



eClassroom

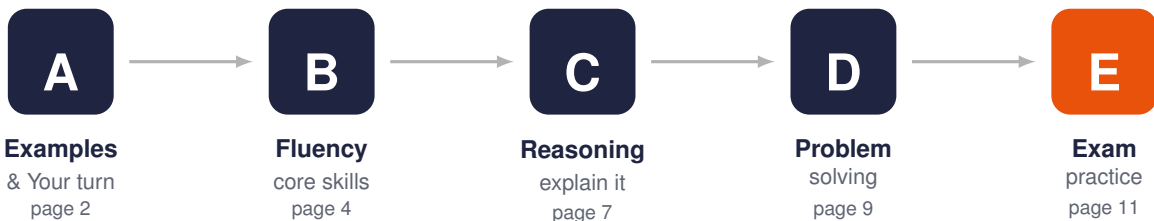
GCSE MATHEMATICS • GEOMETRY

Vectors

Worksheet

Pearson Edexcel GCSE (1MA1) & International GCSE (4MA1) • No calculator

Free Vectors worksheet for GCSE and iGCSE Maths, with full worked solutions



Tap a section to jump straight to it

By the end of this worksheet you can: add, subtract and multiply column vectors • read and draw vectors on a grid • solve vector equations and find magnitudes • *Higher:* write vectors in terms of **a** and **b**, and prove that lines are parallel and points are collinear.

Which questions are for me?

FOUNDATION Everyone starts here. These skills appear on both tiers.

HIGHER Higher only: vector geometry with **a** and **b**, ratios, parallel lines and collinear points. Grades 6 to 9.

Every section has both, clearly labelled. Questions get harder as you go, and every skill in Sections A to D is needed in Section E.

Track your progress

Section	B	C	D	E
Foundation marks	23	10	13	15
Higher marks	16	14	17	15
My score				

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A

Worked examples & Your turn

Read each example carefully. Cover it up, then answer the question beside it using the same method.

FOUNDATION

Everyone starts here

Worked example A1

$$\mathbf{a} = \begin{pmatrix} 3 \\ -1 \end{pmatrix} \text{ and } \mathbf{b} = \begin{pmatrix} -2 \\ 4 \end{pmatrix}. \text{ Work out } \mathbf{a} + \mathbf{b}$$

Add the top numbers, then the bottom numbers.

$$\begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

Your turn A1

$$\mathbf{p} = \begin{pmatrix} 5 \\ 2 \end{pmatrix} \text{ and } \mathbf{q} = \begin{pmatrix} -3 \\ 6 \end{pmatrix}. \text{ Work out } \mathbf{p} - \mathbf{q}$$

Worked example A2

$$\mathbf{a} = \begin{pmatrix} 4 \\ 1 \end{pmatrix} \text{ and } \mathbf{b} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}. \text{ Work out } 2\mathbf{a} - 3\mathbf{b}$$

$$2\mathbf{a} = \begin{pmatrix} 8 \\ 2 \end{pmatrix} \text{ and } 3\mathbf{b} = \begin{pmatrix} 3 \\ -6 \end{pmatrix}, \text{ then subtract.}$$

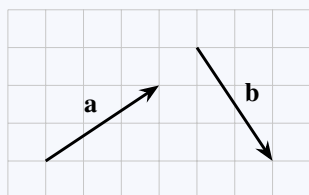
$$\begin{pmatrix} 5 \\ 8 \end{pmatrix}$$

Your turn A2

$$\mathbf{p} = \begin{pmatrix} -1 \\ 3 \end{pmatrix} \text{ and } \mathbf{q} = \begin{pmatrix} 2 \\ -4 \end{pmatrix}. \text{ Work out } 3\mathbf{p} + 2\mathbf{q}$$

Worked example A3

Write $\mathbf{a}$ and $\mathbf{b}$ as column vectors.

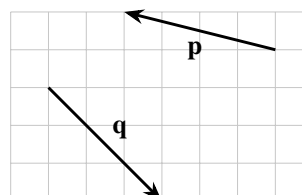


Count across (right +), then up (+) or down (-).

$$\mathbf{a} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$$

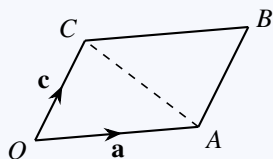
Your turn A3

Write $\mathbf{p}$ and $\mathbf{q}$ as column vectors.



HIGHERHigher only: vector geometry with **a** and **b****Worked example A4: a route through a shape**

$OABC$ is a parallelogram. $\vec{OA} = \mathbf{a}$ and $\vec{OC} = \mathbf{c}$. Write $\vec{AC}$ in terms of **a** and **c**



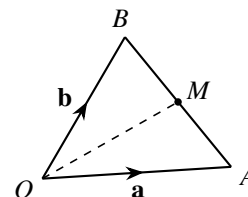
Go $A \rightarrow O \rightarrow C$: $\vec{AC} = -\mathbf{a} + \mathbf{c}$

$\mathbf{c} - \mathbf{a}$

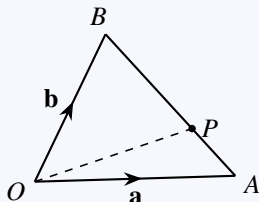
Your turn A4

$\vec{OA} = \mathbf{a}$ and $\vec{OB} = \mathbf{b}$. M is the midpoint of AB .

Write $\vec{OM}$ in terms of **a** and **b**

**Worked example A5: a point dividing a line**

P is on AB with $AP : PB = 1 : 2$. Write $\vec{OP}$ in terms of **a** and **b**



$\vec{AB} = \mathbf{b} - \mathbf{a}$. $\vec{OP} = \mathbf{a} + \frac{1}{3}(\mathbf{b} - \mathbf{a})$

$\frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{b}$

Your turn A5

P is on AB , where $\vec{OA} = \mathbf{a}$ and $\vec{OB} = \mathbf{b}$, with $AP : PB = 3 : 1$. Write $\vec{OP}$ in terms of **a** and **b**

Worked example A6: parallel vectors

Show that $2\mathbf{a} - \mathbf{b}$ and $6\mathbf{a} - 3\mathbf{b}$ are parallel.

$6\mathbf{a} - 3\mathbf{b} = 3(2\mathbf{a} - \mathbf{b})$: one is a multiple of the other.

parallel

Your turn A6

Are $4\mathbf{a} + 6\mathbf{b}$ and $6\mathbf{a} + 9\mathbf{b}$ parallel? Show how you decide.

Worked example A7: collinear points

$\vec{AB} = \mathbf{a} + 2\mathbf{b}$ and $\vec{BC} = 2\mathbf{a} + 4\mathbf{b}$. Show that A , B and C are collinear.

$\vec{BC} = 2\vec{AB}$, so they are parallel, and they share the point B

collinear

Your turn A7

$\vec{PQ} = 3\mathbf{a} - \mathbf{b}$ and $\vec{QR} = 6\mathbf{a} - 2\mathbf{b}$. Show that P , Q and R are collinear.



B**Fluency**

Each question is a designed sequence. From one part to the next, ask yourself: what changed, and what did it do to the answer?

FOUNDATION

Everyone starts here

B1 $\mathbf{a} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -3 \\ 1 \end{pmatrix}$. Work out

(a) $\mathbf{a} + \mathbf{b}$

(b) $\mathbf{a} - \mathbf{b}$

(c) $\mathbf{b} - \mathbf{a}$

(d) $\mathbf{a} + 2\mathbf{b}$

[4 marks]

B2 $\mathbf{a} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -3 \\ 1 \end{pmatrix}$. Work out

(a) $3\mathbf{a}$

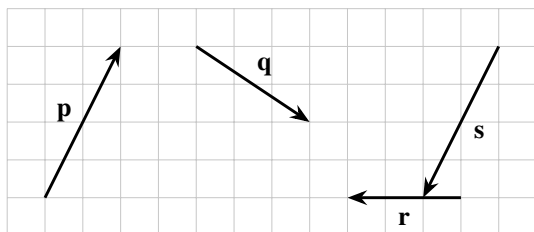
(b) $-2\mathbf{b}$

(c) $2\mathbf{a} + 3\mathbf{b}$

(d) $\frac{1}{2}(\mathbf{a} + \mathbf{b})$

[4 marks]

B3 Write each vector as a column vector.



[4 marks]

B4 (a) $\begin{pmatrix} 3 \\ k \end{pmatrix} + \begin{pmatrix} m \\ 2 \end{pmatrix} = \begin{pmatrix} 7 \\ 5 \end{pmatrix}$. Find m and k

(b) $2 \begin{pmatrix} x \\ 3 \end{pmatrix} = \begin{pmatrix} 8 \\ y \end{pmatrix}$. Find x and y

(c) $\mathbf{a} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$ and $\mathbf{a} + 2\mathbf{c} = \begin{pmatrix} 1 \\ 9 \end{pmatrix}$. Find $\mathbf{c}$

(d) $\begin{pmatrix} 6 \\ k \end{pmatrix}$ is parallel to $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$. Find k

[4 marks]



FOUNDATION

continued: magnitude and routes

B5 Work out the magnitude of each vector. Give (c) to 1 decimal place.

(a) $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$

(b) $\begin{pmatrix} -5 \\ 12 \end{pmatrix}$

(c) $\begin{pmatrix} 2 \\ -6 \end{pmatrix}$

[3 marks]

B6 $\vec{AB} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$, $\vec{BC} = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$ and $\vec{CD} = \begin{pmatrix} -5 \\ 2 \end{pmatrix}$. Work out

(a) $\vec{AC}$

(b) $\vec{BD}$

(c) $\vec{AD}$

(d) $\vec{DA}$

[4 marks]

HIGHER

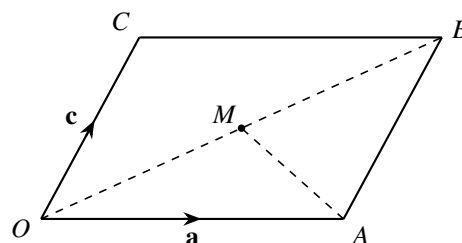
Higher tier only

B7 $OABC$ is a parallelogram. $\vec{OA} = \mathbf{a}$ and $\vec{OC} = \mathbf{c}$. M is the midpoint of OB

Write in terms of $\mathbf{a}$ and $\mathbf{c}$

(a) $\vec{AB}$ (b) $\vec{OB}$

(c) $\vec{AC}$ (d) $\vec{AM}$



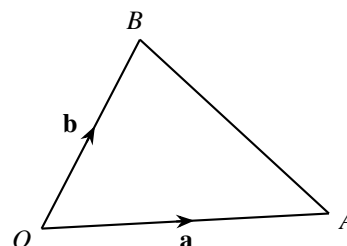
[4 marks]

B8 $\vec{OA} = \mathbf{a}$ and $\vec{OB} = \mathbf{b}$. Write $\vec{OP}$ in terms of $\mathbf{a}$ and $\mathbf{b}$ when P is on AB and

(a) $AP : PB = 1 : 1$ (b) $AP : PB = 1 : 3$

(c) $AP : PB = 2 : 1$

(d) Point Q is on AB extended with $\vec{AQ} = 2\vec{AB}$. Write $\vec{OQ}$



[4 marks]



HIGHER*continued: parallel, collinear and a hexagon*

B9 (a) Are $2\mathbf{a} + 3\mathbf{b}$ and $6\mathbf{a} + 9\mathbf{b}$ parallel? (b) Are $\mathbf{a} - 2\mathbf{b}$ and $3\mathbf{a} + 6\mathbf{b}$ parallel?

(c) $\vec{PQ} = \mathbf{a} + \mathbf{b}$ and $\vec{QR} = 3\mathbf{a} + 3\mathbf{b}$. Are P, Q, R collinear?

(d) $\vec{AB} = 2\mathbf{a} - \mathbf{b}$ and $\vec{BC} = -4\mathbf{a} + 2\mathbf{b}$. Are A, B, C collinear?

Give a reason each time.

[4 marks]

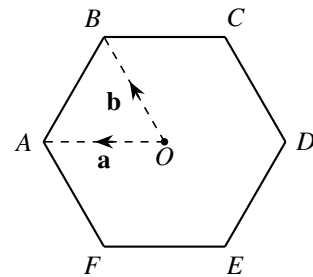
B10 $ABCDEF$ is a regular hexagon with centre O .

$\vec{OA} = \mathbf{a}$ and $\vec{OB} = \mathbf{b}$

Write in terms of $\mathbf{a}$ and $\mathbf{b}$

(a) $\vec{AB}$ (b) $\vec{BC}$

(c) $\vec{AD}$ (d) $\vec{CE}$



[4 marks]



C**Reasoning**

Show your working and explain your thinking. A correct answer without a reason does not score full marks.

FOUNDATION

Everyone starts here

C1 $\mathbf{a} = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$. Tom writes $\mathbf{a} - \mathbf{b} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

Explain Tom's mistake, and give the correct answer.

[2 marks]

C2 Explain the difference between $\overrightarrow{AB}$ and $\overrightarrow{BA}$

[2 marks]

C3 Sam says that $\begin{pmatrix} 2 \\ 6 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 3 \end{pmatrix}$ are parallel. Explain why Sam is right. Is $\begin{pmatrix} 3 \\ 9 \end{pmatrix}$ parallel to them too?

[3 marks]

C4 Show that $\mathbf{a} = \begin{pmatrix} 4 \\ -2 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -6 \\ 3 \end{pmatrix}$ are parallel but point in opposite directions.

[3 marks]



HIGHER

Higher tier only

- C5** $\vec{OA} = \mathbf{a}$ and $\vec{OB} = \mathbf{b}$. M is the midpoint of AB . Jo writes $\vec{OM} = \frac{1}{2}\mathbf{b} - \frac{1}{2}\mathbf{a}$

Explain Jo's mistake, and find $\vec{OM}$

[3 marks]

- C6** Kai shows that $\vec{AB}$ is parallel to $\vec{CD}$ and says that A, B, C and D are on a straight line.

Explain why this is not enough.

[3 marks]

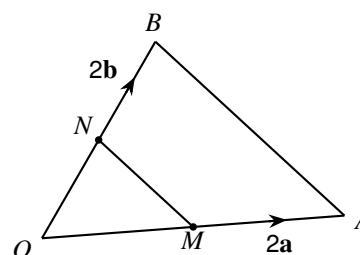
- C7** $\vec{AB} = k\vec{CD}$ where k is a number. Explain what this tells you about the lines AB and CD , and about their lengths.

[3 marks]

- C8** $\vec{OA} = 2\mathbf{a}$ and $\vec{OB} = 2\mathbf{b}$. M is the midpoint of OA and N is the midpoint of OB

(a) Find $\vec{MN}$ and $\vec{AB}$ in terms of $\mathbf{a}$ and $\mathbf{b}$

(b) Prove that MN is parallel to AB and half its length.



[5 marks]



D**Problem solving**

You decide the method. Read the whole question first, and show every step.

FOUNDATION

Everyone starts here

- D1** A boat moves $\begin{pmatrix} 5 \\ 2 \end{pmatrix}$ and then $\begin{pmatrix} -3 \\ 4 \end{pmatrix}$, in kilometres.

Write its overall movement as a column vector, and work out how far it is from its start, to 1 decimal place.

[3 marks]

- D2** $ABCD$ is a parallelogram with $A(1, 2)$, $B(5, 3)$ and $C(6, 7)$

Use vectors to find the coordinates of D

[4 marks]

- D3** $\mathbf{a} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 2 \\ -5 \end{pmatrix}$. Find the number k for which $3\mathbf{a} + k\mathbf{b} = \begin{pmatrix} 7 \\ -4 \end{pmatrix}$

[3 marks]

- D4** $\mathbf{u} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$. Find the numbers m and n for which $m\mathbf{u} + n\mathbf{v} = \begin{pmatrix} 7 \\ 11 \end{pmatrix}$

[3 marks]



HIGHER

Higher tier only

- D5** $OABC$ is a trapezium. $\vec{OA} = \mathbf{a}$, $\vec{OC} = \mathbf{c}$ and $\vec{CB} = 2\mathbf{a}$.
 M is the midpoint of AB

(a) Find $\vec{AB}$ (b) Find $\vec{OM}$

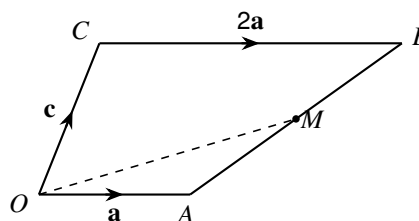


Diagram NOT accurately drawn

[4 marks]

- D6** $\vec{OA} = \mathbf{a}$ and $\vec{OB} = \mathbf{b}$. The point C has $\vec{OC} = 3\mathbf{b} - 2\mathbf{a}$
 Prove that A , B and C are collinear.

[4 marks]

- D7** $\vec{OA} = \mathbf{a}$ and $\vec{OB} = \mathbf{b}$. P is on AB with $\vec{OP} = \frac{1}{3}\mathbf{a} + \frac{2}{3}\mathbf{b}$
 Find the ratio $AP : PB$

[4 marks]

- D8** $OABC$ is a parallelogram. $\vec{OA} = \mathbf{a}$ and $\vec{OC} = \mathbf{c}$. M is the midpoint of AB . The lines OM and AC meet at X
 Find the ratio $OX : XM$

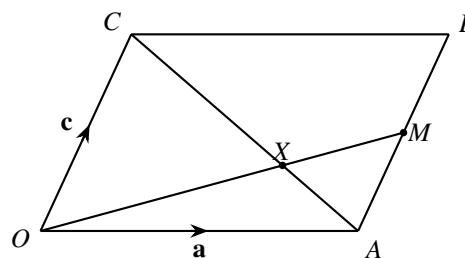


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[5 marks]



E**Exam practice**

Exam-style questions. Write your answers in the spaces. Aim for about one minute per mark.

FOUNDATION

Foundation and Higher crossover

E1 $\mathbf{a} = \begin{pmatrix} 4 \\ -1 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$

Work out $2\mathbf{a} + \mathbf{b}$ as a column vector.

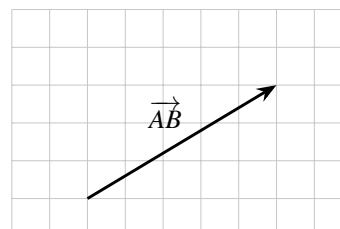
.....
(Total for Question E1 is 2 marks)

E2 (a) Write $\vec{AB}$ as a column vector.

.....
(1)

(b) Write $\vec{BA}$ as a column vector.

.....
(1)



(Total for Question E2 is 2 marks)

E3 $\begin{pmatrix} 3 \\ y \end{pmatrix} + 2 \begin{pmatrix} x \\ -1 \end{pmatrix} = \begin{pmatrix} 11 \\ 5 \end{pmatrix}$

Find the values of x and y

.....
(Total for Question E3 is 3 marks)



- E4** A particle moves $\begin{pmatrix} 6 \\ 1 \end{pmatrix}$ and then $\begin{pmatrix} -2 \\ 2 \end{pmatrix}$

Work out the magnitude of its overall movement.

.....
(Total for Question E4 is 3 marks)

- E5** $\mathbf{p} = \begin{pmatrix} 6 \\ -4 \end{pmatrix}$. Which of these vectors are parallel to $\mathbf{p}$? $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$ $\begin{pmatrix} -9 \\ 6 \end{pmatrix}$ $\begin{pmatrix} 4 \\ 6 \end{pmatrix}$

.....
(Total for Question E5 is 2 marks)

- E6** $\vec{AB} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$, $\vec{BC} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$ and $\vec{CD} = \begin{pmatrix} -5 \\ 2 \end{pmatrix}$

(a) Work out $\vec{AD}$

.....
(2)

(b) Work out the length of AD , to 1 decimal place.

.....
(1)

(Total for Question E6 is 3 marks)

HIGHER

Higher tier only

- E7** $\vec{OA} = \mathbf{a}$ and $\vec{OB} = \mathbf{b}$. P is on AB with $AP : PB = 3 : 2$
Write $\vec{OP}$ in terms of $\mathbf{a}$ and $\mathbf{b}$. Give your answer in its simplest form.

.....

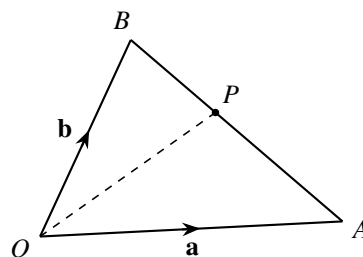


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(Total for Question E7 is 3 marks)



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E8 $\vec{OX} = 3\mathbf{a} + \mathbf{b}$ and $\vec{OY} = \mathbf{a} + 3\mathbf{b}$

Show that XY is parallel to the vector $\mathbf{b} - \mathbf{a}$

(Total for Question E8 is 3 marks)

E9 $\vec{OP} = 2\mathbf{a} + \mathbf{b}$, $\vec{OQ} = 5\mathbf{a} + 4\mathbf{b}$ and $\vec{OR} = 7\mathbf{a} + 6\mathbf{b}$

(a) Prove that P , Q and R are collinear.

(3)

(b) Find the ratio $PQ : QR$

(1)

(Total for Question E9 is 4 marks)

E10 $\vec{OA} = \mathbf{a}$ and $\vec{OB} = \mathbf{b}$. M is the midpoint of OB . N is on AB with $AN : NB = 2 : 1$. X is on AM with $AX : XM = 4 : 1$

(a) Prove that O , X and N are collinear.

(b) Find the ratio $OX : XN$

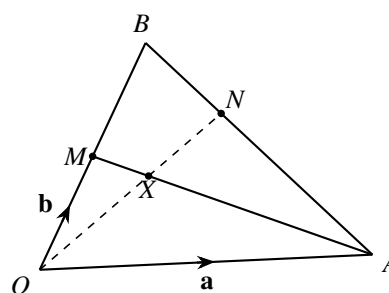


Diagram NOT accurately drawn

(Total for Question E10 is 5 marks)

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