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INTERNATIONAL A LEVEL MATHEMATICS • PURE 3 (WMA13)

Pure 3 Practice Paper 2

Mark Scheme

AIMING FOR A

Written in the style of the Pearson Edexcel International Advanced Level paper (WMA13/01) • 75 marks

Every question marked line by line the way an examiner marks it, with notes on the common errors

Mark every line, not just the answer. Method marks are given for a correct method even when the final answer is wrong. If you lost marks, find the *first* line where your working differs from the scheme. That is the thing to fix.

Code	Meaning
M1	Method mark: for knowing a correct method and attempting to apply it. Small slips in the working do not lose this mark.
dM1	Dependent method mark: can only be given if the method mark before it has been given.
A1	Accuracy mark: for a correct answer or line of working. It depends on the method mark before it.
B1	Independent mark: for a correct result or statement, whatever method was used.
*	The answer is given in the question. Every step must be shown, and the answer must follow from correct working (cso).
cao / cso	Correct answer only / correct solution only, with no errors anywhere in the working.
ft / oe	Follow through from your own earlier answer / or any correct equivalent form.

"Solutions relying on calculator technology are not acceptable." Questions 2, 5, 7 and 8 say this. There, a correct final answer with no algebraic working scores **no marks**.

Rough grade guide (out of 75): A* **60** • A **53** • B **47** • C **41**

Based on recent WMA13 grade boundaries. Real boundaries change every series, so treat this as a guide, not a grade.

Question paper **Exam questions by topic** **More practice papers**

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What to revise next

Write in your score for each question. Where you dropped marks, tap the chapter to practise ten more exam questions on it.

Q	What it tested	Marks	Practise it	Score
1	An inverse and its domain; sketching f and f^{-1}; a root by sign change	10	Ch 2 Functions and Graphs Ch 5 Exponentials and Logarithms Ch 8 Numerical Methods	
2	Proving a double angle form; solving in a doubled interval; an exact value of $\tan\frac{\pi}{12}$	8	Ch 4 Trig Addition Formulae Ch 3 Trigonometric Functions	
3	Reducing $R = am^n$ to a straight line; a rate of change in a model	8	Ch 5 Exponentials and Logarithms Ch 6 Differentiation	
4	Differentiating arctan through $\frac{dx}{dy}$; a normal; points with a given gradient	8	Ch 3 Trigonometric Functions Ch 6 Differentiation	
5	Spotting $\sec^2 x$ with a double angle formula; area between a curve and a line	8	Ch 4 Trig Addition Formulae Ch 7 Integration	
6	A stationary point in an exponential model; a sign change; a rate	8	Ch 5 Exponentials and Logarithms Ch 6 Differentiation Ch 8 Numerical Methods	
7	Modulus graph of $e^x - 4$; solving on each branch and checking	9	Ch 2 Functions and Graphs Ch 5 Exponentials and Logarithms	
8	Chain rule with sec and tan; integrating $\sec x$ by recognition	8	Ch 6 Differentiation Ch 7 Integration	
9	Simplifying a rational function; an inverse with two excluded values; quotient rule	8	Ch 1 Algebraic Methods Ch 2 Functions and Graphs Ch 6 Differentiation	
	Total	**75**		

Where did the marks go? If you lost the first marks of a question, the method needs work: go back to that chapter's key skills page. If you lost the final marks, the method is fine and the gap is accuracy, a domain or a rejected root. Paper 3 gives almost no help between the parts and adds a trap to most questions, so check every answer against the conditions in the question.

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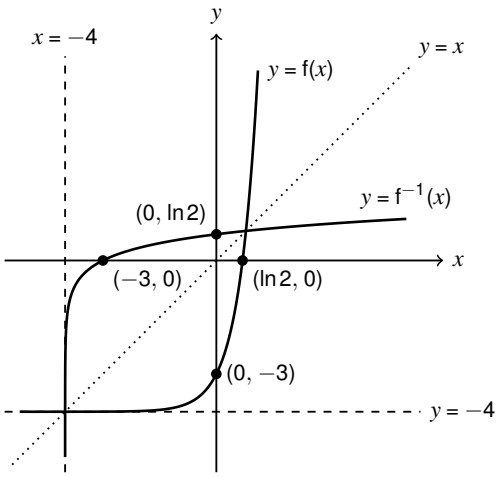
Question 1

10 marks

Part	Scheme	Marks
(a)	$y = e^{2x} - 4 \Rightarrow e^{2x} = y + 4 \Rightarrow x = \frac{1}{2}\ln(y+4)$ (attempts to change the subject) $\boxed{f^{-1}(x) = \frac{1}{2}\ln(x+4)}$ Domain $\boxed{x > -4}$ (the range of f) *The domain of f^{-1} is the range of f. $x \geqslant -4$ scores B0, since $\ln 0$ is undefined.*	**M1** **A1** **B1** **(3)**
(b)	 $y = f(x)$: exponential shape through $(0, -3)$ and $(\ln 2, 0)$ oe, such as $\left(\frac{1}{2}\ln 4, 0\right)$ $y = f^{-1}(x)$: logarithmic shape through $(-3, 0)$ and $(0, \ln 2)$ Asymptotes $y = -4$ for f and $x = -4$ for f^{-1}, the curves reflections of each other in $y = x$	**B1** **B1** **B1** **(3)**
(c)	$y = f^{-1}(x)$ is the reflection of $y = f(x)$ in $y = x$ (and f is increasing), so the curves meet on $y = x$: $f(\alpha) = \alpha$ $e^{2\alpha} - 4 = \alpha \Rightarrow e^{2x} - x - 4 = 0$ * *Setting $e^{2x} - 4 = \frac{1}{2}\ln(x+4)$ and stopping scores M0. The link to the line $y = x$ is the mark.*	**M1** **A1*** **(2)**
(d)	$g(x) = e^{2x} - x - 4$: $g(0.78245) = -0.00025\ldots$ and $g(0.78255) = 0.00060\ldots$ Change of sign and g is continuous, so $0.78245 < \alpha < 0.78255$, so $\alpha = 0.7825$ to 4 decimal places *The interval must be* $[0.78245, 0.78255]$. *Values at 0.7825 and 0.7826, or an iteration alone, score M0.*	**M1** **A1** **(2)**
		Total for Question 1: 10 marks

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Question 2

8 marks

Part	Scheme	Marks
(a)	$\frac{\cos\theta}{\sin\theta} - \frac{\sin\theta}{\cos\theta} = \frac{\cos^2\theta - \sin^2\theta}{\sin\theta\cos\theta}$	M1
	Uses $\cos 2\theta = \cos^2\theta - \sin^2\theta$ and $\sin 2\theta = 2\sin\theta\cos\theta$: $= \frac{\cos 2\theta}{\frac{1}{2}\sin 2\theta}$	M1
	$\boxed{2\cot 2\theta}$, so $k = 2$	A1
		(3)
(b)	$2\cot 2\theta = 4 \Rightarrow \tan 2\theta = \frac{1}{2}$	M1
	$2\theta = 0.4636\ldots$, so $\boxed{\theta = 0.232}$	A1
	$2\theta = \pi + 0.4636\ldots$, so $\boxed{\theta = 1.803}$, and no others for $0 < 2\theta < 2\pi$	A1
	The range for 2θ is $0 < 2\theta < 2\pi$. Stopping at $\theta = 0.232$ is the most common loss.	
		(3)
(c)	$\theta = \frac{\pi}{12}$, $t = \tan\frac{\pi}{12}$: $\frac{1}{t} - t = 2\cot\frac{\pi}{6} = 2\sqrt{3}$, so $t^2 + 2\sqrt{3}t - 1 = 0$	M1
	$t = -\sqrt{3} \pm 2$; $\tan\frac{\pi}{12} > 0$, so $\boxed{\tan\frac{\pi}{12} = 2 - \sqrt{3}}$	A1
	A1 needs the negative root $-2 - \sqrt{3}$ rejected with a reason. A calculator decimal (0.268) scores M0 A0.	
		(2)
		Total for Question 2: 8 marks

Question 3

8 marks

Part	Scheme	Marks
(a)	$\log_{10} R = \log_{10} a + n\log_{10} m$	M1
	Gradient $n = \frac{2.03 - 0.53}{2 - 0} = \boxed{0.75}$	B1
	Intercept: $\log_{10} a = 0.53$, so $a = 10^{0.53}$	M1
	$\boxed{a = 3.39}$	A1
	$a = 0.53$ (the intercept itself) and $a = e^{0.53}$ are the common errors and score M0 A0.	
		(4)
(b)	$R = 3.39 \times 70^{0.75} = \boxed{82.0}$ kJ per hour (accept 82.0 to 82.1)	B1
		(1)
(c)	$\frac{dR}{dm} = 0.75am^{-0.25}$	M1
	Sets = 1: $m^{0.25} = 0.75a$, so $m = (0.75a)^4$	M1
	$\boxed{m = 41.7}$ kg (accept 41.8 from $a = 3.39$)	A1
		(3)
		Total for Question 3: 8 marks

Question 4

8 marks

Part	Scheme	Marks
(a)	$x = 2\tan y \Rightarrow \dfrac{dx}{dy} = 2\sec^2 y$	M1
	Uses $\sec^2 y = 1 + \tan^2 y = 1 + \frac{x^2}{4}$, so $\dfrac{dx}{dy} = \dfrac{4+x^2}{2}$	M1
	$\boxed{\dfrac{dy}{dx} = \dfrac{2}{4+x^2}}$	A1
	An answer left in terms of y, such as $\frac{1}{2\sec^2 y}$, scores M1 M0 A0.	(3)
(b)	At $x = 2$: $y = \arctan 1 = \frac{\pi}{4}$ and gradient $= \frac{2}{8} = \frac{1}{4}$	M1
	Normal gradient -4: $y - \frac{\pi}{4} = -4(x-2)$	M1
	$y = 0$: $\boxed{x = 2 + \dfrac{\pi}{16}}$	A1
		(3)
(c)	$\dfrac{2}{4+x^2} = \dfrac{1}{5} \Rightarrow x^2 = 6$, so $x = \pm\sqrt{6}$	M1
	$\boxed{\left(\sqrt{6}, \arctan\frac{\sqrt{6}}{2}\right) \text{ and } \left(-\sqrt{6}, -\arctan\frac{\sqrt{6}}{2}\right)}$	A1
	Both points are needed, with exact y coordinates. $x = -\sqrt{6}$ is often lost.	(2)
		Total for Question 4: 8 marks

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Question 5

8 marks

Part	Scheme	Marks
(a)	$\dfrac{2}{1+\cos 2x} = 4 \Rightarrow \cos 2x = -\frac{1}{2}$	M1
	$2x = \frac{2\pi}{3}$, so $\boxed{x = \dfrac{\pi}{3}}$	A1
		(2)
(b)	Uses $\cos 2x = 2\cos^2 x - 1$, so $1 + \cos 2x = 2\cos^2 x$	M1
	$y = \dfrac{2}{2\cos^2 x} = \sec^2 x$	A1
	Area of $R = \displaystyle\int_0^{\frac{\pi}{3}} \left(4 - \sec^2 x\right) \mathrm{d}x$ (or rectangle $\frac{4\pi}{3}$ minus the area under C)	M1
	$\displaystyle\int \sec^2 x \, \mathrm{d}x = \tan x$	M1
	$[4x - \tan x]_0^{\frac{\pi}{3}}$ with both limits used	dM1
	$\boxed{\dfrac{4\pi}{3} - \sqrt{3}}$	A1
	Finding only the area under C ($\sqrt{3}$) and not subtracting it from the rectangle scores M1 A1 M0 M1 dM0 A0.	
		(6)
		Total for Question 5: 8 marks

Question 6

8 marks

Part	Scheme	Marks
(a)	$\dfrac{\mathrm{d}C}{\mathrm{d}t} = 12\left(-0.2\mathrm{e}^{-0.2t} + 0.8\mathrm{e}^{-0.8t}\right)$	M1
	Sets = 0: $0.8\mathrm{e}^{-0.8t} = 0.2\mathrm{e}^{-0.2t} \Rightarrow \mathrm{e}^{0.6t} = 4$	M1
	$\boxed{t = \dfrac{5}{3}\ln 4}$ oe, such as $\frac{10}{3}\ln 2$	A1
		(3)
(b)	Substitutes their t: $\mathrm{e}^{-0.2t} = 4^{-\frac{1}{3}}$, $\mathrm{e}^{-0.8t} = 4^{-\frac{4}{3}}$, so $C = 12 \times 4^{-\frac{1}{3}} \times \frac{3}{4}$	M1
	$\boxed{5.67}$ mg per litre	A1
		(2)
(c)	$C(8) = 2.40\ldots$ and $C(9) = 1.97\ldots$	M1
	$C - 2$ changes sign and C is continuous (and decreasing after M), so C falls below 2 between $t = 8$ and $t = 9$	A1
		(2)
(d)	$\boxed{-0.322}$ mg per litre per hour (the concentration is falling at 0.322 mg per litre per hour)	B1
		(1)
		Total for Question 6: 8 marks

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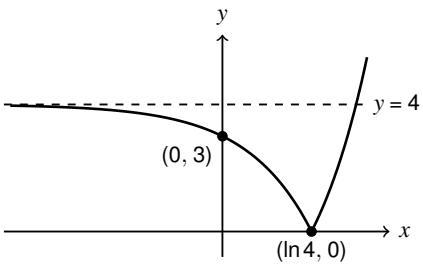
Question 7

9 marks

Part	Scheme	Marks
(a)	 A curve meeting the x-axis at a cusp, falling to the left of it and rising to the right	B1
	$(0, 3)$ and $(\ln 4, 0)$	B1
	Asymptote $\boxed{y = 4}$	B1
		(3)
(b)	$e^x - 4 = 3$ or $e^x - 4 = -3$, so $e^x = 7$ or $e^x = 1$	M1
	$\boxed{x = \ln 7 \text{ and } x = 0}$	A1
		(2)
(c)	$e^x > 4$: $e^x - 4 = 2e^{-x} \Rightarrow e^{2x} - 4e^x - 2 = 0 \Rightarrow e^x = 2 \pm \sqrt{6}$	M1
	$2 - \sqrt{6} < 0$ is rejected, and $2 + \sqrt{6} > 4$ is valid: $\boxed{x = \ln\left(2 + \sqrt{6}\right)}$	A1
	$e^x < 4$: $4 - e^x = 2e^{-x} \Rightarrow e^{2x} - 4e^x + 2 = 0 \Rightarrow e^x = 2 \pm \sqrt{2}$	M1
	Both are positive and less than 4: $\boxed{x = \ln\left(2 + \sqrt{2}\right) \text{ and } x = \ln\left(2 - \sqrt{2}\right)}$	A1
	Three solutions in total. The final A1 needs both answers from the second branch and no $\ln\left(2 - \sqrt{6}\right)$ *anywhere.*	
		(4)
	Total for Question 7: 9 marks	

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Question 8

8 marks

Part	Scheme	Marks
(a)	Chain rule: $\frac{dy}{dx} = \frac{\sec x \tan x + \sec^2 x}{\sec x + \tan x}$	M1
	Both derivatives correct	A1
	$= \frac{\sec x(\tan x + \sec x)}{\sec x + \tan x} = \boxed{\sec x}$	A1
		(3)
(b)	$\int_0^{\frac{\pi}{3}} \sec x \, dx = [\ln(\sec x + \tan x)]_0^{\frac{\pi}{3}} = \ln\left(2+\sqrt{3}\right) - \ln 1$	M1
	$\boxed{\ln\left(2+\sqrt{3}\right)}$	A1
		(2)
(c)	Expands and uses $\tan^2 x = \sec^2 x - 1$: $2\sec^2 x + 2\sec x \tan x - 1$	M1
	Integrates to $2\tan x + 2\sec x - x$	M1
	$\left(2\sqrt{3} + 4 - \frac{\pi}{3}\right) - (0 + 2 - 0) = \boxed{2\sqrt{3} + 2 - \frac{\pi}{3}}$	A1
	Integrating $(\sec x + \tan x)^2$ *as* $\frac{1}{3}(\sec x + \tan x)^3$ *scores M0.* $\sec 0 = 1$*, so the lower limit gives 2, not 0.*	
		(3)
	Total for Question 8: 8 marks	

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Question 9

8 marks

Part	Scheme	Marks
(a)	$\frac{(2x-1)(x+2)}{(x-2)(x+2)}$	M1
	$\boxed{f(x) = \frac{2x-1}{x-2}}$	A1
		(2)
(b)	$y = \frac{2x-1}{x-2} \Rightarrow xy - 2y = 2x - 1 \Rightarrow x(y-2) = 2y - 1$	M1
	$\boxed{f^{-1}(x) = \frac{2x-1}{x-2}}$	A1
	Domain: $\boxed{x \in \mathbb{R},\ x \neq 2,\ x \neq \frac{5}{4}}$	B1
	The domain of f^{-1} is the range of f. f never takes the value 2, and since $x = -2$ is excluded, f never takes the value $\frac{-5}{-4} = \frac{5}{4}$ either. $x \neq 2$ alone scores B0.	
		(3)
(c)	Quotient rule: $f'(x) = \frac{2(x-2) - (2x-1)}{(x-2)^2} = \frac{-3}{(x-2)^2}$	M1
	$(x-2)^2 = 1$, so $x = 1$ or $x = 3$	M1
	$\boxed{(1, -1) \text{ and } (3, 5)}$	A1
		(3)
		Total for Question 9: 8 marks

TOTAL FOR PAPER: 75 MARKS