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A-LEVEL MATHEMATICS • FURTHER PURE 1

Matrices

Worksheet

Pearson Edexcel International A Level (WFM01) • Calculator allowed: show every step

Free Matrices worksheet for A-Level and International A-Level Maths, with full worked solutions

A **Examples** & Your turn page 2

B **Fluency** core skills page 4

C **Reasoning** explain it page 6

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Tap a section to jump straight to it

By the end of this worksheet you can: multiply matrices • find determinants • find the inverse of a 2×2 matrix • identify singular matrices • *Stretch:* constants in matrices, matrix equations, and powers of a matrix.

Which questions are for me?

CORE The methods every A-level student must secure. Start here.

STRETCH Determinants with a constant, inverses in terms of a letter, and matrix equations in the right order.

Every section has both, clearly labelled. Questions get harder as you go, and every skill in Sections A to D is needed in Section E.

Track your progress

Section	B	C	D	E
Core marks	23	10	13	13
Stretch marks	16	14	17	9
My score				

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Name ..... Class ..... Date .....

A Worked examples & Your turn

Read each example carefully. Cover it up, then answer the question beside it using the same method.

CORE *Secure the method*

Worked example A1: multiplying

Find $\begin{pmatrix} 2 & 1 \\ 3 & -1 \end{pmatrix}\begin{pmatrix} 1 & 4 \\ 0 & 2 \end{pmatrix}$

Row by column

$$\begin{pmatrix} 2+0 & 8+2 \\ 3+0 & 12-2 \end{pmatrix}$$

$$\boxed{\begin{pmatrix} 2 & 10 \\ 3 & 10 \end{pmatrix}}$$

Your turn A1

Find $\begin{pmatrix} 3 & -2 \\ 1 & 4 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ -1 & 3 \end{pmatrix}$

Worked example A2: non-square products

Find $\begin{pmatrix} 1 & 2 & 0 \\ 3 & -1 & 2 \end{pmatrix}\begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix}$

$(2\times 3)(3\times 1)$ *gives* 2×1

$$\begin{pmatrix} 2+2+0 \\ 6-1+8 \end{pmatrix}$$

$$\boxed{\begin{pmatrix} 4 \\ 13 \end{pmatrix}}$$

Your turn A2

Find $\begin{pmatrix} 2 & -1 \end{pmatrix}\begin{pmatrix} 3 & 1 \\ 4 & 0 \end{pmatrix}$

Worked example A3: a determinant

Find $\det\begin{pmatrix} 5 & 3 \\ 4 & 2 \end{pmatrix}$

$ad-bc = 10-12$

$$\boxed{-2}$$

Your turn A3

Find $\det\begin{pmatrix} -3 & 2 \\ 5 & -4 \end{pmatrix}$

STRETCH

Worked example A4: an inverse

Find the inverse of $\begin{pmatrix} 4 & 3 \\ 2 & 2 \end{pmatrix}$

Swap a and d, change the signs of b and c, divide by the determinant

$\det = 2$: $\frac{1}{2}\begin{pmatrix} 2 & -3 \\ -2 & 4 \end{pmatrix}$

$$\begin{pmatrix} 1 & -1.5 \\ -1 & 2 \end{pmatrix}$$

Your turn A4

Find the inverse of $\begin{pmatrix} 5 & 2 \\ 7 & 3 \end{pmatrix}$

Worked example A5: singular values

Find k for which $\begin{pmatrix} k & 3 \\ 4 & k+4 \end{pmatrix}$ is singular.

$k(k+4) - 12 = 0$, $(k+6)(k-2) = 0$

$k = 2$ or $k = -6$

Your turn A5

Find k for which $\begin{pmatrix} 2k & 5 \\ k+1 & 3 \end{pmatrix}$ is singular.

Worked example A6: a matrix equation

$\mathbf{A} = \begin{pmatrix} 2 & 1 \\ 5 & 3 \end{pmatrix}$ and $\mathbf{AX} = \begin{pmatrix} 1 & 0 \\ 2 & 4 \end{pmatrix}$. Find $\mathbf{X}$

$\mathbf{X} = \mathbf{A}^{-1}(\mathbf{AX})$, *with* $\mathbf{A}^{-1}$ *on the left*

$\mathbf{A}^{-1} = \begin{pmatrix} 3 & -1 \\ -5 & 2 \end{pmatrix}$

$$\mathbf{X} = \begin{pmatrix} 1 & -4 \\ -1 & 8 \end{pmatrix}$$

Your turn A6

$\mathbf{A} = \begin{pmatrix} 3 & 1 \\ 2 & 1 \end{pmatrix}$ and $\mathbf{XA} = \begin{pmatrix} 5 & 2 \\ 1 & 0 \end{pmatrix}$. Find $\mathbf{X}$

Worked example A7: non-singular for all k

Show that $\begin{pmatrix} k & 2 \\ -1 & k \end{pmatrix}$ is non-singular for all real k

$\det = k^2 + 2 \geqslant 2 > 0$

never zero, so always non-singular

Your turn A7

Show that $\begin{pmatrix} k & 1 \\ -3 & k+2 \end{pmatrix}$ is non-singular for all real k

B Fluency

Each question is a designed sequence. From one part to the next, ask yourself: what changed, and what did it do to the answer?

CORE *Secure the method*

B1 $\mathbf{A} = \begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 2 & 0 \\ 1 & -1 \end{pmatrix}$. Find

(a) $\mathbf{AB}$

(b) $\mathbf{BA}$

(c) $\mathbf{A}^2$

(d) $\mathbf{A} + 2\mathbf{B}$

[4 marks]

B2 $\mathbf{A}$ is 2×3, $\mathbf{B}$ is 3×2 and $\mathbf{C}$ is 2×2. State the order of each product, or explain why it does not exist.

(a) $\mathbf{AB}$

(b) $\mathbf{BA}$

(c) $\mathbf{AC}$

(d) $\mathbf{CA}$

[4 marks]

B3 Find the determinant of

(a) $\begin{pmatrix} 3 & 7 \\ 1 & 4 \end{pmatrix}$

(b) $\begin{pmatrix} -2 & 5 \\ 3 & -6 \end{pmatrix}$

(c) $\begin{pmatrix} k & 2 \\ 3 & k \end{pmatrix}$

(d) $\begin{pmatrix} a & b \\ 2a & 2b \end{pmatrix}$

[4 marks]

B4 Find the inverse of

(a) $\begin{pmatrix} 3 & 5 \\ 1 & 2 \end{pmatrix}$

(b) $\begin{pmatrix} 4 & 2 \\ 3 & 2 \end{pmatrix}$

(c) $\begin{pmatrix} -2 & 1 \\ 4 & -3 \end{pmatrix}$

(d) $\begin{pmatrix} 2 & 0 \\ 0 & 5 \end{pmatrix}$

[4 marks]

B5 Find the values of k for which each matrix is singular.

(a) $\begin{pmatrix} k & 4 \\ 1 & k \end{pmatrix}$

(b) $\begin{pmatrix} k-1 & 2 \\ 3 & k \end{pmatrix}$

(c) $\begin{pmatrix} 2k & 3 \\ k & 4 \end{pmatrix}$

[3 marks]

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B6 $\mathbf{A} = \begin{pmatrix} 2 & -1 \\ 3 & 1 \end{pmatrix}$

(a) Find $\mathbf{A}^{-1}$

(b) Use $\mathbf{A}^{-1}$ to solve $2x - y = 9$, $3x + y = 1$

(c) Find $\mathbf{X}$ with $\mathbf{AX} = \begin{pmatrix} 3 & 0 \\ 2 & 5 \end{pmatrix}$

(d) Find $\mathbf{Y}$ with $\mathbf{YA} = 5\mathbf{I}$

[4 marks]

STRETCH

B7 $\mathbf{A} = \begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix}$. Find

(a) $\det \mathbf{A}$

(b) $\det \mathbf{B}$

(c) $\det(\mathbf{AB})$

(d) $\det(3\mathbf{A})$

[4 marks]

B8 $\mathbf{A} = \begin{pmatrix} k & 2 \\ 3 & k-1 \end{pmatrix}$

(a) Find $\det \mathbf{A}$ in terms of k

(b) Find the values of k for which $\mathbf{A}$ is singular.

(c) Find the values of k for which $\det \mathbf{A} = 6$

(d) Find $\mathbf{A}^{-1}$ when $k = 4$

[4 marks]

B9 $\mathbf{A} = \begin{pmatrix} 3 & 2 \\ 4 & 3 \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$

(a) Find $\mathbf{X}$ with $\mathbf{AX} = \mathbf{C}$

(b) Find $\mathbf{Y}$ with $\mathbf{YA} = \mathbf{C}$

(c) Explain why $\mathbf{X} \neq \mathbf{Y}$

(d) Find $\mathbf{Z}$ with $\mathbf{AZA} = \mathbf{C}$

[4 marks]

B10 $\mathbf{M} = \begin{pmatrix} 3 & -4 \\ 2 & -3 \end{pmatrix}$

(a) Find $\mathbf{M}^2$

(b) Hence write down $\mathbf{M}^{-1}$

(c) Find $\mathbf{M}^5$

(d) Find $\mathbf{M}^{10}$

[4 marks]

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C Reasoning

Show your working and explain your thinking. A correct answer without a reason does not score full marks.

CORE *Secure the method*

C1 Amir writes $\det\begin{pmatrix}3 & -2\\ 4 & 1\end{pmatrix} = 3-8 = -5$. Explain his error, and find the determinant. **[2 marks]**

C2 Bea writes the inverse of $\begin{pmatrix}5 & 2\\ 2 & 1\end{pmatrix}$ as $\begin{pmatrix}5 & 2\\ 2 & 1\end{pmatrix}$, since the determinant is 1. Explain her error, and find the inverse. **[2 marks]**

C3 $\mathbf{A} = \begin{pmatrix}1 & 2\\ 0 & 1\end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix}3 & 0\\ 1 & 1\end{pmatrix}$. Asked for $\mathbf{AB}$, Cal writes $\begin{pmatrix}3 & 6\\ 1 & 3\end{pmatrix}$

Explain his error, and find $\mathbf{AB}$ **[3 marks]**

C4 Dev says that $\begin{pmatrix}k & 4\\ 9 & k\end{pmatrix}$ is singular only when $k = 6$

Explain why he is not fully correct. **[3 marks]**

STRETCH

C5 $\mathbf{A} = \begin{pmatrix}2 & 1\\ 1 & 1\end{pmatrix}$ and $\mathbf{XA} = \begin{pmatrix}4 & 3\\ 1 & 1\end{pmatrix}$. Eve finds $\mathbf{X} = \mathbf{A}^{-1}\begin{pmatrix}4 & 3\\ 1 & 1\end{pmatrix}$

Explain her error, and find $\mathbf{X}$ **[3 marks]**

C6 Fay says that if $\mathbf{AB} = \mathbf{0}$, then $\mathbf{A} = \mathbf{0}$ or $\mathbf{B} = \mathbf{0}$

Show that she is wrong using $\mathbf{A} = \begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix}1 & -1\\ -1 & 1\end{pmatrix}$ **[3 marks]**

C7 Gus says that a 2×2 matrix whose entries are all non-zero always has an inverse.

Give a counter-example, and explain what condition is needed. **[3 marks]**

C8 $\mathbf{A} = \begin{pmatrix}a & b\\ c & d\end{pmatrix}$

(a) Show that $\mathbf{A}^2 - (a+d)\mathbf{A} + (ad-bc)\mathbf{I} = \mathbf{0}$

(3)

(b) Hence show that, when $ad - bc \neq 0$, $\mathbf{A}^{-1} = \dfrac{1}{ad-bc}\left((a+d)\mathbf{I} - \mathbf{A}\right)$

(2)

[5 marks]

D Problem solving

You decide the method. Read the whole question first, and show every step.

CORE *Secure the method*

D1 The matrix $\mathbf{M}$ is given by $\mathbf{M} = \begin{pmatrix} x & 3 \\ 2 & x+1 \end{pmatrix}$, where x is a real number.

Find, in terms of x, the determinant of the matrix $\mathbf{M}$, and hence find the range of values of x for which the determinant of $\mathbf{M}$ is positive, showing your working clearly. **[3 marks]**

D2 The matrix $\mathbf{P}$ is given by $\mathbf{P} = \begin{pmatrix} p & 2 \\ -3 & p-1 \end{pmatrix}$, where p is a constant.

(a) Determine $\det \mathbf{P}$ in simplest form in terms of p

(1)

(b) Hence, by completing the square or otherwise, show that $\mathbf{P}$ is non-singular for all real values of p

(1)

(c) Determine $\mathbf{P}^{-1}$ in terms of p, giving your answer in its simplest form.

(2)

[4 marks]

D3 $\mathbf{M} = \begin{pmatrix} 1 & 2 \\ 3 & a \end{pmatrix}$, where a is a constant and $a \neq 6$. Given that $\mathbf{M}^{-1} = -\mathbf{M} + q\mathbf{I}$, where q is a constant and $\mathbf{I}$ is the 2×2 identity matrix, determine the value of a and the value of q. You should first find $\mathbf{M}^{-1}$ in terms of a, and then compare the entries of the two matrices. **[3 marks]**

D4 The matrix $\mathbf{A}$ is given by $\mathbf{A} = \begin{pmatrix} 2k-1 & 3 \\ k & 4 \end{pmatrix}$, where k is a constant.

(a) Determine the value of k for which $\mathbf{A}$ is singular.

(1)

Given that $k = 2$, so that $\mathbf{A}$ is non-singular,

(b) determine $\mathbf{A}^{-1}$, giving each entry as an exact fraction.

(2)

[3 marks]

STRETCH

D5 Given that $\mathbf{A} = \begin{pmatrix} 1 & k & 0 \\ 2 & 1 & -1 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} k & 1 \\ 1 & 0 \\ 3 & 2 \end{pmatrix}$, where k is a constant,

(a) determine the matrix $\mathbf{AB}$ in terms of k

(2)

(b) determine the value of k for which $\det(\mathbf{AB}) = 0$, and explain what this tells you about the matrix $\mathbf{AB}$ for this value of k

(2)

[4 marks]

D6 The matrices $\mathbf{B}$ and $\mathbf{C}$ are such that $\mathbf{B} = \begin{pmatrix} a & 3 \\ 2 & 1 \end{pmatrix}$ and $\mathbf{BC} = \begin{pmatrix} 1 & 4 \\ 3 & 2 \end{pmatrix}$, where a is a constant and $a \neq 6$

Determine, in terms of a, the matrix $\mathbf{C}$, giving your answer in the form $\frac{1}{a-6}\mathbf{N}$, where $\mathbf{N}$ is a 2×2 matrix whose entries are in terms of a

[4 marks]

D7 The matrix $\mathbf{M}$ is given by $\mathbf{M} = \begin{pmatrix} k & 2 \\ 3 & k \end{pmatrix}$, where k is a constant. Given that the matrix $\mathbf{M}$ maps the point with coordinates $(1, k)$ onto the point with coordinates $(12, 19)$,

(a) determine the value of k

(2)

(b) determine $\mathbf{M}^{-1}$ for this value of k

(2)

[4 marks]

D8 The matrices $\mathbf{A}$ and $\mathbf{B}$ are given by $\mathbf{A} = \begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 1 & -1 \\ 2 & -1 \end{pmatrix}$, and the matrix $\mathbf{R}$ is defined by $\mathbf{R} = \mathbf{AB}$

(a) Determine the matrix $\mathbf{R}$

(1)

(b) The matrix $3\mathbf{R}$ maps a point P to a point P'. Determine the matrix that maps P' back to P

(2)

(c) Given that P' is the point $(6, 9)$, determine the coordinates of P

(2)

[5 marks]

E Exam practice

Exam-style questions in the style of WFM01. Write your answers in the spaces, and show every step.

CORE *Exam questions for everyone*

E1 The matrix $\mathbf{A}$ is defined by

$$\mathbf{A} = \begin{pmatrix} k+1 & 4 \\ 2 & k-1 \end{pmatrix}$$

where k is a constant.

(a) Determine $\det \mathbf{A}$ in simplest form in terms of k

(1)

(b) Determine the values of k for which $\mathbf{A}$ is singular.

(2)

Given that $\mathbf{A}$ is non-singular,

(c) determine $\mathbf{A}^{-1}$ in terms of k, giving your answer in its simplest form.

(2)

(d) Hence write down the matrix $\mathbf{A}^{-1}$ in the case when $k = 4$

(1)

(Total for Question E1 is 6 marks)

E2 The matrix $\mathbf{M}$ is given by

$$\mathbf{M} = \begin{pmatrix} 1 & a \\ 2 & 4 \end{pmatrix}$$

where a is a constant and $a \neq 2$

(a) Determine $\mathbf{M}^{-1}$ in terms of a, giving your answer in its simplest form.

(3)

Given that $\mathbf{M}^{-1} = \frac{1}{2}\mathbf{M} + q\mathbf{I}$, where q is a constant and $\mathbf{I}$ is the 2×2 identity matrix,

(b) determine the value of a and the value of q, showing clearly how you compare the entries of the two sides.

(4)

(Total for Question E2 is 7 marks)

STRETCH

E3 The matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{pmatrix} 2 & -1 \\ 5 & -2 \end{pmatrix}$$

(a) Show that $\mathbf{A}^2 = -\mathbf{I}$, where $\mathbf{I}$ is the 2×2 identity matrix.

(2)

(b) Hence write down $\mathbf{A}^{-1}$ in terms of $\mathbf{A}$

(1)

(c) Determine the matrix $\mathbf{A}^{2027}$

(2)

(d) Using your answer to part (b), and showing all stages of your working, solve the simultaneous equations $2x - y = 3$ and $5x - 2y = 4$

(2)

(e) Determine the matrix $\mathbf{X}$ such that $\mathbf{XA} = \begin{pmatrix} 1 & 0 \\ 3 & 1 \end{pmatrix}$, explaining the order in which you multiply the matrices.

(2)

(Total for Question E3 is 9 marks)

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