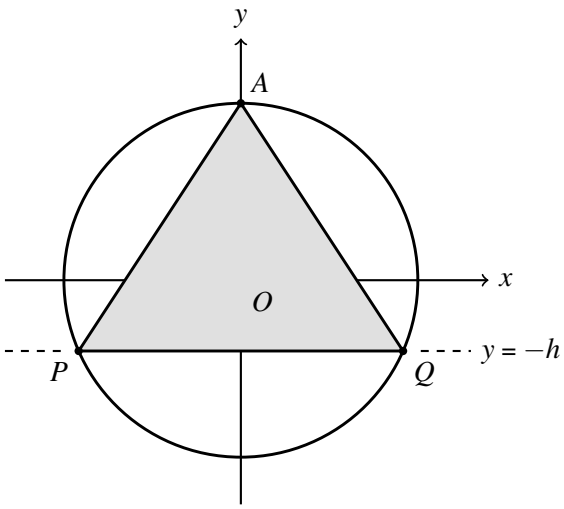

Diagram not drawn to scale

Figure 2

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

Figure 2 shows the circle with equation $x^2 + y^2 = 9$

The point A has coordinates (0, 3).

The points P and Q lie on the circle and on the line with equation $y = -h$, where $0 < h < 3$

The triangle APQ is shown shaded in Figure 2.

(a) Find an expression, in terms of h, for the area of triangle APQ.

(3)

(b) Use calculus to find the exact maximum area of triangle APQ, justifying that your answer is a maximum.

(7)

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Question 5 continued

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(Total for Question 5 is 10 marks)

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6. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

(a) Factorise $n^5 - n$ completely.

(2)

(b) Prove that, for every integer n, the number $n^5 - n$ is divisible by 30

(6)

(c) Show, by means of a counter-example, that $n^5 - n$ is not divisible by 60 for every integer n.

(1)

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Question 6 continued

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(Total for Question 6 is 9 marks)

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7. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

(a) Solve, for $0 < x < 2\pi$,

$$\log_2(\sin x) + \log_2(\cos x) = -2$$

giving your answers to 3 significant figures.

(7)

(b) Find the greatest value of the constant k for which the equation

$$\log_2(\sin x) + \log_2(\cos x) = k$$

has a solution.

(3)

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Question 7 continued

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(Total for Question 7 is 10 marks)

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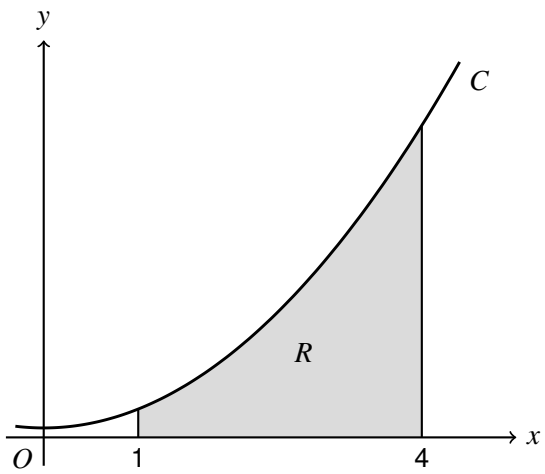

Figure 3

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

Figure 3 shows part of the curve C with equation

$$y = ax^2 + b$$

where a and b are positive constants.

The region R, shown shaded in Figure 3, is bounded by C, the x-axis and the lines $x = 1$ and $x = 4$

Using the trapezium rule with three strips of equal width gives an estimate of 46 for the area of R.

The exact area of R is 45

(a) Find the value of a and the value of b.

(5)

(b) Use the trapezium rule with six strips of equal width to estimate the area of R, and show that the error is one quarter of the error when three strips are used.

(3)

(c) Assuming that this pattern continues, write down the estimate given by the trapezium rule with twelve strips of equal width.

(1)

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Question 8 continued

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(Total for Question 8 is 9 marks)

TOTAL FOR PAPER IS 75 MARKS

END

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