



eClassroom

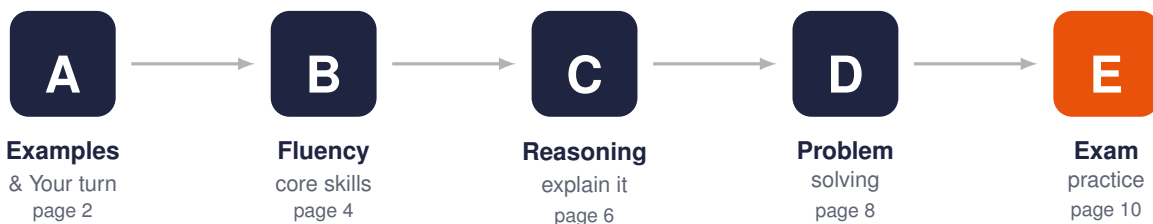
GCSE MATHEMATICS • RATIO & PROPORTION

Direct & Inverse Proportion

Worksheet

Pearson Edexcel GCSE (1MA1) & International GCSE (4MA1) • Calculator only where shown

Free Direct and Inverse Proportion worksheet for GCSE and iGCSE Maths, with full worked solutions



Tap a section to jump straight to it

By the end of this worksheet you can: scale recipes and costs, and compare best buys • solve inverse proportion problems such as workers and time • use tables and graphs of direct proportion • *Higher:* form and use equations of proportion, find the effect of changing x , and link two proportion statements.

Which questions are for me?

FOUNDATION Everyone starts here. These skills appear on both tiers.

HIGHER Higher only: equations such as $y = kx^2$ and $y = k/\sqrt{x}$, the effect of changing x , and chains of proportion. Grades 6 to 9.

Every section has both, clearly labelled. Questions get harder as you go, and every skill in Sections A to D is needed in Section E.

Track your progress

Section	B	C	D	E
Foundation marks	27	9	14	19
Higher marks	16	14	16	16
My score				

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A**Worked examples & Your turn**

Read each example carefully. Cover it up, then answer the question beside it using the same method.

FOUNDATION*Everyone starts here***Worked example A1**

5 notebooks cost £3.60

Work out the cost of 8 notebooks.

Find the cost of 1 notebook first.

$$3.60 \div 5 = 0.72$$

Then multiply by 8. $0.72 \times 8 = 5.76$

£5.76

Your turn A1

7 m of cable costs £9.45

Work out the cost of 4 m of the same cable.

Worked example A2

Pack A has 6 rolls for £2.70

Pack B has 9 rolls for £3.87

Which pack is better value?

Find the cost of 1 roll in each pack.

$$\text{A: } 270 \div 6 = 45\text{p} \quad \text{B: } 387 \div 9 = 43\text{p}$$

The lower price for 1 roll is better value.

Pack B

Your turn A2

Jar A has 340 g of jam for £2.21

Jar B has 450 g of jam for £2.97

Which jar is better value?

Worked example A3

4 pumps empty a tank in 9 hours.

How long would 6 pumps take?

More pumps take less time: this is inverse proportion.

Find the time for 1 pump.

$$1 \text{ pump: } 4 \times 9 = 36 \text{ hours}$$

Share that between 6 pumps. $36 \div 6 = 6$

6 hours

Your turn A3

5 cleaners clean a school in 12 hours.

How long would 3 cleaners take?



HIGHER*Higher only: equations of proportion***Worked example A4: write $y = kx^2$ and find k** $y \propto x^2$, $y = 48$ when $x = 4$. Find y when $x = 6$

$$y = kx^2 \quad 48 = k \times 16, \text{ so } k = 3$$

$$\text{Use } y = 3x^2 \quad 3 \times 36 = 108$$

$$y = 108$$

Your turn A4 $p \propto q^3$, $p = 40$ when $q = 2$. Find p when $q = 3$ **Worked example A5: inverse, so divide by $\sqrt{x}$** $y \propto \frac{1}{\sqrt{x}}$ and $y = 6$ when $x = 16$. Find x when $y = 8$

$$y = \frac{k}{\sqrt{x}} \quad 6 = \frac{k}{4}, \text{ so } k = 24$$

$$8 = \frac{24}{\sqrt{x}}, \text{ so } \sqrt{x} = 3$$

$$x = 9$$

Your turn A5 $F \propto \frac{1}{d^2}$ and $F = 20$ when $d = 3$. Find F when $d = 6$ **Worked example A6: the multiplier method** $y \propto \frac{1}{x^2}$ and x is multiplied by 3. What happens to y ?*Replace x by $3x$ in $y = \frac{k}{x^2}$*

$$\frac{k}{(3x)^2} = \frac{k}{9x^2} = \frac{1}{9} \times \frac{k}{x^2}$$

 y is divided by 9**Your turn A6** $y \propto x^3$ and x is halved. What happens to y ?**Worked example A7: linking two statements** $a \propto b^2$ and $b \propto c$ $a = 12$ when $c = 2$. Find a when $c = 5$ $b = mc$, so $a = k(mc)^2 = Kc^2$

$$12 = K \times 4, \text{ so } K = 3 \quad 3 \times 25 = 75$$

$$a = 75$$

Your turn A7 $p \propto \frac{1}{q}$ and $q \propto r^2$ $p = 9$ when $r = 2$. Find p when $r = 3$ 

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B**Fluency**

Each question is a designed sequence. From one part to the next, ask yourself: what changed, and what did it do to the answer?

FOUNDATION

Everyone starts here

B1 3 pens cost £2.40 Work out the cost of

- (a) 6 pens (b) 15 pens (c) 1 pen (d) 10 pens

[4 marks]

B2 A recipe for 4 people uses 300 g of pasta, 180 ml of cream and 60 g of cheese. Work out

- (a) the pasta for 8 people (b) the cream for 6 people
(c) the cheese for 10 people (d) the pasta for 3 people

[4 marks]

B3 For each pair, which is better value? You must show your working.

- (a) 4 bottles for £3.20 (b) 500 g for £1.85 (c) 12 eggs for £2.64
or 6 bottles for £4.50 or 750 g for £2.70 or 30 eggs for £6.75

[3 marks]

B4 It takes 6 workers 8 days to build a wall. At the same rate, how many days would it take

- (a) 12 workers (b) 3 workers (c) 4 workers (d) 5 workers

[4 marks]

B5 Is each pair of quantities in direct proportion, inverse proportion or neither?

- (a) The cost of petrol and the number of litres bought
(b) The number of people sharing a £60 bill equally and the amount each person pays
(c) A child's age and the child's height
(d) The speed of a car and the time it takes to drive 100 km

[4 marks]

B6 Work out the missing values in each table.

y is directly proportional to x

x	4	10	18
y	14	(a)	(b)

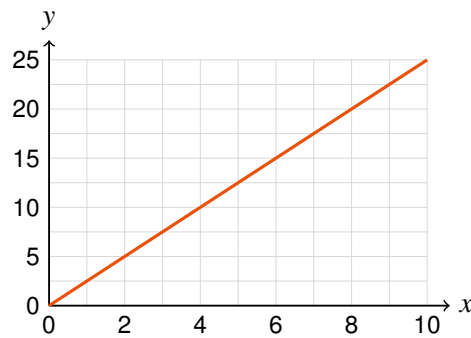
y is inversely proportional to x

x	4	10	(d)
y	15	(c)	2

[4 marks]



B7 The graph shows that y is directly proportional to x



(a) Find y when $x = 8$

(b) Find x when $y = 35$

(c) Write a formula for y in terms of x

(d) Find y when $x = 100$

[4 marks]

HIGHER

Higher tier only

B8 Find a formula for y in terms of x in each case.

(a) y is directly proportional to x , and $y = 12$ when $x = 4$

(b) y is directly proportional to x^2 , and $y = 50$ when $x = 5$

(c) y is inversely proportional to x , and $y = 6$ when $x = 4$

(d) y is inversely proportional to x^2 , and $y = 2$ when $x = 3$

[4 marks]

B9 (a) $y = 3x^2$ Find y when $x = 4$

(b) $y = 3x^2$ Find the positive value of x when $y = 75$

(c) $y = \frac{36}{\sqrt{x}}$ Find y when $x = 9$

(d) $y = \frac{36}{\sqrt{x}}$ Find x when $y = 4$

[4 marks]

B10 Describe what happens to y in each case.

(a) y is proportional to x^2 and x is doubled.

(b) y is proportional to x^3 and x is multiplied by 3

(c) y is inversely proportional to x and x is multiplied by 5

(d) y is inversely proportional to $\sqrt{x}$ and x is multiplied by 9

[4 marks]

B11 Work out the percentage change in y in each case.

(a) $y \propto x^2$ and x increases by 10%

(b) $y \propto \frac{1}{x}$ and x increases by 25%

(c) $y \propto x^3$ and x decreases by 20%

(d) $y \propto \sqrt{x}$ and x increases by 44%

[4 marks]



C**Reasoning**

Show your working and explain your thinking. A correct answer without a reason does not score full marks.

FOUNDATION

Everyone starts here

C1 Kate says, “3 workers take 12 days to paint a house, so 6 workers will take 24 days.”

(a) Explain why Kate is wrong. (b) Work out how long 6 workers will take.

[2 marks]

C2 Here is a table of values.

x	2	5	8
y	6	15	25

Is y directly proportional to x ? You must show how you decide.

[2 marks]

C3 A recipe for 4 people uses 3 eggs.

Jo says, “For 6 people I need 5 eggs, because 6 is 2 more than 4.”

Is Jo correct? Give a reason for your answer.

[2 marks]

C4 A journey takes 45 minutes at an average speed of 40 mph.

Show that the same journey takes 36 minutes at an average speed of 50 mph.

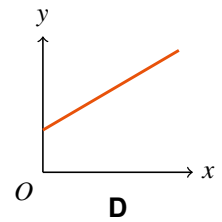
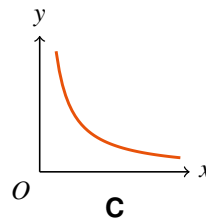
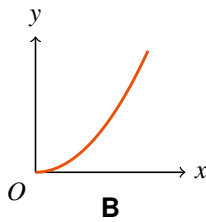
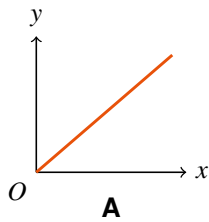
[3 marks]



HIGHER

Higher tier only

C5 Here are sketches of four graphs.



(a) Write down the letter of the graph that shows y is directly proportional to x

(b) Write down the letter of the graph that could show $y \propto x^2$, and the graph that could show $y \propto \frac{1}{x}$

(c) Explain why graph D does not show direct proportion.

[3 marks]

C6 y is inversely proportional to x^2 . When $x = 2$, $y = 8$

Ben writes $y = \frac{k}{x^2}$ $8 = \frac{k}{4}$ so $k = 2$

(a) Explain Ben's mistake. (b) Find the value of y when $x = 4$

[3 marks]

C7 y is proportional to x^n , where n is a positive whole number.

When x is doubled, y is multiplied by 8. When $x = 3$, $y = 54$

(a) Find the value of n . Give a reason for your answer. (b) Find the value of y when $x = 5$ [4 marks]

C8 y is inversely proportional to x

The points $(a, 12)$ and $(a+3, 8)$ both lie on the graph of y against x

Find the value of a , and the value of y when $x = 16$

[4 marks]



D**Problem solving**

You decide the method. Read the whole question first, and show every step.

FOUNDATION

Everyone starts here

- D1** The time taken to cook a chicken is directly proportional to its mass.
A chicken of mass 1.6 kg takes 72 minutes to cook.
How long does a chicken of mass 2.2 kg take to cook? Give your answer in hours and minutes. **[3 marks]**
- D2** Shop A sells tins of soup in packs of 4 for £3.00
Shop B sells the same tins in packs of 6 for £4.35
Tom needs at least 20 tins. He can buy packs from either shop, or from both.
What is the least amount Tom can pay? You must show your working. **[4 marks]**
- D3** 8 workers can build a wall in 15 days.
They start work together. After 6 days, 2 of the workers leave. The others finish the wall at the same rate.
How many days does it take to build the wall altogether? **[4 marks]**
- D4** (*calculator*) A car uses 6 litres of fuel for every 80 km it travels.
Fuel costs £1.42 per litre. Work out the cost of the fuel for a journey of 520 km. **[3 marks]**



HIGHER*Higher tier only*

- D5** The distance an object falls is directly proportional to the square of the time it has been falling.
An object falls 20 m in 2 seconds.
A ball is dropped from the top of a cliff 180 m high. A second ball is dropped from the same point 2 seconds later.
How high above the ground is the second ball when the first ball hits the ground? [4 marks]
- D6** The brightness of a lamp at a sensor is inversely proportional to the square of the distance between them.
The sensor is 2 m from the lamp. The sensor is moved further away until the brightness has fallen by 75%.
How far has the sensor been moved? [3 marks]
- D7** (*calculator*) A café prices its pizzas so that the cost is directly proportional to the square of the diameter.
A pizza of diameter 10 inches costs £6.00
(a) Work out the cost of a pizza of diameter 14 inches.
(b) Sam has £20. Diameters are whole numbers of inches. What is the largest pizza Sam can buy? [4 marks]
- D8** The time taken to fill a tank through one pipe is inversely proportional to the square of the radius of the pipe.
Through a pipe of radius 2 cm, the tank fills in 45 minutes.
The empty tank is filled using a pipe of radius 3 cm and a pipe of radius 1 cm at the same time.
How long does it take to fill the tank? [5 marks]





Exam practice

Exam-style questions. Write your answers in the spaces. Aim for about one minute per mark.

FOUNDATION

Foundation and Higher crossover

E1 Here are the ingredients for a cake for 6 people.

450 g flour	180 g butter	120 g sugar	300 ml milk
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Marta makes a cake for 15 people. Work out the amount of each ingredient she needs.

..... g flour g butter

..... g sugar ml milk

(Total for Question E1 is 3 marks)

E2 (*calculator*) Washing powder is sold in two sizes.

Small box: 1.5 kg for £4.20 Large box: 2.4 kg for £6.48

Which box is better value for money? You **must** show your working.

.....

(Total for Question E2 is 3 marks)

E3 6 builders take 10 days to build a garage.

(a) How many days would 4 builders take to build the same garage?

.....

(2)

(b) Write down one assumption you made in your answer to part (a).

.....

(1)

(Total for Question E3 is 3 marks)



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E4 Here is a recipe for 8 flapjacks.

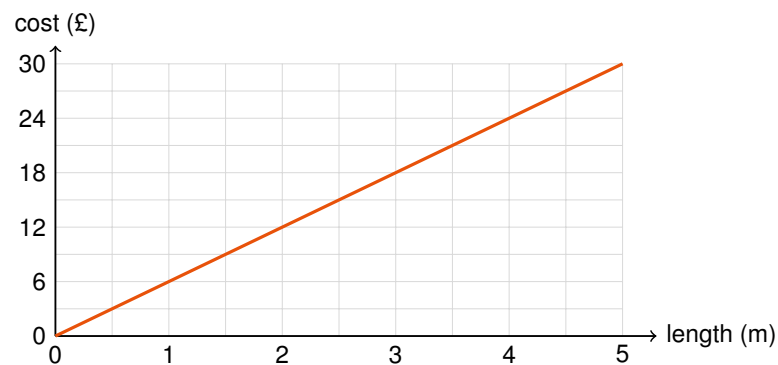
200 g oats	120 g butter	90 g syrup
------------	--------------	------------

Leah has 500 g of oats, 270 g of butter and 250 g of syrup.

What is the greatest number of flapjacks Leah can make?

.....
(Total for Question E4 is 4 marks)

E5 The cost of fabric is directly proportional to its length. The graph shows the cost of up to 5 m of fabric.



(a) Use the graph to find the cost of 2.5 m of fabric.

£
(1)

(b) Work out the cost of 40 m of fabric.

£
(2)

(Total for Question E5 is 3 marks)



E6 3 decorators take 8 hours to paint 4 rooms.

How long would 4 decorators take to paint 6 rooms? Assume they all work at the same rate.

..... hours

(Total for Question E6 is 3 marks)

HIGHER

Higher tier only

E7 y is directly proportional to x^2 . When $x = 3$, $y = 45$

(a) Find a formula for y in terms of x

.....
(3)

(b) Find the value of y when $x = 5$

.....
(1)

(Total for Question E7 is 4 marks)

E8 p is inversely proportional to $\sqrt{q}$. When $q = 16$, $p = 5$

(a) Find the value of p when $q = 100$

$p =$
(3)

(b) Find the value of q when $p = 10$

$q =$
(1)

(Total for Question E8 is 4 marks)



- E9** The power produced by a wind turbine is directly proportional to the cube of the wind speed.
The wind speed increases by 20%
Work out the percentage increase in the power produced.

..... %

(Total for Question E9 is 3 marks)

- E10** P is directly proportional to Q^3
 Q is inversely proportional to R
When $R = 2$, $Q = 6$ and $P = 54$
(a) Find a formula for P in terms of R

.....
(4)

- (b) Find the value of P when $R = 3$

.....
(1)

(Total for Question E10 is 5 marks)

Finished? Mark your work with the **full worked solutions**, fill in your scores on the front page, then keep the skill sharp with unlimited questions on the **Ratio revision engine**.

