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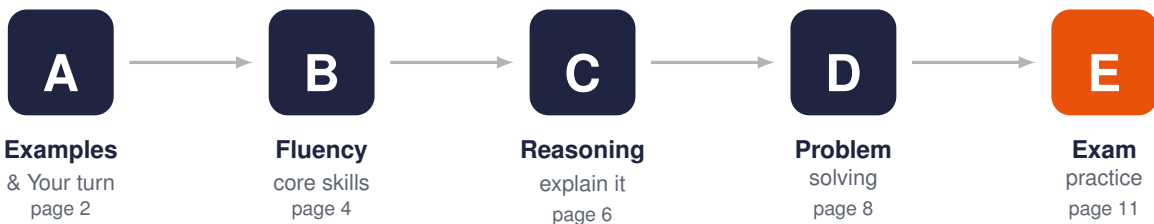
A-LEVEL MATHEMATICS • PURE 2

Algebraic Methods

Worksheet

Pearson Edexcel International A Level (WMA12) & A Level (9MA0) • Calculator allowed: show every step

Free Algebraic Methods worksheet for A-Level and International A-Level Maths, with full worked solutions



Tap a section to jump straight to it

By the end of this worksheet you can: simplify algebraic fractions by factorising • divide a polynomial by $(x \pm a)$, giving the quotient and the remainder • use the factor theorem to show a factor and to factorise a cubic completely • use the remainder theorem to find remainders and unknown constants • *Stretch:* solve for two or three unknowns, justify the number of real roots and use a factorisation in a new equation.

Which questions are for me?

CORE The methods every A-level student must secure. Start here.

STRETCH Two unknowns from factor and remainder conditions, three remainders, the number of real roots, and substitutions.

Every section has both, clearly labelled. Questions get harder as you go, and every skill in Sections A to D is needed in Section E.

Track your progress

Section	B	C	D	E
Core marks	23	10	13	11
Stretch marks	16	14	17	8
My score				

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Name

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A

Worked examples & Your turn

Read each example carefully. Cover it up, then answer the question beside it using the same method.

CORE

Secure the method

Worked example A1

Simplify fully $\frac{x^2 - 9}{x^2 + x - 6}$

Factorise the top and the bottom, then cancel common factors.

$$\frac{(x-3)(x+3)}{(x+3)(x-2)}$$

$$\frac{x-3}{x-2}$$

Your turn A1

Simplify fully $\frac{2x^2 + 7x + 3}{x^2 - 9}$

Worked example A2

Find the quotient and the remainder when $x^3 + 2x^2 - 5x - 6$ is divided by $(x - 2)$

Divide the leading term each time, multiply back, subtract.

$$x^3 + 2x^2 - 5x - 6 = (x - 2)(x^2 + 4x + 3) + 0$$

$$x^2 + 4x + 3, \text{ remainder } 0$$

Your turn A2

Find the quotient and the remainder when $2x^3 - 3x^2 - 4x + 5$ is divided by $(x + 1)$

Worked example A3

Show that $(x - 3)$ is a factor of $f(x) = x^3 - 4x^2 + x + 6$

The factor theorem: substitute the value that makes the factor zero.

$$f(3) = 27 - 36 + 3 + 6 = 0$$

$$f(3) = 0, \text{ so } (x - 3) \text{ is a factor}$$

Your turn A3

Show that $(2x + 1)$ is a factor of $f(x) = 2x^3 + 5x^2 - 4x - 3$



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STRETCH

Worked example A4: a remainder gives a constant

$f(x) = x^3 + kx^2 - 4x + 1$. When $f(x)$ is divided by $(x - 2)$ the remainder is 5. Find k

Remainder theorem: the remainder is $f(2)$

$$8 + 4k - 8 + 1 = 5$$

$$k = 1$$

Your turn A4

$f(x) = 2x^3 - x^2 + kx - 6$. When $f(x)$ is divided by $(x + 2)$ the remainder is -2 . Find k

Worked example A5: factorise completely

Factorise completely $x^3 - 7x - 6$

Try small values: $f(-1) = -1 + 7 - 6 = 0$, so $(x + 1)$ is a factor.

$$x^3 - 7x - 6 = (x + 1)(x^2 - x - 6)$$

$$(x + 1)(x - 3)(x + 2)$$

Your turn A5

Factorise completely $2x^3 + x^2 - 13x + 6$

Worked example A6: two unknowns

$f(x) = x^3 + ax^2 + bx - 6$. $(x - 1)$ is a factor, and the remainder on division by $(x - 3)$ is 12. Find a and b

$$f(1) = 0: a + b = 5; \quad f(3) = 12: 9a + 3b = -9$$

Simplify the second to $3a + b = -3$, then subtract.

$$a = -4, \quad b = 9$$

Your turn A6

$f(x) = 2x^3 + ax^2 + bx + 3$. $(x + 1)$ is a factor, and the remainder on division by $(x - 2)$ is 15. Find a and b

Worked example A7: check for excluded values

$$\text{Solve } \frac{x^2 - 4}{x^2 - x - 2} = 3$$

$$\frac{(x - 2)(x + 2)}{(x - 2)(x + 1)} = \frac{x + 2}{x + 1} = 3, \text{ so } x + 2 = 3x + 3$$

The original fraction is undefined at $x = 2$ and $x = -1$: check the answer is neither.

$$x = -\frac{1}{2}$$

Your turn A7

$$\text{Solve } \frac{x^2 + 2x - 3}{x^2 - 1} = 2$$



B**Fluency**

Each question is a designed sequence. From one part to the next, ask yourself: what changed, and what did it do to the answer?

CORE*Secure the method***B1** Simplify fully

(a) $\frac{x^2 + 3x}{x^2 - 9}$

(b) $\frac{x^2 - 5x + 6}{x^2 - 4}$

(c) $\frac{2x^2 - x - 1}{4x^2 - 1}$

(d) $\frac{3 - x}{x^2 - 9}$

[4 marks]**B2** $f(x) = x^3 + 2x^2 - x + 3$. Find the quotient and the remainder when $f(x)$ is divided by

(a) $(x - 1)$

(b) $(x + 1)$

(c) $(x - 2)$

(d) $(x + 3)$

[4 marks]**B3** $f(x) = 2x^3 - x^2 + 3x - 4$. Use the remainder theorem to find the remainder when $f(x)$ is divided by

(a) $(x - 1)$

(b) $(x + 1)$

(c) $(x - 2)$

(d) $(2x - 1)$

[4 marks]**B4** $f(x) = x^3 - 2x^2 - 5x + 6$. Decide whether each is a factor of $f(x)$, giving a reason.

(a) $(x - 1)$

(b) $(x + 1)$

(c) $(x - 3)$

(d) $(x + 2)$

[4 marks]

CORE *continued: factorising and unknowns***B5** Factorise completely

(a) $x^3 - 6x^2 + 11x - 6$

(b) $x^3 + x^2 - 8x - 12$

(c) $2x^3 - 7x^2 + 2x + 3$

[3 marks]

B6 Find the value of the constant k in each case.

(a) $(x - 2)$ is a factor of $x^3 + kx - 4$

(b) $(x + 2)$ is a factor of $x^3 + kx - 4$

(c) the remainder when $x^3 + kx - 4$ is divided by $(x - 2)$ is 6

(d) $(2x - 1)$ is a factor of $4x^3 + kx - 4$

[4 marks]

STRETCH**B7** Simplify fully

(a) $\frac{x^2 - 1}{x + 2} \times \frac{x + 2}{x - 1}$

(b) $\frac{x^2 - 4}{x^2 + 3x} \div \frac{x + 2}{x}$

(c) $\frac{2x^2 + 3x - 2}{x^2 - 4}$

(d) $\frac{x^3 - 8}{x^2 - 4}$

[4 marks]

B8 Find the quotient when

(a) $x^3 - 8$ is divided by $(x - 2)$

(b) $x^3 + 27$ is divided by $(x + 3)$

(c) $2x^3 - 5x + 3$ is divided by $(x - 1)$

(d) $x^4 - 1$ is divided by $(x + 1)$

[4 marks]

B9 $f(x) = x^3 + ax^2 + bx + 6$, where a and b are constants. $(x - 1)$ and $(x - 2)$ are both factors of $f(x)$

(a) Show that $a + b = -7$

(b) Show that $2a + b = -7$

(c) Find a and b

(d) Factorise $f(x)$ completely.

[4 marks]

B10 State, with a reason, the number of distinct real roots of each equation.

(a) $(x - 1)(x^2 + 4x + 5) = 0$

(b) $(x + 2)(x^2 - 2x + 1) = 0$

(c) $(x - 3)(x^2 - 5) = 0$

(d) $(2x - 1)(x^2 + x + 1) = 0$

[4 marks]



C**Reasoning**

Show your working and explain your thinking. A correct answer without a reason does not score full marks.

CORE

Secure the method

- C1** Sam wants to show that $(x+2)$ is a factor of $f(x) = x^3 + 3x^2 - 4$, and works out $f(2)$
Explain Sam's mistake, and show correctly that $(x+2)$ is a factor. [2 marks]
- C2** Asked to show that $(x-3)$ is a factor of $x^3 - 4x^2 + x + 6$, Lily writes only: $27 - 36 + 3 + 6$
Explain what Lily must add to earn full marks. [2 marks]
- C3** Kai tests whether $(2x-1)$ is a factor of $f(x) = 2x^3 + x^2 - 5x + 2$ by working out $f\left(-\frac{1}{2}\right)$
Explain Kai's mistake, and show correctly that $(2x-1)$ is a factor. [3 marks]
- C4** Jo is asked to factorise $2x^3 + 7x^2 - 7x - 12$ completely. She shows that $(x+1)$ is a factor and writes
$$2x^3 + 7x^2 - 7x - 12 = (x+1)(2x^2 + 5x - 12)$$

Explain why Jo's answer would not score full marks, and complete it. [3 marks]



STRETCH

- C5** $f(x) = (x - 2)(x^2 + 2x + 5)$. Ali says the equation $f(x) = 0$ has three real roots, because $f(x)$ is a cubic.
Explain why Ali is wrong, and state the number of real roots with a reason. [3 marks]
- C6** Max simplifies $\frac{x^2 + 5x}{x^2 + 2x}$ by cancelling the x^2 terms, to get $\frac{5x}{2x} = \frac{5}{2}$
Explain Max's mistake, and simplify the fraction correctly. [3 marks]
- C7** $f(x) = (x + 1)(x^2 - 3x + k) + 9$, where k is a constant.
Write down, without dividing, the remainder when $f(x)$ is divided by $(x + 1)$, giving a reason. Given that $(x - 2)$ is a factor of $f(x)$, find k [3 marks]
- C8** (a) Prove that $(x - a)$ is a factor of $x^n - a^n$ for every positive integer n and every constant a (2)
- (b) Hence factorise $x^3 - 27$ into a linear and a quadratic factor, and show that the equation $x^3 - 27 = 0$ has exactly one real root. (3)
- [5 marks]



D**Problem solving**

You decide the method. Read the whole question first, and show every step.

CORE*Secure the method*

D1 In this question you must show all stages of your working.

$$f(x) = ax^3 + 3x^2 - 5x + 4, \text{ where } a \text{ is a constant.}$$

Given that when $f(x)$ is divided by $(x - 2)$ the remainder is 22,

(a) use the remainder theorem to find the value of a , showing your working clearly,

(2)

(b) hence find the remainder when $f(x)$ is divided by $(x + 1)$, stating which theorem you use.

(1)

[3 marks]

D2 In this question you must show all stages of your working.

$$f(x) = 2x^3 + x^2 + kx - 6, \text{ where } k \text{ is a constant.}$$

Given that $(2x - 3)$ is a factor of $f(x)$,

(a) use the factor theorem to show that $k = -2$

(2)

(b) Using algebra, write $f(x)$ as the product of a linear and a quadratic factor, and show that the equation $f(x) = 0$ has exactly one real root.

(2)

[4 marks]

D3 Using algebra, find the quotient and the remainder when

$$3x^3 - 2x^2 + 4x - 7$$

is divided by $(x - 2)$, giving the quotient in the form $ax^2 + bx + c$ and stating the remainder clearly. Then use the remainder theorem to check your remainder.

[3 marks]



D4 By dividing $x^2 + 3x - 1$ by $(x - 2)$, write

$$\frac{x^2 + 3x - 1}{x - 2} \text{ in the form } Ax + B + \frac{C}{x - 2}$$

where A , B and C are constants to be found. You must show your division clearly, and you should check your answer by multiplying out.

[3 marks]

STRETCH

D5 In this question you must show all stages of your working.

$$f(x) = x^3 + ax^2 + bx - 12, \text{ where } a \text{ and } b \text{ are constants.}$$

Given that $(x + 2)$ is a factor of $f(x)$, and that the remainder when $f(x)$ is divided by $(x - 1)$ is -18 ,

(a) find the value of a and the value of b ,

(3)

(b) hence factorise $f(x)$ completely, writing any repeated factor as a power.

(1)

[4 marks]

D6 $f(x) = x^3 - 3x^2 - 4x + 12$

(a) Show that $(x - 3)$ is a factor of $f(x)$

(1)

(b) Factorise $f(x)$ completely.

(2)

(c) Hence, using your answer to part (b) and making your method clear, solve the equation $8y^3 - 12y^2 - 8y + 12 = 0$

(1)

[4 marks]



D7 In this question you must show all stages of your working.

$$g(x) = x^3 - 6x^2 + 13x - 10$$

(a) Use the factor theorem to show that $(x - 2)$ is a factor of $g(x)$

(1)

(b) Using algebra, show that the equation $g(x) = 0$ has exactly one real root, giving a reason for your answer.

(2)

(c) Hence, making your reasoning clear, solve the equation $g(x + 1) = 0$

(1)

[4 marks]

D8 In this question you must show all stages of your working.

$f(x) = 2x^3 - 3x^2 + ax + 6$, where a is a constant. Given that $(x - 2)$ is a factor of $f(x)$,

(a) Find the value of a

(1)

(b) Factorise $f(x)$ completely.

(2)

(c) Hence, using algebra and giving a reason for any value you reject, solve the equation $2 \times 8^y - 3 \times 4^y - 5 \times 2^y + 6 = 0$

(2)

[5 marks]



E**Exam practice**

Exam-style questions in the style of WMA12. Write your answers in the spaces, and show every step.

CORE

Exam questions for everyone

E1

$$f(x) = 2x^3 - 5x^2 + ax + b$$

where a and b are constants.

Given that $(x - 3)$ is a factor of $f(x)$,

(a) show that $3a + b = -9$

(1)

Given further that when $f(x)$ is divided by $(x - 2)$ the remainder is -9 ,

(b) find the value of a and the value of b .

(2)

(c) Hence, using algebra, factorise $f(x)$ completely. You must make your method clear.

(2)

(Total for Question E1 is 5 marks)



E2

$$f(x) = x^3 + px^2 + 7x - 6$$

where p is a constant. Given that $(x - 1)$ is a factor of $f(x)$,

(a) show that $p = -2$

(1)

(b) Using algebra, find the quotient and the remainder when $f(x)$ is divided by $(x - 3)$, stating each of them clearly.

.....
(3)

(c) Show that the equation $f(x) = 0$ has exactly one real root, giving a reason for your answer.

(2)

(Total for Question E2 is 6 marks)



STRETCH

E3

$$f(x) = x^3 + ax^2 + bx + c$$

where a , b and c are constants. When $f(x)$ is divided by $(x - 1)$, $(x - 2)$ and $(x - 3)$, the remainders are 2, 5 and 14 respectively.

(a) Write down three equations in a , b and c , simplifying each one.

(3)

(b) Hence find the values of a , b and c .

(2)

(c) Find the remainder when $f(x)$ is divided by $(2x - 1)$

(1)

(d) Show that $(x - 1)$ is a factor of $f(x) - 2$, and hence show that the equation $f(x) = 2$ has exactly one real root.

(2)

(Total for Question E3 is 8 marks)

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