



Coordinate Geometry

Worked Solutions

Suitable for:

Pearson Edexcel International A-Level (iAL) — Pure Mathematics 2



Question 1 — Worked Solution

(a) Complete the square

Group x and y terms and complete the square:

$$x^2 - 6x + y^2 + 4y = 12$$

$$(x - 3)^2 - 9 + (y + 2)^2 - 4 = 12$$

$$\therefore (x - 3)^2 + (y + 2)^2 = 25$$

(b) Centre and radius

Centre = (3, -2), radius = 5



Question 2 — Worked Solution

(a) Equation of the circle

Centre $(-3, 4)$, radius 6:

$$\therefore (x + 3)^2 + (y - 4)^2 = 36$$

(b) Expand to general form

$$x^2 + 6x + 9 + y^2 - 8y + 16 = 36$$

$$x^2 + y^2 + 6x - 8y + 25 = 36$$

$$\therefore x^2 + y^2 + 6x - 8y - 11 = 0$$



Question 3 — Worked Solution

(a) Find the radius

Use the distance formula from centre $(2, -1)$ to $(5, 3)$:

$$r^2 = (5 - 2)^2 + (3 - (-1))^2 = 9 + 16 = 25$$

$$\therefore r = 5$$

(b) Equation of the circle

$$\therefore (x - 2)^2 + (y + 1)^2 = 25$$

(c) Verify $(7, -1)$ lies on C

$$(7 - 2)^2 + (-1 + 1)^2 = 25 + 0 = 25 \quad \checkmark$$

$(7, -1)$ satisfies the equation so it lies on C. $\checkmark$



Question 4 — Worked Solution

(a) Show P(7, 5) lies on C

$$(7 - 1)^2 + (5 - 2)^2 = 36 + 9 = 45 \quad \checkmark$$

(b) Gradient of radius to P

Centre of C is (1, 2). Gradient of OP:

$$m_{\text{radius}} = \frac{5 - 2}{7 - 1} = \frac{3}{6} = \frac{1}{2}$$

$$\therefore m_{\text{radius}} = \frac{1}{2}$$

(c) Equation of tangent at P

Tangent is perpendicular to radius, so tangent gradient = -2.

$$y - 5 = -2(x - 7) = -2x + 14$$

$$\therefore y = -2x + 19$$



Question 5 — Worked Solution

(a) Show $5x^2 - 20x = 0$

Substitute $y = 2x - 5$ into $x^2 + y^2 = 25$:

$$x^2 + (2x - 5)^2 = 25$$

$$x^2 + 4x^2 - 20x + 25 = 25$$

$$5x^2 - 20x = 0 \quad \checkmark$$

(b) Coordinates of intersection

$$5x(x - 4) = 0 \Rightarrow x = 0 \text{ or } x = 4$$

$$x = 0 : y = 2(0) - 5 = -5 \Rightarrow (0, -5)$$

$$x = 4 : y = 2(4) - 5 = 3 \Rightarrow (4, 3)$$

$\therefore$ Intersection points: $(0, -5)$ and $(4, 3)$





Question 6 — Worked Solution

(a) Show L is tangent to C

The centre of C is (3, 4) and $r = 5$. Rewrite L as $3x + 4y - 50 = 0$.

Find the perpendicular distance from (3, 4) to the line:

$$d = \frac{|3(3) + 4(4) - 50|}{\sqrt{3^2 + 4^2}} = \frac{|9 + 16 - 50|}{5} = \frac{25}{5} = 5$$

Since $d = r = 5$, the line is a tangent. ✓

(b) Point of tangency

The tangent point lies on the line from the centre, perpendicular to L. The direction of L is (4, -3) so the normal direction is (3, 4).

Parametric point on normal: (3 + 3t, 4 + 4t). Substitute into $3x + 4y = 50$:

$$3(3 + 3t) + 4(4 + 4t) = 50 \Rightarrow 9 + 9t + 16 + 16t = 50 \Rightarrow 25t = 25$$

$$t = 1 \Rightarrow \text{point} = (3 + 3, 4 + 4) = (6, 8)$$

$$\therefore \text{Point of tangency} = (6, 8)$$



Question 7 — Worked Solution

(a) Verify A(0, 6) and B(4, 8) lie on C

$$A : (0 - 4)^2 + (6 - 3)^2 = 16 + 9 = 25 \checkmark$$

$$B : (4 - 4)^2 + (8 - 3)^2 = 0 + 25 = 25 \checkmark$$

(b) Perpendicular bisector of AB

Midpoint M of AB:

$$M = \left(\frac{0 + 4}{2}, \frac{6 + 8}{2} \right) = (2, 7)$$

Gradient of AB:

$$m_{AB} = \frac{8 - 6}{4 - 0} = \frac{2}{4} = \frac{1}{2}$$

Perpendicular bisector has gradient -2 and passes through (2, 7):

$$y - 7 = -2(x - 2) \Rightarrow y = -2x + 11$$

$$\therefore y = -2x + 11$$

(c) Passes through centre (4, 3)

$$y = -2(4) + 11 = -8 + 11 = 3 \checkmark$$

The perpendicular bisector passes through (4, 3), the centre of C. ✓





Question 8 — Worked Solution

(a) Complete the square

$$x^2 - 6x + y^2 - 4y = 0$$

$$(x - 3)^2 - 9 + (y - 2)^2 - 4 = 0$$

$$\therefore (x - 3)^2 + (y - 2)^2 = 13$$

Centre = (3, 2), radius = $\sqrt{13}$

(b) Intersections with the x-axis ($y = 0$)

$$(x - 3)^2 + (0 - 2)^2 = 13 \Rightarrow (x - 3)^2 = 9$$

$$x - 3 = \pm 3 \Rightarrow x = 0 \text{ or } x = 6$$

$\therefore$ x-axis intersections: (0, 0) and (6, 0)

(c) Intersections with the y-axis ($x = 0$)

$$(0 - 3)^2 + (y - 2)^2 = 13 \Rightarrow (y - 2)^2 = 4$$

$$y - 2 = \pm 2 \Rightarrow y = 0 \text{ or } y = 4$$

$\therefore$ y-axis intersections: (0, 0) and (0, 4)

