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A-LEVEL MATHEMATICS • FURTHER PURE 3

Vectors

Worked Solutions

Pearson Edexcel International A Level (WFM03) • Calculator allowed: show every step

Full worked solutions for the free Vectors worksheet, A-Level and International A-Level Maths

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Tap a section to jump straight to it

Full worked solutions for every question on the Further Pure 3 Vectors worksheet, in the same order and with the same question numbers.

How to mark your work. Tick each step you got right, not just the final answer. At A-level most marks are for method, so a wrong final answer can still earn most of the marks. If you lost marks, find the *first* line where your working is different from ours. That is the thing to fix.

In Section E: **M** = method mark, **dM** = method mark that depends on the previous M mark, **A** = accuracy mark (needs the M mark before it), **B** = independent mark.

Question sheet | Exam questions by topic | Further Pure 3 exam questions | More worksheets

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A Your turn answers

Compare each line of your working with the solution.

CORE

A1 $(0-2,\ 1+6,\ 6-0)$: $\boxed{\begin{pmatrix}-2\\7\\6\end{pmatrix}}$

A2 $\begin{pmatrix}2\\0\\1\end{pmatrix}\times\begin{pmatrix}0\\3\\1\end{pmatrix}=\begin{pmatrix}-3\\-2\\6\end{pmatrix}$, length 7: $\boxed{\frac{7}{2}}$

A3 $2+0-5=-3$: $\boxed{x+4y-z=-3}$

STRETCH

A4 $\begin{pmatrix}1\\3\\0\end{pmatrix}\times\begin{pmatrix}1\\0\\4\end{pmatrix}=\begin{pmatrix}12\\-4\\-3\end{pmatrix}$, $\begin{pmatrix}2\\1\\1\end{pmatrix}\cdot\begin{pmatrix}12\\-4\\-3\end{pmatrix}=17$: $\boxed{\frac{17}{6}}$

A5 $\begin{pmatrix}0\\1\\-1\end{pmatrix}\times\begin{pmatrix}1\\1\\2\end{pmatrix}=\begin{pmatrix}3\\-1\\-1\end{pmatrix}$: $\boxed{\mathbf{r}\times\begin{pmatrix}1\\1\\2\end{pmatrix}=\begin{pmatrix}3\\-1\\-1\end{pmatrix}}$

A6 $(2+\lambda)-(-1+2\lambda)+3\lambda=9$, $\lambda=3$: $\boxed{(5,5,9)}$

A7 $\dfrac{|1-4-8-1|}{3}$: $\boxed{4}$

B Fluency solutions

One mark for each correct part. The notes in italics explain why each sequence was chosen.

CORE

B1 **(a)** $\boxed{\mathbf{k}}$ **(b)** $\boxed{\begin{pmatrix}1\\-1\\1\end{pmatrix}}$ **(c)** $\boxed{\mathbf{0}}$: the vectors are parallel **(d)** $\left|\begin{pmatrix}-6\\6\\-3\end{pmatrix}\right|$: $\boxed{9}$

Parallel vectors have a zero vector product.

B2 **(a)** $\boxed{\begin{pmatrix}1\\2\\0\end{pmatrix}, \begin{pmatrix}0\\1\\3\end{pmatrix}}$ **(b)** $\boxed{\begin{pmatrix}6\\-3\\1\end{pmatrix}}$ **(c)** $\boxed{\frac{\sqrt{46}}{2}}$ **(d)** $\boxed{\sqrt{46}}$

A parallelogram is two triangles.

B3 **(a)** $\boxed{\begin{pmatrix}1\\1\\2\end{pmatrix}, \begin{pmatrix}-1\\2\\0\end{pmatrix}}$ **(b)** $\boxed{\begin{pmatrix}4\\2\\-3\end{pmatrix}}$ **(c)** $\boxed{4x+2y-3z=1}$ **(d)** $\boxed{\mathbf{r}\cdot\begin{pmatrix}4\\2\\-3\end{pmatrix}=1}$

The normal is the vector product of two directions in the plane.

B4 **(a)** $\boxed{\begin{pmatrix}2\\1\\-2\end{pmatrix}}$ **(b)** $\boxed{\mathbf{r}=\begin{pmatrix}2\\-1\\3\end{pmatrix}+\lambda\begin{pmatrix}2\\1\\-2\end{pmatrix}}$ **(c)** $\boxed{\frac{x-2}{2}=\frac{y+1}{1}=\frac{z-3}{-2}}$ **(d)** $\boxed{\mathbf{r}\times\begin{pmatrix}2\\1\\-2\end{pmatrix}=\begin{pmatrix}-1\\10\\4\end{pmatrix}}$

Three ways of writing one line.

B5 **(a)** normals perpendicular: $\boxed{90°}$ **(b)** $\sin\theta=\frac{1}{\sqrt{2}}$: $\boxed{45°}$ **(c)** $\cos\theta=\frac{8}{9}$: $\boxed{27.3°}$

Between planes use cos of the normals; line and plane use sin.

B6 **(a)** $\boxed{2}$ **(b)** $2+12-0=14$, so it lies on the plane **(c)** $\frac{28}{7}$: $\boxed{4}$ **(d)** $(3,5,-2)-\frac{4}{7}\begin{pmatrix}2\\6\\-3\end{pmatrix}$: $\boxed{\left(\frac{13}{7},\frac{11}{7},-\frac{2}{7}\right)}$

$|\mathbf{n}|=7$ *throughout.*

STRETCH

B7 **(a)** $\boxed{\begin{pmatrix}-2\\6\\-2\end{pmatrix}}$ **(b)** $\boxed{10}$ **(c)** $\boxed{\frac{5}{3}}$ **(d)** $k-3+12=0$: $\boxed{k=-9}$

A zero triple product means the points are coplanar.

B8 **(a)** $5+4\lambda=21$, $\lambda=4$: $\boxed{(9,-2,8)}$ **(b)** $\boxed{\frac{4}{9}}$ **(c)** $\boxed{26.4^\circ}$ **(d)** $\boxed{\frac{16}{3}}$

Substitute into the full plane equation.

B9 **(a)** $\boxed{\begin{pmatrix}0\\-3\\-3\end{pmatrix}}$ **(b)** $\boxed{(1,1,0)}$ **(c)** $\boxed{\mathbf{r}=\begin{pmatrix}1\\1\\0\end{pmatrix}+t\begin{pmatrix}0\\1\\1\end{pmatrix}}$ **(d)** $\mathbf{n}_1\cdot\mathbf{n}_2=0$: $\boxed{90^\circ}$

The line lies in both planes, so it is perpendicular to both normals.

B10 **(a)** $\boxed{\begin{pmatrix}1\\-1\\1\end{pmatrix}}$ **(b)** $\boxed{\begin{pmatrix}-1\\1\\3\end{pmatrix}}$ **(c)** $\frac{|-1-1+3|}{\sqrt{3}}$: $\boxed{\frac{\sqrt{3}}{3}}$ **(d)** $\begin{pmatrix}1\\1\\0\end{pmatrix}$ is not a multiple of $\begin{pmatrix}0\\1\\1\end{pmatrix}$

Project the join onto the common perpendicular.

C Reasoning solutions

Your wording can be different. Check that your reason makes the same point.

CORE

C1 The vector product is not commutative: $\mathbf{j}\times\mathbf{i}=-\mathbf{i}\times\mathbf{j}=\boxed{-\mathbf{k}}$

C2 With the normal this ratio is $\sin\theta$, not $\cos\theta$: $\boxed{\theta=\arcsin\frac{4}{9}=26.4^\circ}$

C3 The plane is $=4$, not $=0$: $-1-5\lambda=4$, $\lambda=-1$: $\boxed{(0,-1,1)}$

C4 A normal is $\begin{pmatrix}1\\0\\1\end{pmatrix}\times\begin{pmatrix}0\\1\\0\end{pmatrix}=\begin{pmatrix}-1\\0\\1\end{pmatrix}$; $\begin{pmatrix}1\\1\\1\end{pmatrix}\cdot\begin{pmatrix}-1\\0\\1\end{pmatrix}=0$, so **the line is parallel to the plane: angle 0°**

STRETCH

C5 A vector equation gives the position vector $\mathbf{r}$ of a general point: $\boxed{\mathbf{r}=\begin{pmatrix}1\\2\\3\end{pmatrix}+t\begin{pmatrix}1\\0\\1\end{pmatrix}}$

C6 That is the parallelepiped; the tetrahedron is one sixth: $\boxed{2}$

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C7 He must divide by $|\mathbf{n}| = 3$: $\boxed{2}$

C8 **(a)** $\mathbf{a} \times \mathbf{b} = \begin{pmatrix} a_2b_3 - a_3b_2 \\ a_3b_1 - a_1b_3 \\ a_1b_2 - a_2b_1 \end{pmatrix}$; the dot product with $\mathbf{a}$ cancels term by term to 0, as required

(b) $|(\mathbf{p} - \mathbf{a}) \times \mathbf{b}| = |\mathbf{p} - \mathbf{a}||\mathbf{b}| \sin\theta$, and $|\mathbf{p} - \mathbf{a}| \sin\theta$ is the perpendicular distance, as required

D Problem solving solutions

There is often more than one correct method. Any complete, correct method scores full marks.

CORE

D1 **(a)** $\begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix} \times \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$: $\boxed{\begin{pmatrix} -3 \\ 5 \\ 1 \end{pmatrix}}$

(b) $\begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} -3 \\ 5 \\ 1 \end{pmatrix} = -1$: $\boxed{3x - 5y - z = 1}$

D2 **(a)** $\overrightarrow{BA} = \begin{pmatrix} 3 \\ 1 \\ 6 \end{pmatrix}$, $\mathbf{n} = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$: $\sin\theta = \dfrac{|3 + 2 - 12|}{3\sqrt{46}}$: $\boxed{20.1^\circ}$

(b) $\dfrac{|4 + 6 - 12 - 5|}{3}$: $\boxed{\dfrac{7}{3}}$; and $\dfrac{7/3}{\sqrt{46}} = \sin\theta$, which agrees

D3 Eliminating x: $-7y - z = 4$ and $-3y - 5z = -12$, so $32y = -32$, $y = -1$, $z = 3$, $x = 2$: $\boxed{(2, -1, 3)}$

D4 $\begin{pmatrix} -1 \\ 2 \\ 0 \end{pmatrix} \cdot \left(\begin{pmatrix} -1 \\ 0 \\ 3 \end{pmatrix} \times \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} \right) = \begin{pmatrix} -1 \\ 2 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} -6 \\ 5 \\ -2 \end{pmatrix} = 16$: $\boxed{\dfrac{8}{3}}$

STRETCH

D5 **(a)** $\mathbf{b} = \begin{pmatrix}1\\1\\1\end{pmatrix} \times \begin{pmatrix}1\\-1\\2\end{pmatrix} = \begin{pmatrix}3\\-1\\-2\end{pmatrix}$: $\boxed{\left(\mathbf{r} - \begin{pmatrix}4\\0\\-1\end{pmatrix}\right) \times \begin{pmatrix}3\\-1\\-2\end{pmatrix} = \mathbf{0}}$

(b) $\dfrac{|\mathbf{a}\times\mathbf{b}|}{|\mathbf{b}|} = \dfrac{\left|\begin{pmatrix}-1\\5\\-4\end{pmatrix}\right|}{\sqrt{14}} = \dfrac{\sqrt{42}}{\sqrt{14}}$: $\boxed{\sqrt{3}}$

D6 **(a)** $\mathbf{r} = \begin{pmatrix}6\\-2\\7\end{pmatrix} + t\begin{pmatrix}2\\-1\\2\end{pmatrix}$; $9t + 27 = 1$ gives $t = -3$: $\boxed{(0,1,1)}$

(b) $t = -6$: $\boxed{(-6,4,-5)}$

D7 $\overrightarrow{AB} \times \overrightarrow{AC} = \begin{pmatrix}-3\\-4\\6\end{pmatrix}$, $\overrightarrow{AD} = \begin{pmatrix}1\\k-1\\3\end{pmatrix}$: $\dfrac{1}{6}|19 - 4k| = 2$, $19 - 4k = \pm 12$: $\boxed{k = \dfrac{7}{4} \text{ or } \dfrac{31}{4}}$

D8 **(a)** $2 + 2 + 2 = 6$, as required

(b) $A + t\begin{pmatrix}1\\1\\1\end{pmatrix}$ meets Π at $t = 1$; double it: $\boxed{A'(3,4,2)}$

(c) through B and A': $\boxed{\mathbf{r} = \begin{pmatrix}2\\2\\2\end{pmatrix} + \mu\begin{pmatrix}1\\2\\0\end{pmatrix}}$

E Exam practice solutions

Mark-scheme notes are shown on the right of each line.

CORE *Exam questions for everyone*

E1 **(a)** $\mathbf{n} = \begin{pmatrix}2\\1\\0\end{pmatrix} \times \begin{pmatrix}0\\1\\3\end{pmatrix} = \begin{pmatrix}3\\-6\\2\end{pmatrix}$ *M1*

$\boxed{3x - 6y + 2z = 7}$ *A1*

(b) $\dfrac{|12 + 18 + 18 - 7|}{7}$ *M1*

$\boxed{\dfrac{41}{7}}$ *A1*

(c) $\sin\alpha = \dfrac{|3 - 6 + 2|}{7\sqrt{3}}$ *M1*

$\boxed{\alpha = 4.7^\circ}$ *A1*

E2 **(a)** $\mathbf{b} = \begin{pmatrix}1\\-1\\2\end{pmatrix} \times \begin{pmatrix}2\\1\\-1\end{pmatrix}$ *M1*

$= \begin{pmatrix}-1\\5\\3\end{pmatrix}$ *A1*

$A(1,0,1)$ lies on both: $1 + 2 = 3$, $2 - 1 = 1$ *B1*

$\boxed{\mathbf{r} = \begin{pmatrix}1\\0\\1\end{pmatrix} + t\begin{pmatrix}-1\\5\\3\end{pmatrix}}$ *A1*

(b) $\cos\theta = \dfrac{|2 - 1 - 2|}{\sqrt{6}\sqrt{6}}$ *M1 A1*

$\boxed{80.4^\circ}$ *A1*

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STRETCH

E3 **(a)** l_2: $\mathbf{r} = \begin{pmatrix}3\\0\\1\end{pmatrix} + t\begin{pmatrix}1\\2\\0\end{pmatrix}$; y: $2 = 2t$, $t = 1$; x: $1 + s = 4$, $s = 3$ *M1 A1*

z: $3 \neq 1$, so no intersection; $\begin{pmatrix}1\\0\\1\end{pmatrix}$ is not a multiple of $\begin{pmatrix}1\\2\\0\end{pmatrix}$ *A1*

(b) $\mathbf{b}_1 \times \mathbf{b}_2 = \begin{pmatrix}-2\\1\\2\end{pmatrix}$, $\mathbf{a}_2 - \mathbf{a}_1 = \begin{pmatrix}2\\-2\\1\end{pmatrix}$ *M1*

$\dfrac{|-4-2+2|}{3}$ *M1*

$\boxed{\dfrac{4}{3}}$ *A1*

(c) $\mathbf{n} = \begin{pmatrix}-2\\1\\2\end{pmatrix}, \begin{pmatrix}1\\2\\0\end{pmatrix} \cdot \mathbf{n} = 0$ *M1 A1*

$\boxed{-2x + y + 2z = 0}$; the distance of A from it is $\dfrac{|-6+0+2|}{3} = \dfrac{4}{3}$, confirming (b) *A1*