eClassroom

A-LEVEL MATHEMATICS • MECHANICS 2

Projectiles

Worksheet

Pearson Edexcel International A Level (WME02) • Calculator allowed: take $g = 9.8\,\text{m}\,\text{s}^{-2}$ and give answers to 2 or 3 s.f.

Free Projectiles worksheet for A-Level and International A-Level Maths, with full worked solutions

A **Examples** & Your turn page 2

B **Fluency** core skills page 4

C **Reasoning** explain it page 6

D **Problem** solving page 7

E **Exam** practice page 10

Tap a section to jump straight to it

By the end of this worksheet you can: resolve a velocity of projection into components • find the time of flight, greatest height and range • find the speed and direction at any instant • use the equation of the path • *Stretch:* launches from a height, intervals of time, and finding U and α from the motion.

Which questions are for me?

CORE The methods every A-level student must secure. Start here.

STRETCH Projection from a height, two particles at once, times when a condition holds, and working back from the path.

Every section has both, clearly labelled. Questions get harder as you go, and every skill in Sections A to D is needed in Section E.

Track your progress

Section	B	C	D	E
Core marks	23	10	13	21
Stretch marks	16	14	17	14
My score				

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A Worked examples & Your turn

Read each example carefully. Cover it up, then answer the question beside it using the same method.

CORE *Secure the method*

Worked example A1: horizontal projection

A ball is thrown horizontally at $12\,\text{m s}^{-1}$ from the top of a vertical cliff 19.6 m high. Find the time it takes to reach the ground below and how far from the foot of the cliff it lands.

Vertically: $u = 0$, $a = 9.8$ down. Horizontally: constant speed.

$19.6 = 4.9t^2$, so $t = 2$; $x = 12 \times 2$

$t = 2\,\text{s}$, $x = 24\,\text{m}$

Your turn A1

A ball is thrown horizontally at $9\,\text{m s}^{-1}$ from the top of a vertical cliff 78.4 m high. Find the time it takes to reach the ground below and how far from the foot of the cliff it lands.

Worked example A2: components

A particle is projected with speed $25\,\text{m s}^{-1}$ at an angle α above the horizontal, where $\tan\alpha = \frac{4}{3}$

Find the components of its velocity of projection and the time it takes to reach its highest point.

$\tan\alpha = \frac{4}{3}$ *gives* $\sin\alpha = \frac{4}{5}$, $\cos\alpha = \frac{3}{5}$

$25 \times \frac{3}{5} = 15$; $25 \times \frac{4}{5} = 20$; $0 = 20 - 9.8t$

$15\,\text{m s}^{-1}$, $20\,\text{m s}^{-1}$, $2.04\,\text{s}$

Your turn A2

A particle is projected with speed $26\,\text{m s}^{-1}$ at an angle α above the horizontal, where $\tan\alpha = \frac{5}{12}$

Find the components of its velocity of projection and the time it takes to reach its highest point.

Worked example A3: on level ground

A ball is projected from a point on horizontal ground with speed $19.6\,\text{m s}^{-1}$ at 30° above the horizontal. Find the time of flight, the greatest height and the range.

Components 16.97 and 9.8. It lands when $y = 0$

$9.8t - 4.9t^2 = 0$, $t = 2$; $\dfrac{9.8^2}{2(9.8)}$; 16.97×2

2 s, 4.9 m, 33.9 m

Your turn A3

A ball is projected from a point on horizontal ground with speed $28\,\text{m s}^{-1}$ at 45° above the horizontal. Find the time of flight, the greatest height and the range.

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STRETCH

Worked example A4: velocity at an instant

A particle is projected at $20\,\text{m s}^{-1}$, 60° above the horizontal. Find its speed and direction of motion 3 seconds later.

Only v_y changes: $v_x = 10$, $v_y = 17.32 - 29.4 = -12.08$

$\sqrt{10^2 + 12.08^2}$; $\tan^{-1}\dfrac{12.08}{10}$

$15.7\,\text{m s}^{-1}$, 50.4° below the horizontal

Your turn A4

A particle is projected with velocity $(8\mathbf{i} + 14\mathbf{j})\,\text{m s}^{-1}$ ($\mathbf{i}$ horizontal, $\mathbf{j}$ vertically upwards). Find its speed and direction of motion 2 seconds later.

Worked example A5: the equation of the path

A particle is projected from O with speed $14\,\text{m s}^{-1}$ at α above the horizontal, where $\tan\alpha = \frac{3}{4}$. Find the equation of its path, and its height when $x = 10$

$y = x\tan\alpha - \dfrac{gx^2}{2U^2}(1+\tan^2\alpha)$: $\dfrac{9.8}{2(196)} \times \dfrac{25}{16} = \dfrac{5}{128}$

$y = \frac{3}{4}x - \frac{5}{128}x^2$, $3.59\,\text{m}$

Your turn A5

A particle is projected from O with speed $21\,\text{m s}^{-1}$ at α above the horizontal, where $\tan\alpha = \frac{4}{3}$. Find the equation of its path, and its height when $x = 18$

Worked example A6: from a height

A stone is thrown from the top of a 20 m tower at $10\,\text{m s}^{-1}$, 30° above the horizontal. Find when it hits the ground and how far from the tower it lands.

The ground is 20 m below *the start:* $-20 = 5t - 4.9t^2$, positive root $t = 2.594$; 8.660×2.594

2.59 s, 22.5 m

Your turn A6

A stone is thrown from the top of a 15 m cliff at $14\,\text{m s}^{-1}$, 30° *below* the horizontal. Find when it hits the ground and how far from the cliff it lands.

Worked example A7: finding U and α

From level ground, a particle reaches a greatest height of 10 m and lands 40 m away. Find its speed and angle of projection.

$u_y^2 = 2(9.8)(10)$, $u_y = 14$; $T = \dfrac{28}{9.8}$, $u_x = \dfrac{40}{T} = 14$

$U = 19.8\,\text{m s}^{-1}$, $\alpha = 45^\circ$

Your turn A7

From level ground, a particle reaches a greatest height of 4.9 m and lands 39.2 m away. Find its speed and angle of projection.

B Fluency

Each question is a designed sequence. From one part to the next, ask yourself: what changed, and what did it do to the answer?

CORE *Secure the method*

B1 A stone is thrown horizontally at $7\,\text{m s}^{-1}$ from the edge of a vertical drop.

(a) The drop is 4.9 m. Find the time to fall.

(b) Find the horizontal distance travelled.

(c) The drop is now 19.6 m. Find the time to fall.

(d) Find the horizontal distance travelled.

[4 marks]

B2 A particle is projected with speed $20\,\text{m s}^{-1}$ at an angle α above the horizontal. Find the horizontal and vertical components of its velocity of projection when

(a) $\alpha = 30^\circ$

(b) $\alpha = 60^\circ$

(c) $\tan\alpha = \frac{3}{4}$

(d) $\tan\alpha = \frac{4}{3}$

[4 marks]

B3 A ball is projected from horizontal ground with velocity $(6\mathbf{i} + 9.8\mathbf{j})\,\text{m s}^{-1}$

(a) Find the time to its highest point.

(b) Find its greatest height.

(c) Find its time of flight.

(d) Find its range.

[4 marks]

B4 A particle is projected from horizontal ground with speed $14\,\text{m s}^{-1}$

(a) Find its range when projected at 45°

(b) Find its range when projected at 30°

(c) Find its range when projected at 60°

(d) Find its greatest height when projected at 60°

[4 marks]

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CORE *continued: velocity and the path*

B5 A particle is projected from horizontal ground with velocity $(5\mathbf{i} + 14.7\mathbf{j})\,\text{m s}^{-1}$

(a) Find its velocity after 1 second, and after 2 seconds.

(b) Explain why its speed is the same at both times.

(c) Find its speed after 3 seconds. **[3 marks]**

B6 A particle is projected from O with velocity $(10\mathbf{i} + 20\mathbf{j})\,\text{m s}^{-1}$. At time t seconds it is at (x, y) relative to O

(a) Write x and y in terms of t

(b) Eliminate t to find y in terms of x

(c) Find y when $x = 20$

(d) Find x where it returns to the level of O

[4 marks]

STRETCH

B7 From a point 24.5 m above horizontal ground, a stone is thrown with speed $14.7\,\text{m s}^{-1}$

(a) horizontally: find the time to reach the ground.

(b) vertically upwards: find the time to reach the ground.

(c) vertically downwards: find the time to reach the ground.

(d) Show that the stone reaches the ground with the same speed in all three cases.

[4 marks]

B8 A particle is projected from horizontal ground with velocity $(12\mathbf{i} + 16\mathbf{j})\,\text{m s}^{-1}$

(a) Find the time when it is moving at 45° above the horizontal.

(b) Find the time when it is moving at 45° below the horizontal.

(c) Find its speed at each of these times.

(d) Find the time between these two instants.

[4 marks]

B9 A particle is projected from O with speed $14\,\text{m s}^{-1}$ at an angle α above the horizontal.

(a) Find the equation of its path when $\tan\alpha = 1$

(b) Find the equation of its path when $\tan\alpha = 2$

(c) Find the range in each case.

(d) Find the values of $\tan\alpha$ for which the path passes through $(10, 5)$

[4 marks]

B10 A particle is projected from horizontal ground with speed $U\,\text{m s}^{-1}$ at 30° above the horizontal and lands 40 m away.

(a) Find the time of flight in terms of U

(b) Find the range in terms of U

(c) Find U

(d) Find the greatest height.

[4 marks]

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C Reasoning

Show your working and explain your thinking. A correct answer without a reason does not score full marks.

CORE *Secure the method*

C1 A particle is projected at $20\,\text{m s}^{-1}$, 30° above the horizontal. Amir says its least speed is zero, because it stops at its highest point.

Explain his error, and find the least speed. **[2 marks]**

C2 A ball is projected with speed $25\,\text{m s}^{-1}$ at α above the horizontal, where $\tan\alpha = \frac{3}{4}$. To find the time to the highest point, Bea writes $0 = 25 - 9.8t$

Explain her error, and find the correct time. **[2 marks]**

C3 A ball is projected from 15 m above horizontal ground with a vertical component of velocity of $9.8\,\text{m s}^{-1}$ upwards. For the time to reach the ground, Cal writes $15 = 9.8t - 4.9t^2$

Explain his error, and find the correct time. **[3 marks]**

C4 Two particles are projected from the same point on horizontal ground with the same speed U, at 30° and at 60° above the horizontal. Dev says the one at 60° lands further away, because it is in the air for longer.

Show that Dev is wrong. **[3 marks]**

STRETCH

C5 A particle is projected with velocity $(3\mathbf{i} + 4\mathbf{j})\,\text{m s}^{-1}$. For the time when it moves perpendicular to its initial direction, Eve says its velocity is then parallel to $(3\mathbf{i} - 4\mathbf{j})$

Explain her error, and find the correct time. **[3 marks]**

C6 Particles P and Q are projected together from the same point with velocities $(6\mathbf{i} + 8\mathbf{j})\,\text{m s}^{-1}$ and $(3\mathbf{i} + 12\mathbf{j})\,\text{m s}^{-1}$. Fay says that after 1 second they are 4 m apart, because Q is 4 m higher than P

Explain her error, and find how far apart they are. **[3 marks]**

C7 A ball is projected from the top of a 14.7 m cliff with a vertical component of velocity of $9.8\,\text{m s}^{-1}$ upwards. It is highest after 1 second, so Gus says it reaches the ground below after 2 seconds.

Explain his error, and find the correct time. **[3 marks]**

C8 A particle is projected from horizontal ground with speed U at an angle α above the horizontal. It reaches a greatest height H and lands a distance R from its starting point.

(a) Show that $H = \dfrac{U^2 \sin^2\alpha}{2g}$ and $R = \dfrac{U^2 \sin 2\alpha}{g}$

(3)

(b) Hence show that $\tan\alpha = \dfrac{4H}{R}$

(2)

[5 marks]

D Problem solving

You decide the method. Read the whole question first, and show every step.

CORE *Secure the method*

D1 A ball is projected horizontally from the point A at the top of a vertical wall AB of height h metres, where B is on horizontal ground. The ball moves freely under gravity and hits the ground at the point C, where $BC = 12\,\text{m}$. At the instant it hits the ground, the ball is moving at 45° below the horizontal, as shown in Figure 1.

(a) Show that the speed of projection is $\sqrt{117.6}\,\text{m s}^{-1}$ **(2)**

(b) Find the value of h **(1)**

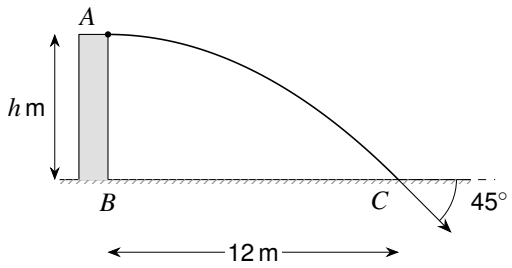


Figure 1

[3 marks]

D2 A football is kicked from the point O on horizontal ground with speed $17.5\,\text{m s}^{-1}$ at an angle α above the horizontal, where $\tan\alpha = \frac{3}{4}$. The ball moves towards a goal whose crossbar DE is 2.44 m above the ground and 19.6 m from O, as shown in Figure 2. The ball is modelled as a particle moving freely under gravity.

(a) Show that the ball passes over the crossbar, and find by how much it clears it. **(3)**

(b) Determine whether the ball is rising or falling as it passes over the crossbar. **(1)**

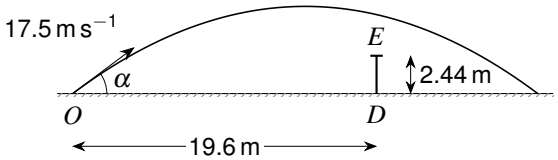


Figure 2

[4 marks]

D3 [In this question **i** is horizontal and **j** is vertically upwards.] A stone is projected from the point O on horizontal ground with velocity $(3u\mathbf{i} + 4u\mathbf{j})\,\text{m s}^{-1}$, where u is a positive constant. It moves freely under gravity and first hits the ground at the point A, where $OA = 30\,\text{m}$, as shown in Figure 3.

(a) Show that $u = 3.5$ **(2)**

(b) Find the greatest height of the stone above the ground. **(1)**

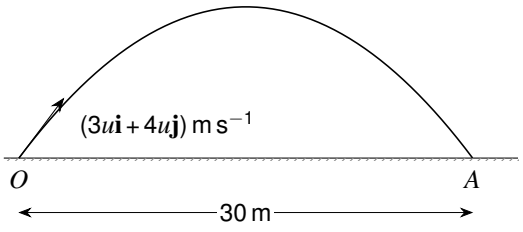


Figure 3

[3 marks]

D4 [In this question **i** is horizontal and **j** is vertically upwards.] At time $t = 0$, two particles are projected from the point O on horizontal ground. Particle P is projected with velocity $(8\mathbf{i} + 9.8\mathbf{j})\,\text{m s}^{-1}$ and particle Q is projected with velocity $(6\mathbf{i} + 14.7\mathbf{j})\,\text{m s}^{-1}$, as shown in Figure 4. Both particles move freely under gravity in the same vertical plane.

Find the distance between P and Q at the instant when P is at its highest point.

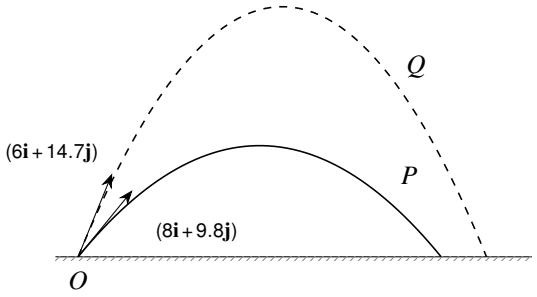


Figure 4

[3 marks]

STRETCH

D5 [In this question **i** is horizontal and **j** is vertically upwards.] A stone is projected from the point A at the top of a vertical cliff AB, where B is at sea level and $AB = 40$ m. The stone is projected with velocity $(7\mathbf{i} + 7\mathbf{j})\,\text{m s}^{-1}$ and moves freely under gravity until it hits the sea, as shown in Figure 5.

(a) Find the time taken for the stone to reach the sea.

(2)

(b) Find the direction of motion of the stone at the instant it hits the sea.

(2)

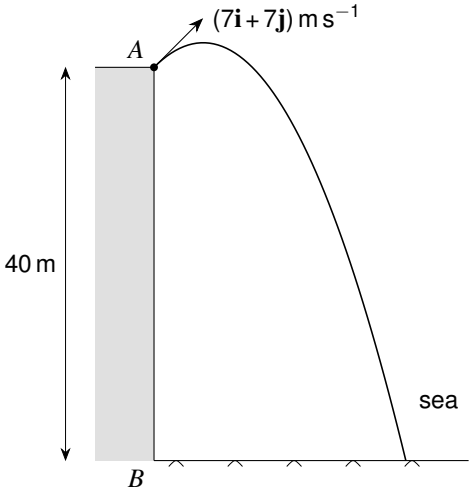


Figure 5

[4 marks]

D6 A ball is projected from the point A, which is 20 m vertically above the point B on horizontal ground. The ball is projected with speed $U\,\text{m s}^{-1}$ at θ° above the horizontal and moves freely under gravity. It hits the ground at the point C, where $BC = 24$ m, 2.5 seconds after it is projected, as shown in Figure 6.

Find the value of U and the value of θ

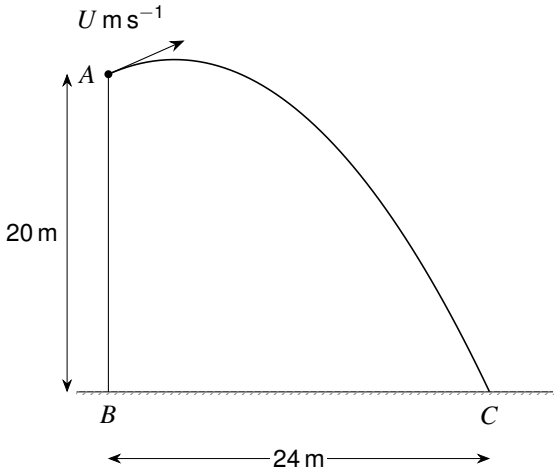


Figure 6

[4 marks]

D7 [In this question **i** is horizontal and **j** is vertically upwards.] A particle is projected from the point O on horizontal ground with velocity $(8\mathbf{i} + 12\mathbf{j})\,\text{m s}^{-1}$ and moves freely under gravity until it hits the ground again, as shown in Figure 7.

Find the total time for which the direction of motion of the particle makes an angle of more than 30° with the horizontal.

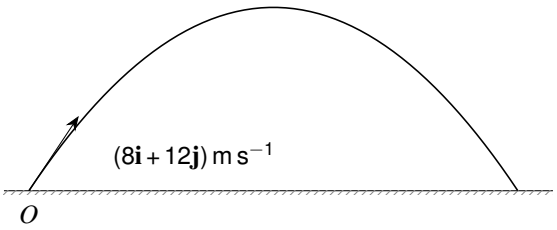


Figure 7

[4 marks]

D8 A ball is projected from the point O on horizontal ground with speed $U\,\text{m s}^{-1}$ at 45° above the horizontal. It moves freely under gravity and lands at the point P on the flat horizontal top of a platform. The top of the platform is 2.5 m above the ground, and P is 10 m horizontally from O, as shown in Figure 8.

(a) Use the equation of the path of the ball to show that $U^2 = \dfrac{392}{3}$

(3)

(b) Find the speed and the direction of motion of the ball at the instant it lands at P

(2)

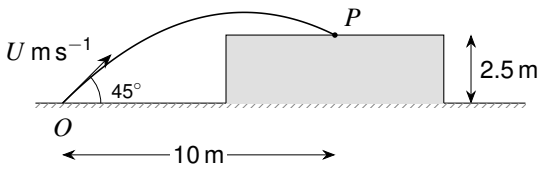


Figure 8

[5 marks]

E Exam practice

Full-size exam questions in the style of WME02. Write your answers in the spaces, and show every step.

CORE *Exam questions for everyone*

E1 [In this question $\mathbf{i}$ and $\mathbf{j}$ are unit vectors, with $\mathbf{i}$ horizontal and $\mathbf{j}$ vertically upwards.]

A ball is kicked from the point O on horizontal ground. The ball is modelled as a particle moving freely under gravity, and air resistance is ignored. The initial velocity of the ball is $(14\mathbf{i} + 9.8\mathbf{j})\,\text{m s}^{-1}$ and the ball first hits the ground again at the point A, as shown in Figure 9.

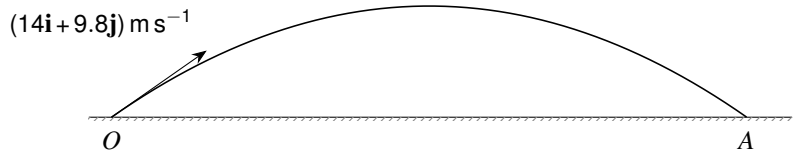


Figure 9

Using the model,

(a) find the time taken for the ball to travel from O to A

(2)

(b) find the distance OA

(1)

(c) find the greatest height of the ball above the ground.

(2)

(d) find the speed and the direction of motion of the ball 1.5 seconds after it is kicked,

(3)

(e) find the length of time, in seconds, for which the ball is at least 3.675 m above the ground.

(2)

(Total for Question E1 is 10 marks)

E2 [In this question **i** and **j** are unit vectors, with **i** horizontal and **j** vertically upwards.]

The fixed points A and C lie on horizontal ground. The point B is vertically above A, with $AB = 19.6\,\text{m}$. At time $t = 0$, a particle P is projected from B with velocity $(x\mathbf{i} + y\mathbf{j})\,\text{m s}^{-1}$, where x and y are positive constants. The particle moves freely under gravity in the vertical plane containing A, B and C, and it hits the ground at C. At the instant before P hits the ground, the velocity of P is $(7\mathbf{i} - 24.5\mathbf{j})\,\text{m s}^{-1}$, as shown in Figure 10.

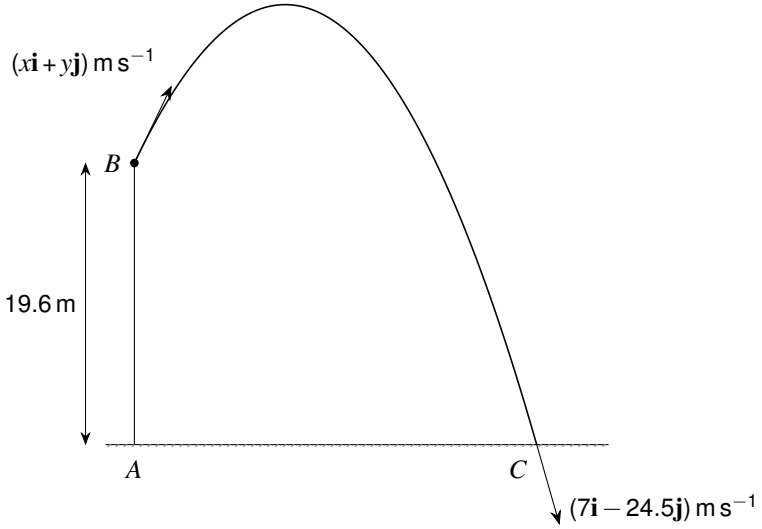


Figure 10

(a) Show that $y = 14.7$

(3)

(b) Find the time taken for P to travel from B to C

(2)

(c) Find the distance AC

(1)

(d) Find the length of time, in seconds, for which P is higher than the point B

(2)

(e) Find the two values of t at which P is moving at 45° to the horizontal.

(3)

(Total for Question E2 is 11 marks)

STRETCH

E3 A particle is projected from a fixed point O with speed $U\,\text{m s}^{-1}$ at an angle α above the horizontal and moves freely under gravity. At the instant when the horizontal distance of the particle from O is x metres, the particle is y metres vertically above the level of O

(a) Show that $y = x\tan\alpha - \dfrac{gx^2}{2U^2}(1+\tan^2\alpha)$

(4)

A ball is projected from the point O on horizontal ground. In a model of its motion, the ball is a particle moving freely under gravity, and its path has equation $y = \frac{4}{3}x - \frac{2}{45}x^2$, where x and y are measured in metres, as shown in Figure 11.

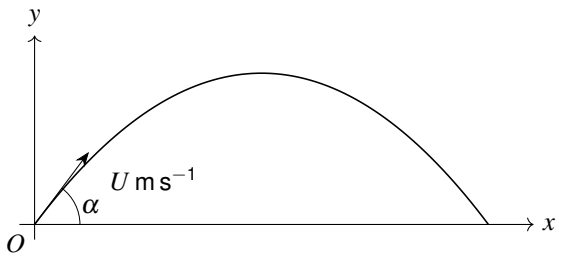


Figure 11

(b) Using the model, find the value of U and the value of $\tan\alpha$

(3)

(c) Find the greatest height of the ball above the ground, according to the model.

(2)

(d) Find the length of time for which the ball is more than 6.4 m above the ground.

(2)

(e) Find the speed and the direction of motion of the ball at the instant when it is 6.4 m above the ground for the second time.

(3)

(Total for Question E3 is 14 marks)

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