eClassroom

INTERNATIONAL A LEVEL MATHEMATICS • FURTHER PURE MATHEMATICS 3

Further Pure 3

Chapter 2: Further Coordinate Systems

Exam Questions by Topic

Pearson Edexcel International Advanced Level (WFM03/01) • Calculator allowed

Multi-step exam questions at the difficulty of the real WFM03 paper, with a full mark scheme

Chapters (tap a chapter to jump to it)		Qs	Marks	Page	My score
1	**Hyperbolic Functions** Definitions, graphs, inverse functions, identities and equations	10	80	separate file	
2	**Further Coordinate Systems** Ellipses and hyperbolas, foci and directrices, tangents, normals and loci	10	80	2	
3	**Differentiation** Hyperbolic and inverse trigonometric and hyperbolic functions	10	80	separate file	
4	**Integration** Substitutions, reduction formulae, arc length and surface area	10	80	separate file	
5	**Vectors** Vector product, triple products, lines and planes, distances and angles	10	80	separate file	
6	**Further Matrix Algebra** 3×3 matrices, inverses, eigenvalues and eigenvectors, diagonalisation	10	80	separate file	

Full mark scheme | Free courses | More worksheets

Name ..... Class ..... Date .....

CHAPTER 2 • KEY SKILLS

2 Further Coordinate Systems

Specification FP3: Further coordinate systems • 10 questions • 80 marks • about 96 minutes

1 Standard forms

Ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, $(a\cos\theta, b\sin\theta)$.

Hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, $(a\cosh t, b\sinh t)$ or $(a\sec\theta, b\tan\theta)$

2 Eccentricity

Ellipse: $b^2=a^2\left(1-e^2\right)$, $0<e<1$.

Hyperbola: $b^2=a^2\left(e^2-1\right)$, $e>1$.

Take the positive root.

3 Foci and directrices

Foci $(\pm ae, 0)$, given as coordinates; directrices $x=\pm\frac{a}{e}$, given as equations.

4 Focus–directrix property

For any point P on the curve, $PS=e\times PM$, where M is the foot of the perpendicular from P to the directrix.

5 Asymptotes

The hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ has asymptotes $y=\pm\frac{b}{a}x$

6 Tangents

Differentiate parametrically, $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{d}y/\mathrm{d}\theta}{\mathrm{d}x/\mathrm{d}\theta}$, or implicitly, then use $y-y_1=m(x-x_1)$

7 Normals

Gradient $-\frac{1}{m}$; multiply out and collect into the form required, showing each step.

8 Intersections

Substitute one line into the other, keeping the x in equations such as $y=kx$

9 Loci

Find the coordinates of the moving point in terms of the parameter, then eliminate it using $\cos^2\theta+\sin^2\theta=1$ or $\cosh^2 t-\sinh^2 t=1$

Skills map every skill above is tested at least once; interleaved skills come from GCSE or other chapters

Skill	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Marks	6	6	6	6	6	10	10	10	10	10
1 Standard forms	●	●	●	●	●	●	●	●	●	●
2 Eccentricity	●	●			●	●			●	●
3 Foci and directrices	●	●				●			●	●
4 Focus–directrix property	●				●				●	●
5 Asymptotes		●					●			
6 Tangents			●			●	●			●
7 Normals				●		●		●		
8 Intersections				●		●	●	●		●
9 Loci					●			●		
Interleaved: parametric and implicit differentiation (P3, P4)			●	●		●	●	●		●
Interleaved: hyperbolic and trigonometric identities (Chapter 1, P3)				●			●	●		
Interleaved: distances, scalar product and areas (P1, P4)	●				●		●		●	●

Exam difficulty. Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WFM03 paper, where one part leads into the next and skills from different topics are combined.

Must learn. Learn which eccentricity formula goes with which curve. Give foci as coordinates and directrices as equations, and show every line when proving a printed tangent or normal.

★★★★★

Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

1. The ellipse E has equation $\frac{x^2}{25} + \frac{y^2}{16} = 1$

(a) Find the eccentricity of E.

(2)

(b) Write down the coordinates of the foci and the equations of the directrices of E.

(2)

(c) Using the focus–directrix property, show that for any point P on E the sum of the distances from P to the two foci is 10

(2)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 1 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

(Total for Question 1 is 6 marks)

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

2. The hyperbola H has equation $\frac{x^2}{9} - \frac{y^2}{16} = 1$

(a) Find the eccentricity of H.

(2)

(b) Find the coordinates of the foci and the equations of the directrices of H.

(2)

(c) Write down the equations of the asymptotes of H.

(2)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

Question 2 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

(Total for Question 2 is 6 marks)

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

3.

(a) Show that the tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at the point $(a\cos\theta, b\sin\theta)$ has equation $bx\cos\theta + ay\sin\theta = ab$

(4)

(b) Find the equation of the tangent to $\frac{x^2}{16} + \frac{y^2}{9} = 1$ at the point where $\theta = \frac{\pi}{3}$, giving it in the form $px + qy = r$

(2)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

Question 3 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

(Total for Question 3 is 6 marks)

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

4. The point $P(4\cosh t, 3\sinh t)$, $t \neq 0$, lies on the hyperbola $\frac{x^2}{16} - \frac{y^2}{9} = 1$

(a) Show that the normal to the hyperbola at P has equation $4x\sinh t + 3y\cosh t = 25\sinh t\cosh t$

(5)

(b) Find, in terms of t, the x-coordinate of the point where this normal meets the x-axis.

(1)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

Question 4 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

(Total for Question 4 is 6 marks)

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

5. The point $P(x, y)$ moves so that its distance from the point $(4, 0)$ is $\frac{2}{3}$ of its distance from the line $x = 9$

(a) Show that the locus of P is the ellipse $\frac{x^2}{36} + \frac{y^2}{20} = 1$

(5)

(b) Verify that the eccentricity of this ellipse is $\frac{2}{3}$

(1)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

Question 5 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

(Total for Question 5 is 6 marks)

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

6. The ellipse E has equation $\frac{x^2}{9} + \frac{y^2}{4} = 1$

(a) Find the exact eccentricity of E, and hence the coordinates of its foci and the equations of its directrices.

(4)

The point $P(3\cos\theta, 2\sin\theta)$ lies on E, where $0 < \theta < \frac{\pi}{2}$

(b) Show that the tangent to E at P has equation $2x\cos\theta + 3y\sin\theta = 6$

(3)

(c) The line l passes through the origin and is perpendicular to this tangent. Find an equation of l, and the x-coordinate of the point where l meets the tangent.

(3)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

Question 6 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

Question 6 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

(Total for Question 6 is 10 marks)

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

7.

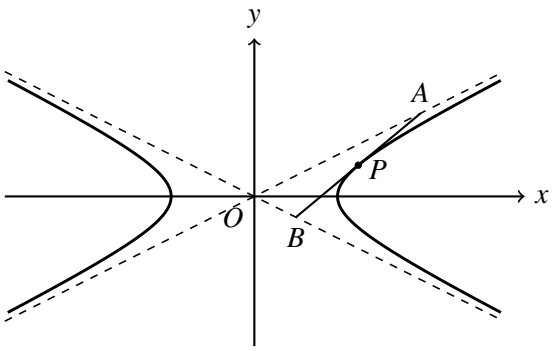

Figure 1

The hyperbola H has equation $\frac{x^2}{4} - y^2 = 1$. The point $P(2\sec\theta, \tan\theta)$, $0 < \theta < \frac{\pi}{2}$, lies on H. The tangent to H at P meets the asymptotes of H at A and B, as shown in Figure 1.

(a) Show that the tangent at P has equation $x\sec\theta - 2y\tan\theta = 2$

(4)

(b) Show that P is the midpoint of AB.

(4)

(c) Show that the area of triangle OAB is the same for every position of P, and state its value.

(2)

DO NOT WRITE IN THIS AREA

Question 7 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

Question 7 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

(Total for Question 7 is 10 marks)

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

8. The point $P(4\cos\theta, 3\sin\theta)$, $0 < \theta < \frac{\pi}{2}$, lies on the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$

(a) Show that the normal to the ellipse at P has equation $\frac{4x}{\cos\theta} - \frac{3y}{\sin\theta} = 7$

(4)

The normal meets the x-axis at A and the y-axis at B, and M is the midpoint of AB.

(b) Find the coordinates of A and B in terms of θ.

(2)

(c) Show that, as θ varies, M lies on the curve $64x^2 + 36y^2 = 49$

(4)

DO NOT WRITE IN THIS AREA

Question 8 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

Question 8 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

(Total for Question 8 is 10 marks)

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

9. The hyperbola H has equation $\frac{x^2}{16} - \frac{y^2}{9} = 1$

(a) Find the eccentricity of H, the coordinates of its foci S and S', where S has positive x-coordinate, and the equations of its directrices.

(3)

(b) The point P on H has x-coordinate 8 and positive y-coordinate. Using the focus–directrix property, find PS, and check your answer by finding the coordinates of P.

(3)

(c) Show that, for any point P on the branch of H with $x > 0$, $PS' - PS = 8$

(4)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 9 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

Question 9 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

(Total for Question 9 is 10 marks)

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

10.

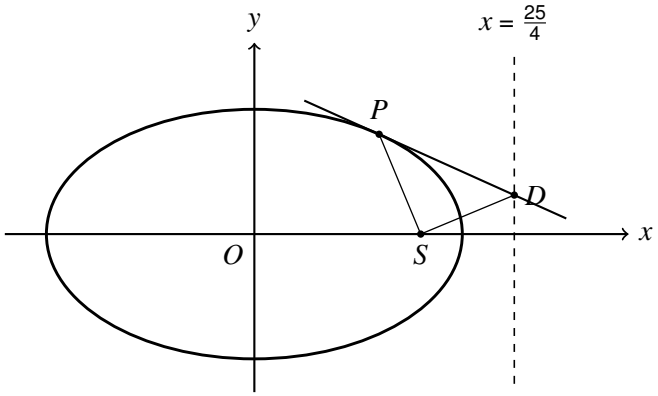

Figure 2

The ellipse E has equation $9x^2 + 25y^2 = 225$. The point $P(5\cos\theta, 3\sin\theta)$, $0 < \theta < \frac{\pi}{2}$, lies on E.

(a) Find the eccentricity of E, and write down the coordinates of the focus S with positive x-coordinate and the equation of the corresponding directrix.

(3)

(b) Show that the tangent to E at P has equation $3x\cos\theta + 5y\sin\theta = 15$

(3)

(c) The tangent meets the x-axis at T, and N is the foot of the perpendicular from P to the x-axis. Show that $OT \times ON = 25$

(2)

(d) The tangent meets the directrix at D, as shown in Figure 2. Show that angle PSD is a right angle.

(2)

DO NOT WRITE IN THIS AREA

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

Question 10 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

Question 10 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

(Total for Question 10 is 10 marks)

TOTAL FOR CHAPTER 2 IS 80 MARKS

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk