eClassroom

A-LEVEL MATHEMATICS • FURTHER PURE 3

Vectors

Worksheet

Pearson Edexcel International A Level (WFM03) • Calculator allowed: show every step

Free Vectors worksheet for A-Level and International A-Level Maths, with full worked solutions

A **Examples** & Your turn page 2

B **Fluency** core skills page 4

C **Reasoning** explain it page 6

D **Problem** solving page 7

E **Exam** practice page 9

Tap a section to jump straight to it

By the end of this worksheet you can: find vector products, areas and volumes • write lines and planes in several forms • *Stretch:* intersections, angles, shortest distances, feet of perpendiculars and reflections.

Which questions are for me?

CORE The methods every A-level student must secure. Start here.

STRETCH Vector and triple products, lines and planes in every form, intersections, angles, distances and reflections.

Every section has both, clearly labelled. Questions get harder as you go, and every skill in Sections A to D is needed in Section E.

Track your progress

Section	B	C	D	E
Core marks	23	10	13	13
Stretch marks	16	14	17	9
My score				

Full worked solutions | Exam questions by topic | Further Pure 3 exam questions | Formula sheet builder

Name ..... Class ..... Date .....

A Worked examples & Your turn

Read each example carefully. Cover it up, then answer the question beside it using the same method.

CORE *Secure the method*

Worked example A1: a vector product

Find $\begin{pmatrix}1\\2\\3\end{pmatrix} \times \begin{pmatrix}2\\-1\\1\end{pmatrix}$

$(2\times1-3\times(-1),\ 3\times2-1\times1,\ 1\times(-1)-2\times2)$

$$\boxed{\begin{pmatrix}5\\5\\-5\end{pmatrix}}$$

Your turn A1

Find $\begin{pmatrix}3\\0\\1\end{pmatrix} \times \begin{pmatrix}1\\2\\-2\end{pmatrix}$

Worked example A2: a triangle's area

Find the area of triangle ABC with $A(1,0,2)$, $B(3,1,2)$, $C(1,2,0)$

$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{pmatrix}2\\1\\0\end{pmatrix} \times \begin{pmatrix}0\\2\\-2\end{pmatrix} = \begin{pmatrix}-2\\4\\4\end{pmatrix}$, *length 6*

$$\boxed{3}$$

Your turn A2

Find the area of triangle ABC with $A(2,1,0)$, $B(4,1,1)$, $C(2,4,1)$

Worked example A3: a plane

Find the Cartesian equation of the plane through $(1,2,-1)$ with normal $\begin{pmatrix}3\\-1\\2\end{pmatrix}$

$3-2-2=-1$

$$\boxed{3x-y+2z=-1}$$

Your turn A3

Find the Cartesian equation of the plane through $(2,0,5)$ with normal $\begin{pmatrix}1\\4\\-1\end{pmatrix}$

STRETCH

Worked example A4: a triple product

Find the volume of the tetrahedron with edges $\begin{pmatrix}1\\0\\2\end{pmatrix}$, $\begin{pmatrix}2\\1\\0\end{pmatrix}$, $\begin{pmatrix}0\\3\\2\end{pmatrix}$ from one vertex.

$$\begin{pmatrix}1\\0\\2\end{pmatrix}\cdot\left(\begin{pmatrix}2\\1\\0\end{pmatrix}\times\begin{pmatrix}0\\3\\2\end{pmatrix}\right)=\begin{pmatrix}1\\0\\2\end{pmatrix}\cdot\begin{pmatrix}2\\-4\\6\end{pmatrix}=14$$

$\frac{14}{6}=\frac{7}{3}$

Your turn A4

Find the volume of the tetrahedron with edges $\begin{pmatrix}2\\1\\1\end{pmatrix}$, $\begin{pmatrix}1\\3\\0\end{pmatrix}$, $\begin{pmatrix}1\\0\\4\end{pmatrix}$ from one vertex.

Worked example A5: the form $\mathbf{r}\times\mathbf{b}=\mathbf{a}\times\mathbf{b}$

Write the line through $(1,2,3)$ with direction $\begin{pmatrix}2\\0\\1\end{pmatrix}$ in the form $\mathbf{r}\times\mathbf{b}=\mathbf{c}$

$$\mathbf{r}\times\begin{pmatrix}2\\0\\1\end{pmatrix}=\begin{pmatrix}2\\5\\-4\end{pmatrix}$$

Your turn A5

Write the line through $(0,1,-1)$ with direction $\begin{pmatrix}1\\1\\2\end{pmatrix}$ in the form $\mathbf{r}\times\mathbf{b}=\mathbf{c}$

Worked example A6: a line meets a plane

Find where $\mathbf{r}=\begin{pmatrix}1\\0\\2\end{pmatrix}+\lambda\begin{pmatrix}1\\1\\-1\end{pmatrix}$ meets $2x+y+z=10$

$2(1+\lambda)+\lambda+(2-\lambda)=10,\ \lambda=3$

$(4,3,-1)$

Your turn A6

Find where $\mathbf{r}=\begin{pmatrix}2\\-1\\0\end{pmatrix}+\lambda\begin{pmatrix}1\\2\\3\end{pmatrix}$ meets $x-y+z=9$

Worked example A7: point to plane

Find the distance from $(3,1,2)$ to $2x-y+2z=3$

$$\frac{|6-1+4-3|}{3}$$

2

Your turn A7

Find the distance from $(1,-2,4)$ to $x+2y-2z=1$

B Fluency

Each question is a designed sequence. From one part to the next, ask yourself: what changed, and what did it do to the answer?

CORE *Secure the method*

B1 Find

(a) $\mathbf{i} \times \mathbf{j}$

(b) $\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$

(c) $\begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} \times \begin{pmatrix} 4 \\ -2 \\ 6 \end{pmatrix}$, and explain the result

(d) $\left| \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} \times \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix} \right|$

[4 marks]

B2 $P(1,1,1)$, $Q(2,3,1)$, $R(1,2,4)$

(a) Find $\overrightarrow{PQ}$ and $\overrightarrow{PR}$

(b) Find $\overrightarrow{PQ} \times \overrightarrow{PR}$

(c) Find the area of triangle PQR

(d) Find the area of parallelogram $PQSR$

[4 marks]

B3 The plane through $A(1,0,1)$, $B(2,1,3)$, $C(0,2,1)$

(a) Find $\overrightarrow{AB}$ and $\overrightarrow{AC}$

(b) Find a normal vector.

(c) Find the Cartesian equation.

(d) Write it in the form $\mathbf{r} \cdot \mathbf{n} = d$

[4 marks]

B4 The line through $A(2,-1,3)$ and $B(4,0,1)$

(a) Find a direction vector.

(b) Write a vector equation.

(c) Write the Cartesian equations.

(d) Write it as $\mathbf{r} \times \mathbf{b} = \mathbf{c}$

[4 marks]

B5 Find the acute angle between

(a) the planes $x+y+z=1$ and $x-y=3$

(b) the line with direction $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ and the plane $z=4$

(c) the planes $2x+y+2z=1$ and $x+2y+2z=4$

[3 marks]

B6 The plane $2x+6y-3z=14$

(a) Find its distance from O

(b) Show that $(1,2,0)$ lies on it.

(c) Find its distance from $(3,5,-2)$

(d) Find the foot of the perpendicular from $(3,5,-2)$

[4 marks]

STRETCH

B7 O, $A(1,2,0)$, $B(0,1,3)$, $C(2,1,1)$

(a) Find $\overrightarrow{OB}\times\overrightarrow{OC}$

(b) Find $\overrightarrow{OA}\cdot\left(\overrightarrow{OB}\times\overrightarrow{OC}\right)$

(c) Find the volume of tetrahedron $OABC$

(d) Find k so that O, A, B and $(2,1,k)$ are coplanar.

[4 marks]

B8 $\mathbf{r}=\begin{pmatrix}1\\2\\0\end{pmatrix}+\lambda\begin{pmatrix}2\\-1\\2\end{pmatrix}$ and the plane $x+2y+2z=21$

(a) Find where they meet.

(b) Find sin of the angle between them.

(c) Find the acute angle.

(d) Find the distance from $(1,2,0)$ to the plane.

[4 marks]

B9 $\Pi_1:\ x+y-z=2$ and $\Pi_2:\ 2x-y+z=1$

(a) Find $\mathbf{n}_1\times\mathbf{n}_2$

(b) Find a point on both planes.

(c) Write the line of intersection.

(d) Find the angle between the planes.

[4 marks]

B10 $l_1:\ \mathbf{r}=\begin{pmatrix}1\\0\\0\end{pmatrix}+s\begin{pmatrix}1\\1\\0\end{pmatrix}$ and $l_2:\ \mathbf{r}=\begin{pmatrix}0\\1\\3\end{pmatrix}+t\begin{pmatrix}0\\1\\1\end{pmatrix}$

(a) Find $\mathbf{b}_1\times\mathbf{b}_2$

(b) Find $\mathbf{a}_2-\mathbf{a}_1$

(c) Find the shortest distance between the lines.

(d) Explain why the lines are not parallel.

[4 marks]

★★★★★
Join students already acing their exams.
Book your FREE lesson at **www.eclassroom.org.uk**

C Reasoning

Show your working and explain your thinking. A correct answer without a reason does not score full marks.

CORE *Secure the method*

C1 Amir says $\mathbf{j} \times \mathbf{i} = \mathbf{k}$ because $\mathbf{i} \times \mathbf{j} = \mathbf{k}$. Explain his error. **[2 marks]**

C2 For a line and a plane, Bea finds $\frac{|\mathbf{b} \cdot \mathbf{n}|}{|\mathbf{b}||\mathbf{n}|} = \frac{4}{9}$ and gives the angle as $\arccos\frac{4}{9} = 63.6°$. Explain her error.

[2 marks]

C3 To find where $\mathbf{r} = \begin{pmatrix}1\\1\\1\end{pmatrix} + \lambda\begin{pmatrix}1\\2\\0\end{pmatrix}$ meets $x - 3y + z = 4$, Cal solves $(1+\lambda) - 3(1+2\lambda) + 1 = 0$

Explain his error, and find the point. **[3 marks]**

C4 For the angle between the line with direction $\begin{pmatrix}1\\1\\1\end{pmatrix}$ and the plane $\mathbf{r} = \lambda\begin{pmatrix}1\\0\\1\end{pmatrix} + \mu\begin{pmatrix}0\\1\\0\end{pmatrix}$, Dev uses $\begin{pmatrix}1\\0\\1\end{pmatrix}$ in place of a normal.

Explain his error, and find the angle. **[3 marks]**

STRETCH

C5 Asked for a vector equation of a line, Eve writes $l_2 = \begin{pmatrix}1\\2\\3\end{pmatrix} + t\begin{pmatrix}1\\0\\1\end{pmatrix}$

Explain why this is not acceptable, and correct it. **[3 marks]**

C6 Fay gives the volume of a tetrahedron as $|\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})| = 12$

Explain her error, and give the volume. **[3 marks]**

C7 Gus gives the distance from $(2, 1, 1)$ to $2x - 2y + z = 9$ as $|4 - 2 + 1 - 9| = 6$

Explain his error, and correct it. **[3 marks]**

C8 **(a)** Show that $\mathbf{a} \cdot (\mathbf{a} \times \mathbf{b}) = 0$ for $\mathbf{a} = \begin{pmatrix}a_1\\a_2\\a_3\end{pmatrix}$, $\mathbf{b} = \begin{pmatrix}b_1\\b_2\\b_3\end{pmatrix}$

(2)

(b) Show that the distance from a point P to the line $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$ is $\frac{|(\mathbf{p} - \mathbf{a}) \times \mathbf{b}|}{|\mathbf{b}|}$

(3)

[5 marks]

D Problem solving

You decide the method. Read the whole question first, and show every step.

CORE *Secure the method*

D1 The plane Π has vector equation

$$\mathbf{r} = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$$

where λ and μ are scalar parameters.

(a) Determine a vector perpendicular to Π, showing your use of the vector product

(1)

(b) Hence determine a Cartesian equation of Π, giving your answer in the form $ax + by + cz = d$, where a, b, c and d are integers, and check that your equation is satisfied by the point with position vector $\begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$

(2)

[3 marks]

D2 The plane Π has equation $x + 2y - 2z = 5$

The line l passes through the point $A(4, 3, 6)$ and meets Π at the point $B(1, 2, 0)$. The acute angle between l and Π is θ

(a) Determine, to one decimal place, the value of θ, making clear which vectors you use

(2)

(b) Determine the exact shortest distance from A to Π, and use your answer to check your value of θ using the length AB

(2)

[4 marks]

D3 *In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.*

The planes Π_1, Π_2 and Π_3 have equations

$$x + 2y + z = 3, \qquad 3x - y + 2z = 13, \qquad 2x + y - 3z = -6$$

respectively. By first finding the line of intersection of two of the planes, or otherwise, determine the coordinates of the single point that lies on all three planes, showing all your working. **[3 marks]**

D4 The points A, B, C and D have coordinates $(1, 0, 0)$, $(0, 2, 0)$, $(0, 0, 3)$ and $(2, 2, 2)$ respectively, and they are the vertices of a tetrahedron.

Use the scalar triple product of the vectors $\overrightarrow{AB}$, $\overrightarrow{AC}$ and $\overrightarrow{AD}$ to determine the exact volume of the tetrahedron $ABCD$, giving your answer as a fraction in its simplest form. **[3 marks]**

STRETCH

D5 The planes Π_1 and Π_2 have equations $x+y+z=3$ and $x-y+2z=2$ respectively, and they intersect in the line l. The point with coordinates $(4,0,-1)$ lies on both planes.

(a) Determine an equation for l, giving your answer in the form $(\mathbf{r}-\mathbf{a})\times\mathbf{b}=\mathbf{0}$, where $\mathbf{a}$ and $\mathbf{b}$ are constant vectors.

(2)

(b) Determine the exact shortest distance from the origin O to the line l, giving your answer as a surd in simplest form.

(2)

[4 marks]

D6 The plane Π has equation $2x-y+2z=1$ and the point P, which does not lie on Π, has coordinates $(6,-2,7)$

(a) By considering the line through P that is perpendicular to Π, determine the coordinates of the foot of the perpendicular from P to Π

(3)

(b) Hence write down the coordinates of the reflection of P in Π, explaining how you obtain it from your answer to part (a)

(1)

[4 marks]

D7 The points $A(1,1,0)$, $B(3,1,1)$, $C(1,4,2)$ and $D(2,k,3)$, where k is a constant, are the vertices of a tetrahedron.

Given that the volume of the tetrahedron is 2 cubic units, use the scalar triple product to determine the possible values of k, explaining why there are two values. **[4 marks]**

D8 The plane Π has equation $x+y+z=6$, and O is the origin.

The line l_1 passes through the points $A(1,2,0)$ and $B(2,2,2)$, where A does not lie on Π

(a) Show that B lies on Π

(1)

(b) Determine the coordinates of the reflection of A in Π

(2)

(c) The line l_2 is the reflection of l_1 in Π. Determine a vector equation for l_2, giving your answer in the form $\mathbf{r}=\mathbf{a}+\mu\mathbf{b}$, where $\mathbf{a}$ and $\mathbf{b}$ are constant vectors.

(2)

[5 marks]

E Exam practice

Exam-style questions in the style of WFM03. Write your answers in the spaces, and show every step.

CORE *Exam questions for everyone*

E1 The plane Π has vector equation

$$\mathbf{r} = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix}$$

where λ and μ are scalar parameters.

(a) Determine a Cartesian equation for Π, giving your answer in the form $ax + by + cz = d$, where a, b, c and d are integers.

(2)

(b) Determine the exact shortest distance from the point $Q(4, -3, 9)$ to Π

(2)

The line l passes through the point Q and is parallel to the vector $\mathbf{i} + \mathbf{j} + \mathbf{k}$. The acute angle between l and Π is α

(c) Determine, to one decimal place, the value of α, making clear which angle you have calculated.

(2)

(Total for Question E1 is 6 marks)

E2 The plane Π_1 has equation $x - y + 2z = 3$ and the plane Π_2 has equation $2x + y - z = 1$

The planes intersect in the line l. The point A, with coordinates $(1, 0, 1)$, lies on both planes.

(a) Determine a vector equation for l, giving your answer in the form $\mathbf{r} = \mathbf{a} + t\mathbf{b}$, where $\mathbf{a}$ and $\mathbf{b}$ are constant vectors and t is a scalar parameter.

(4)

(b) Determine, to one decimal place, the acute angle between the planes Π_1 and Π_2, showing clearly how you obtain it from the normal vectors of the planes.

(3)

(Total for Question E2 is 7 marks)

STRETCH

E3 The line l_1 has equation $\mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} + s\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$, where s is a scalar parameter.

The line l_2 passes through the points $A(3, 0, 1)$ and $B(4, 2, 1)$

(a) By writing down a vector equation for l_2 and considering the components, show that l_1 and l_2 do not intersect, and explain why they are not parallel.

(3)

(b) Determine the exact shortest distance between the lines l_1 and l_2, giving your answer as a fraction.

(3)

(c) The plane Π contains l_1 and is parallel to l_2. Determine a Cartesian equation of Π, and use it to confirm your answer to part (b).

(3)

(Total for Question E3 is 9 marks)

Want more exam practice? The **Exam Questions by Topic** booklets have 10 more full exam questions on Further Pure 3 Chapter 5, Vectors, with a full mark scheme. **Open the booklet**

Finished? Mark your work with the **full worked solutions**, fill in your scores on the front page, then practise real exam questions on the **Further Pure 3 exam questions page**.