eClassroom

A-LEVEL MATHEMATICS • MECHANICS 2

Variable Acceleration

Worksheet

Pearson Edexcel International A Level (WME02) • Calculator allowed: take $g = 9.8\,\text{m}\,\text{s}^{-2}$ and give answers to 2 or 3 s.f.

Free Variable Acceleration worksheet for A-Level and International A-Level Maths, with full worked solutions

A **Examples** & Your turn page 2

B **Fluency** core skills page 4

C **Reasoning** explain it page 6

D **Problem** solving page 7

E **Exam** practice page 10

Tap a section to jump straight to it

By the end of this worksheet you can: differentiate and integrate to move between displacement, velocity and acceleration • use initial conditions • find when a particle is at rest and the distance it travels • work with velocity and acceleration vectors • *Stretch:* greatest and least values, piecewise motion, and directions given by vectors.

Which questions are for me?

CORE The methods every A-level student must secure. Start here.

STRETCH Fractional powers, velocities in pieces, directions from vectors, and exact answers without a calculator.

Every section has both, clearly labelled. Questions get harder as you go, and every skill in Sections A to D is needed in Section E.

Track your progress

Section	B	C	D	E
Core marks	23	10	13	19
Stretch marks	16	14	17	12
My score				

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Name ..... Class ..... Date .....

A Worked examples & Your turn

Read each example carefully. Cover it up, then answer the question beside it using the same method.

CORE *Secure the method*

Worked example A1: differentiating

A particle moves along a straight line. At time t seconds its displacement from O is x metres, where $x = 2t^3 - 15t^2 + 36t$. Find its velocity and acceleration when $t = 1$

$v = \frac{dx}{dt}$, then $a = \frac{dv}{dt}$

$v = 6t^2 - 30t + 36$; $a = 12t - 30$

$v = 12\,\text{m s}^{-1}$, $a = -18\,\text{m s}^{-2}$

Your turn A1

A particle moves along a straight line. At time t seconds its displacement from O is x metres, where $x = t^3 - 6t^2 + 5t$. Find its velocity and acceleration when $t = 3$

Worked example A2: integrating

A particle starts from O with velocity $2\,\text{m s}^{-1}$. At time t seconds its acceleration is $(4 - 6t)\,\text{m s}^{-2}$. Find its velocity and displacement from O when $t = 2$

Integrate, then use $v = 2$ and $x = 0$ when $t = 0$

$v = 4t - 3t^2 + 2$; $x = 2t^2 - t^3 + 2t$

$v = -2\,\text{m s}^{-1}$, $x = 4\,\text{m}$

Your turn A2

At $t = 0$ a particle is $5\,\text{m}$ from O with velocity $-3\,\text{m s}^{-1}$. Its acceleration is $(6t - 4)\,\text{m s}^{-2}$. Find its velocity and its displacement from O when $t = 3$

Worked example A3: at rest

A particle has velocity $v = t^2 - 7t + 10\,\text{m s}^{-1}$ at time t seconds. Find the times when it is at rest, and its acceleration at each of these times.

At rest: $v = 0$. Then $a = 2t - 7$

$(t - 2)(t - 5) = 0$

$t = 2$: $a = -3\,\text{m s}^{-2}$; $t = 5$: $a = 3\,\text{m s}^{-2}$

Your turn A3

A particle has velocity $v = 2t^2 - 9t + 4\,\text{m s}^{-1}$ at time t seconds. Find the times when it is at rest, and its acceleration at each of these times.

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STRETCH

Worked example A4: distance travelled

A particle starts at O with velocity $v = 3t^2 - 12t + 9\,\text{m s}^{-1}$. Find the distance it travels in the first 4 seconds.

Rest at $t = 1$, $t = 3$. $x = t^3 - 6t^2 + 9t$

$x(1) = 4$, $x(3) = 0$, $x(4) = 4$; $4 + 4 + 4$

$12\,\text{m}$

Your turn A4

A particle starts at O with velocity $v = 6t^2 - 30t + 24\,\text{m s}^{-1}$. Find the distance it travels in the first 5 seconds.

Worked example A5: greatest speed

For $0 \leqslant t \leqslant 5$, a particle has velocity $v = 24t - 6t^2\,\text{m s}^{-1}$. Find its greatest velocity and its greatest speed.

$a = 0$ at $t = 2$; check the end points too

$v(2) = 24$; $v(5) = -30$

$24\,\text{m s}^{-1}$; $30\,\text{m s}^{-1}$ (at $t = 5$)

Your turn A5

For $0 \leqslant t \leqslant 8$, a particle has velocity $v = 15 + 4t - t^2\,\text{m s}^{-1}$. Find its greatest velocity and its greatest speed.

Worked example A6: vectors

A particle has position vector $\mathbf{r} = (t^3 - 3t)\mathbf{i} + (4t^2 - 2t)\mathbf{j}$ m. Find its velocity when it is moving parallel to $\mathbf{j}$, and the magnitude of its acceleration then.

Parallel to $\mathbf{j}$: the $\mathbf{i}$ part of $\mathbf{v}$ is 0

$\mathbf{v} = (3t^2 - 3)\mathbf{i} + (8t - 2)\mathbf{j}$, $t = 1$; $\mathbf{a} = 6\mathbf{i} + 8\mathbf{j}$

$6\mathbf{j}\,\text{m s}^{-1}$, $10\,\text{m s}^{-2}$

Your turn A6

A particle has position vector $\mathbf{r} = (2t^2 - 6t)\mathbf{i} + (t^3 - 12t)\mathbf{j}$ m. Find its velocity when it is moving parallel to $\mathbf{i}$, and the magnitude of its acceleration then.

Worked example A7: vector constants

A particle at O has velocity $(2\mathbf{i} + 5\mathbf{j})\,\text{m s}^{-1}$ when $t = 0$. Its acceleration is $(6t\mathbf{i} - 4\mathbf{j})\,\text{m s}^{-2}$. Find its distance from O when $t = 2$

The constants are vectors

$\mathbf{v} = (3t^2 + 2)\mathbf{i} + (5 - 4t)\mathbf{j}$; $\mathbf{r} = (t^3 + 2t)\mathbf{i} + (5t - 2t^2)\mathbf{j}$

$\mathbf{r} = 12\mathbf{i} + 2\mathbf{j}$, $12.2\,\text{m}$

Your turn A7

At $t = 0$ a particle is at $(\mathbf{i} - 2\mathbf{j})$ m with velocity $(-3\mathbf{i} + \mathbf{j})\,\text{m s}^{-1}$. Its acceleration is $(2\mathbf{i} + (6t - 6)\mathbf{j})\,\text{m s}^{-2}$. Find its distance from O when $t = 3$

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B Fluency

Each question is a designed sequence. From one part to the next, ask yourself: what changed, and what did it do to the answer?

CORE *Secure the method*

B1 A particle moves along a straight line so that its displacement from O at time t seconds is $x = t^3 - 6t^2 + 9t + 2$ metres.

(a) Find v in terms of t

(b) Find a in terms of t

(c) Find the times when the particle is at rest.

(d) Find a at each of these times.

[4 marks]

B2 A particle has acceleration $6t\,\text{m s}^{-2}$ at time t seconds. Find v in terms of t when

(a) $v = 0$ when $t = 0$

(b) $v = 4$ when $t = 0$

(c) $v = 4$ when $t = 1$

(d) Find x when $x = 0$ at $t = 0$ and v is as in part (b).

[4 marks]

B3 A particle has velocity $v = t^2 - 5t + 6\,\text{m s}^{-1}$ at time t seconds.

(a) Find the times when it is at rest.

(b) State the interval of time when it moves in the negative direction.

(c) Find its acceleration when $t = 2.5$

(d) Find its least velocity.

[4 marks]

B4 For $0 \leqslant t \leqslant 8$, the displacement of a particle from O is $x = 9t^2 - t^3$ metres.

(a) Find its greatest velocity.

(b) Find its greatest displacement from O

(c) Find its displacement when $t = 8$

(d) Find its velocity when $t = 8$

[4 marks]

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CORE *continued: distance and vectors*

B5 A particle starts from O with velocity $v = 3t^2 - 6t\,\text{m s}^{-1}$

(a) Find its displacement from O when $t = 3$

(b) Find the distance it travels in the first 3 seconds.

(c) Find the distance it travels in the first 4 seconds. **[3 marks]**

B6 A particle has position vector $\mathbf{r} = (t^2 + 1)\mathbf{i} + (t^3 - 2t)\mathbf{j}$ metres at time t seconds.

(a) Find $\mathbf{v}$ in terms of t

(b) Find $\mathbf{a}$ in terms of t

(c) Find its speed when $t = 1$

(d) Find the size of its acceleration when $t = 1$

[4 marks]

STRETCH

B7 A particle moves from O so that its displacement at time t seconds is $x = 8t^{\frac{3}{2}} - 3t^2$ metres, $t \geqslant 0$

(a) Find v in terms of t

(b) Find the value of $t > 0$ when it is at rest.

(c) Show that $a = 0$ when $t = 1$, and find its greatest velocity.

(d) Find its displacement when it is at rest.

[4 marks]

B8 A particle has velocity $\mathbf{v} = (t^2 - 4)\mathbf{i} + 3t\mathbf{j}\,\text{m s}^{-1}$ at time t seconds, $t \geqslant 0$. Find the time when it is moving parallel to

(a) $\mathbf{j}$

(b) $\mathbf{i} - \mathbf{j}$

(c) $4\mathbf{i} + 3\mathbf{j}$, giving your answer exactly

(d) Find its speed when it moves parallel to $\mathbf{i} - \mathbf{j}$

[4 marks]

B9 A particle at O has velocity $(\mathbf{i} + 3\mathbf{j})\,\text{m s}^{-1}$ when $t = 0$. Its acceleration is $(2\mathbf{i} - 6t\mathbf{j})\,\text{m s}^{-2}$

(a) Find $\mathbf{v}$ in terms of t

(b) Find $\mathbf{r}$ in terms of t

(c) Find the time when it moves parallel to $\mathbf{i}$

(d) Find its position vector at that time.

[4 marks]

B10 A particle starts from O with velocity $v = t^3 - 6t^2 + 8t\,\text{m s}^{-1}$

(a) Find the times when it is at rest.

(b) Find its displacement when $t = 2$

(c) Find its displacement when $t = 4$

(d) Find the distance it travels in the first 5 seconds.

[4 marks]

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C Reasoning

Show your working and explain your thinking. A correct answer without a reason does not score full marks.

CORE *Secure the method*

C1 A particle has acceleration $4t\,\text{m s}^{-2}$ and velocity $3\,\text{m s}^{-1}$ when $t = 0$. Amir writes $v = 2t^2$

Explain his error, and write the correct expression. **[2 marks]**

C2 A particle starts from rest with acceleration $6t\,\text{m s}^{-2}$. To find its speed when $t = 2$, Bea writes $v = u + at = 0 + (6 \times 2) \times 2 = 24$

Explain her error, and find the correct speed. **[2 marks]**

C3 A particle starts at O with velocity $v = 3t^2 - 18t + 15\,\text{m s}^{-1}$. Cal says that, since $x = -25$ when $t = 5$, the particle travels 25 m in the first 5 seconds.

Explain his error, and find the distance travelled. **[3 marks]**

C4 A particle has position vector $\mathbf{r} = (t^2 - 4t)\mathbf{i} + (2t + 1)\mathbf{j}$ m. Dev says it moves parallel to $\mathbf{j}$ when $t = 4$, because the $\mathbf{i}$ part of $\mathbf{r}$ is then zero.

Explain his error, and find the correct time. **[3 marks]**

STRETCH

C5 A particle has velocity $\mathbf{v} = (5t - 2)\mathbf{i} + (t^2 + 2)\mathbf{j}\,\text{m s}^{-1}$. To find when it moves parallel to $\mathbf{i} + \mathbf{j}$, Eve writes $5t - 2 = 1$ and $t^2 + 2 = 1$

Explain her error, and find the correct times. **[3 marks]**

C6 For $0 \leqslant t \leqslant 5$, a particle has velocity $v = 8t - 2t^2\,\text{m s}^{-1}$. Fay says its greatest speed is $8\,\text{m s}^{-1}$, because $a = 0$ when $t = 2$

Explain her error, and find the greatest speed. **[3 marks]**

C7 A particle starts from O with velocity $v = 12 - 3t^2\,\text{m s}^{-1}$. Gus works out $\int_0^3 (12 - 3t^2)\,\text{d}t = 9$ and says the particle travels 9 m in the first 3 seconds.

Explain his error, and find the distance travelled. **[3 marks]**

C8 A particle starts from O at $t = 0$ and moves along a straight line with velocity $v = 3t^2 - 2bt\,\text{m s}^{-1}$, where b is a positive constant.

(a) Show that the particle returns to O when $t = b$

(2)

(b) Show that the distance it travels between $t = 0$ and $t = b$ is $\dfrac{8b^3}{27}$ metres.

(3)

[5 marks]

D Problem solving

You decide the method. Read the whole question first, and show every step.

CORE *Secure the method*

D1 *In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.*

In a test of a car on a straight track, the car starts from rest. At time t seconds after the start of the test, the velocity of the car is $v\,\text{m s}^{-1}$, where $v = 0.3t^2 - 0.02t^3$, for $0 \leqslant t \leqslant 10$

(a) Find the greatest acceleration of the car during the test.

(2)

(b) Find the distance travelled by the car during the test.

(1)

[3 marks]

D2 *In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.*

A particle P moves along a straight line. The fixed point O lies on the line. At time t seconds, $t \geqslant 0$, the displacement of P from O is x metres, where $x = 2t^3 - 21t^2 + 60t$

(a) Find the values of t for which P is instantaneously at rest.

(2)

(b) Find the total distance travelled by P in the interval $0 \leqslant t \leqslant 5$

(2)

[4 marks]

D3 At time t seconds, $t \geqslant 0$, a particle P has position vector $\mathbf{r}$ metres relative to a fixed point O, where $\mathbf{r} = 4t^{\frac{3}{2}}\mathbf{i} + (t^2 - 6t)\mathbf{j}$

(a) Find the exact velocity of P at the instant when P is moving parallel to the vector $\mathbf{i}$

(2)

(b) Find the acceleration of P at this instant.

(1)

[3 marks]

D4 [In this question $\mathbf{i}$ and $\mathbf{j}$ are horizontal perpendicular unit vectors.] A particle P of mass 2 kg moves on a smooth horizontal plane under the action of a single force $\mathbf{F}$ newtons. At time t seconds, the velocity of P is $\mathbf{v}\,\text{m s}^{-1}$, where $\mathbf{v} = (3t^2 - 4t)\mathbf{i} + (2t + 1)\mathbf{j}$

Find the magnitude of $\mathbf{F}$ when $t = 2$ **[3 marks]**

STRETCH

D5 *In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.*

A particle P moves along a straight line, starting from the fixed point O at time $t = 0$. At time t seconds, the velocity of P is $v\,\text{m s}^{-1}$, given by the continuous function

$$v = \begin{cases} t^2 + 2t & 0 \leqslant t \leqslant 2 \\ k - 2t^2 & t > 2 \end{cases}$$

where k is a constant.

(a) Show that $k = 16$

(1)

(b) Find the exact value of t when P is instantaneously at rest.

(1)

(c) Find the exact displacement of P from O at this instant.

(2)

[4 marks]

D6 *In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.*

At time $t = 0$, a particle P is at the fixed point O. At time t seconds, $t > 0$, P has position vector $\mathbf{r}$ metres relative to O, where $\mathbf{r} = (t^2 - 4t)\mathbf{i} + (t^3 - 3t)\mathbf{j}$

(a) Find the speed of P at the instant when P is moving in the direction of the vector $\mathbf{j}$

(2)

(b) Show that P never returns to O

(2)

[4 marks]

D7 *In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.*

At time t seconds, $t \geqslant 0$, a particle P is moving with velocity $\mathbf{v}\,\text{m s}^{-1}$, where $\mathbf{v} = (6t - 4)\mathbf{i} + (3t^2 + 2)\mathbf{j}$. At time $t = 0$, P is at the point with position vector $(2\mathbf{i} - 12\mathbf{j})$ m.

(a) Find the position vector of P when $t = 2$

(2)

(b) Find the two values of t at which P is moving in the direction of the vector $4\mathbf{i} + 7\mathbf{j}$

(2)

[4 marks]

D8 A particle P moves along the x-axis. At time $t = 0$, P is at rest at the origin O. At time t seconds, $t \geqslant 0$, the acceleration of P, in the positive x direction, is $k(2-t)\,\text{m s}^{-2}$, where k is a positive constant. Given that the speed of P is $6\,\text{m s}^{-1}$ when $t = 2$

(a) show that $k = 3$

(2)

(b) show that the displacement of P from O at time t is $\frac{1}{2}t^2(6-t)$ metres,

(1)

(c) find the total distance travelled by P before it returns to O

(2)

[5 marks]

E Exam practice

Full-size exam questions in the style of WME02. Write your answers in the spaces, and show every step.

CORE *Exam questions for everyone*

E1 A particle P moves along a straight line. The fixed point O lies on the line. At time $t = 0$, P passes through O with velocity $8\,\text{m s}^{-1}$ in the positive direction. At time t seconds, $t \geqslant 0$, the acceleration of P is $(6t - 14)\,\text{m s}^{-2}$ in the positive direction, and the velocity of P is $v\,\text{m s}^{-1}$ in the positive direction.

(a) Show that $v = 3t^2 - 14t + 8$

(2)

(b) Find the values of t for which P is instantaneously at rest.

(2)

(c) Find the least velocity of P in the interval $0 \leqslant t \leqslant 5$

(2)

(d) Find the total distance travelled by P in the interval $0 \leqslant t \leqslant 5$

(3)

(Total for Question E1 is 9 marks)

E2 *In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.*

[In this question $\mathbf{i}$ and $\mathbf{j}$ are horizontal perpendicular unit vectors.] A particle P of mass 0.5 kg is moving on a smooth horizontal plane. At time $t = 0$, P is at the fixed point O. At time t seconds, $t \geqslant 0$, P has velocity $\mathbf{v}\,\text{m s}^{-1}$, where $\mathbf{v} = (4t - 3)\mathbf{i} + (t^2 - 4t)\mathbf{j}$

(a) Find the speed of P at the instant when $t > 0$ and P is moving parallel to the vector $\mathbf{i}$

(3)

(b) Find the exact magnitude of the resultant force acting on P when $t = 1$

(3)

(c) Find the position vector of P when $t = 3$

(2)

(d) Find the distance of P from O at the instant, after $t = 0$, when P is next due east of O, where $\mathbf{i}$ points due east.

(2)

(Total for Question E2 is 10 marks)

STRETCH

E3 *In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.*

A particle P moves along a straight line. At time $t = 0$, P is at rest at the fixed point O on the line. At time t seconds, $t \geqslant 0$, the acceleration of P is $(6 - 6\sqrt{t})\,\text{m s}^{-2}$ in the positive direction, and the displacement of P from O is x metres.

(a) Show that P is next at rest when $t = \frac{9}{4}$

(2)

(b) Find the greatest speed of P in the interval $0 \leqslant t \leqslant \frac{9}{4}$

(3)

(c) Show that $x = 3t^2 - \frac{8}{5}t^{\frac{5}{2}}$

(2)

(d) Find the exact total distance travelled by P in the interval $0 \leqslant t \leqslant 4$

(4)

(e) Find the greatest speed of P in the interval $0 \leqslant t \leqslant 4$

(1)

(Total for Question E3 is 12 marks)

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