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INTERNATIONAL A LEVEL MATHEMATICS • FURTHER PURE 2 (WFM02)

Further Pure 2 Practice Paper 1

Mark Scheme

AIMING FOR B

Written in the style of the Pearson Edexcel International Advanced Level paper (WFM02/01) • 75 marks

Every question marked line by line the way an examiner marks it, with notes on the common errors

Mark every line, not just the answer. Method marks are given for a correct method even when the final answer is wrong. If you lost marks, find the *first* line where your working differs from the scheme. That is the thing to fix.

Code	Meaning
M1	Method mark: for knowing a correct method and attempting to apply it. Small slips in the working do not lose this mark.
dM1	Dependent method mark: can only be given if the method mark before it has been given.
A1	Accuracy mark: for a correct answer or line of working. It depends on the method mark before it.
B1	Independent mark: for a correct result or statement, whatever method was used.
*	The answer is given in the question. Every step must be shown, and the answer must follow from correct working (cso).
cao / cso	Correct answer only / correct solution only, with no errors anywhere in the working.
ft / oe	Follow through from your own earlier answer / or any correct equivalent form.

"Solutions relying on calculator technology are not acceptable." Questions 1 and 9 say this. There, a correct final answer with no algebraic working scores **no marks**. Elsewhere a calculator may be used, but the method must still be seen.

Rough grade guide (out of 75): A* **69** • A **64** • B **57** • C **49**

Based on recent WFM02 grade boundaries. Real boundaries change every series, so treat this as a guide, not a grade.

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What to revise next

Write in your score for each question. Where you dropped marks, tap the chapter to practise ten more exam questions on it.

Q	What it tested	Marks	Practise it	Score
1	An inequality with two fractions, solved by multiplying by squares	6	Ch 1 Inequalities	
2	Partial fractions and the method of differences	7	Ch 2 Series	
3	Exponential form and the fifth roots of a complex number	7	Ch 3 Complex Numbers	
4	A first-order equation solved with an integrating factor	8	Ch 5 First-Order DEs	
5	A Maclaurin series by differentiation, then an estimate	8	Ch 7 Maclaurin and Taylor	
6	A circle and a half-line in the Argand diagram	8	Ch 4 Further Argand Diagrams	
7	A particular integral, the general solution and a particular solution	9	Ch 6 Second-Order DEs	
8	Tangents parallel to the initial line; an area in polar coordinates	10	Ch 8 Polar Coordinates	
9	de Moivre's theorem for $\cos^5\theta$; an exact integral	12	Ch 3 Complex Numbers	
	Total	**75**		

Where did the marks go? In Further Pure 2 the method is usually known, and the marks go on the details: an inequality multiplied by a denominator that could be negative, an argument outside the range asked for, a differential equation answer not written as $y = \ldots$, or a missing factor of $\frac{1}{2}$ in a polar area. Find the first line where your working differs from the scheme, then practise that chapter.

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Question 1

6 marks

Part	Scheme	Marks
	Multiply both sides by $(x-2)^2(x+1)^2$: $x(x-2)(x+1)^2 < 2x(x+1)(x-2)^2$	M1
	$x(x+1)(x-2)[(x+1)-2(x-2)] < 0$, so $x(x+1)(x-2)(5-x) < 0$	M1 A1
	Critical values -1, 0, 2, 5	B1
	$\boxed{x < -1,\ 0 < x < 2,\ x > 5}$	dM1 A1
	Multiplying by $(x-2)(x+1)$ only, without considering its sign, scores M0. Including $x = 0$ or $x = 5$ loses the final A1.	
		(6)
		Total for Question 1: 6 marks

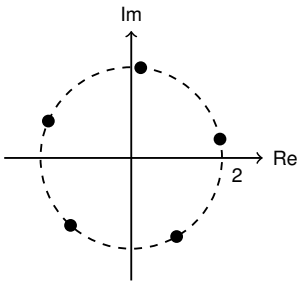
Question 2

7 marks

Part	Scheme	Marks
(a)	$\frac{A}{r+1} + \frac{B}{r+3}$, $4 = A(r+3) + B(r+1)$	M1
	$\boxed{\frac{2}{r+1} - \frac{2}{r+3}}$	A1
		(2)
(b)	$r = 1$: $\frac{2}{2} - \frac{2}{4}$; $r = 2$: $\frac{2}{3} - \frac{2}{5}$; …; $r = n$: $\frac{2}{n+1} - \frac{2}{n+3}$ (at least the first two and the last term)	M1
	Terms left: $\frac{2}{2} + \frac{2}{3} - \frac{2}{n+2} - \frac{2}{n+3}$	A1
	$= \frac{5}{3} - \frac{2(2n+5)}{(n+2)(n+3)}$	M1
	$= \frac{5(n+2)(n+3) - 6(2n+5)}{3(n+2)(n+3)} = \frac{5n^2+13n}{3(n+2)(n+3)}$	dM1
	$= \frac{n(5n+13)}{3(n+2)(n+3)}$	A1*
		(5)
		Total for Question 2: 7 marks

Question 3

7 marks

Part	Scheme	Marks
(a)	$\lvert w\rvert = \sqrt{16^2 + (16\sqrt{3})^2} = 32,\ \arg w = \frac{\pi}{3}$	M1
	$w = 32e^{i\pi/3}$	A1
		(2)
(b)	$z = 32^{\frac{1}{5}} e^{i\left(\frac{\pi}{3}+2k\pi\right)/5},\ \lvert z\rvert = 2$	M1 B1
	$\arg z = \frac{\pi}{15} + \frac{2k\pi}{5}$	M1
	$z = 2e^{i\theta},\ \theta = -\frac{11\pi}{15}, -\frac{\pi}{3}, \frac{\pi}{15}, \frac{7\pi}{15}, \frac{13\pi}{15}$	A1
	All five roots with arguments in $-\pi < \theta \leqslant \pi$ are needed for the final A1.	(4)
(c)	Five points equally spaced on a circle of radius 2, one at angle $\frac{\pi}{15}$ 	B1
		(1)
		Total for Question 3: 7 marks

Question 4

8 marks

Part	Scheme	Marks
(a)	$\frac{dy}{dx} + y\tan x = \cos^2 x$	M1
	Integrating factor $e^{\int \tan x\,dx} = e^{\ln\sec x} = \sec x$	M1 A1
	$y\sec x = \int \cos x\,dx = \sin x + c$	M1 A1
	$y = \sin x\cos x + c\cos x$	A1
	The answer must be given as $y = \ldots$ for the final A1.	(6)
(b)	$x = 0, y = 2$: $c = 2,\ y = \cos x(\sin x + 2)$	M1
	$\frac{\sqrt{3}}{4} + 1$	A1
		(2)
		Total for Question 4: 8 marks

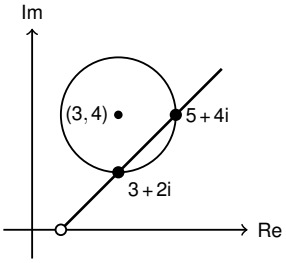
Question 5

8 marks

Part	Scheme	Marks
(a)	$f'(x) = \cos x e^{\sin x}$	B1
	$f''(x) = -\sin x e^{\sin x} + \cos x \times \cos x e^{\sin x}$ (product rule)	M1
	$= \left(\cos^2 x - \sin x\right) e^{\sin x}$	A1*
		(3)
(b)	$f'''(x) = (-2\sin x \cos x - \cos x)e^{\sin x} + \left(\cos^2 x - \sin x\right)\cos x e^{\sin x}$	M1
	$f(0) = 1, f'(0) = 1, f''(0) = 1, f'''(0) = -1 + 1 = 0$	A1
	$e^{\sin x} \approx 1 + x + \frac{1}{2}x^2 + 0x^3$	A1
		(3)
(c)	$\int_0^{0.2} \left(1 + x + \frac{1}{2}x^2\right) dx = 0.2 + 0.02 + \frac{0.008}{6}$	M1
	0.2213	A1
		(2)
		Total for Question 5: 8 marks

Question 6

8 marks

Part	Scheme	Marks
(a)	Circle, centre (3, 4), radius 2	B1
	Half-line from (1, 0) at 45° to the real axis	B1
	Both correctly placed, with (1, 0) an open end point	B1
		
		(3)
(b)	Half-line $y = x - 1, x > 1$; $(x-3)^2 + (x-5)^2 = 4$	M1
	$x^2 - 8x + 15 = 0$, $x = 3$ or $x = 5$	A1
	$3 + 2i$ and $5 + 4i$	A1
		(3)
(c)	$\lvert 3 + 4i \rvert + 2$	M1
	7	A1
		(2)
		Total for Question 6: 8 marks

Question 7

9 marks

Part	Scheme	Marks
(a)	$y' = -\lambda\sin x + \mu\cos x,\ \ y'' = -\lambda\cos x - \mu\sin x$	M1
	$(4\lambda + 2\mu)\cos x + (4\mu - 2\lambda)\sin x = 10\cos x$	M1 A1
	$\boxed{\lambda = 2,\ \mu = 1}$	A1
		(4)
(b)	$m^2 + 2m + 5 = 0,\ \ m = -1 \pm 2\text{i}$	M1
	$\boxed{y = \text{e}^{-x}(A\cos 2x + B\sin 2x) + 2\cos x + \sin x}$	A1
		(2)
(c)	$x = 0,\ y = 0\text{:}\ \ A + 2 = 0,\ \ A = -2$	B1
	$y' = -\text{e}^{-x}(A\cos 2x + B\sin 2x) + \text{e}^{-x}(-2A\sin 2x + 2B\cos 2x) - 2\sin x + \cos x;$ $-A + 2B + 1 = 0$	M1
	$\boxed{y = \text{e}^{-x}\left(-2\cos 2x - \frac{3}{2}\sin 2x\right) + 2\cos x + \sin x}$	A1
		(3)
		Total for Question 7: 9 marks

Question 8

10 marks

Part	Scheme	Marks
(a)	$y = r\sin\theta = 4\sin\theta + 4\sin\theta\cos\theta = 4\sin\theta + 2\sin 2\theta$	M1
	$\dfrac{\text{d}y}{\text{d}\theta} = 4\cos\theta + 4\cos 2\theta = 0$	M1 A1
	$2\cos^2\theta + \cos\theta - 1 = 0,\ \ \cos\theta = \frac{1}{2}\ \ (\cos\theta = -1$ gives the pole)	M1
	$\boxed{P\left(6, \frac{\pi}{3}\right),\ Q\left(6, -\frac{\pi}{3}\right)}$	A1
		(5)
(b)	$\frac{1}{2}\displaystyle\int_{-\pi/3}^{\pi/3} 16(1+\cos\theta)^2\,\text{d}\theta = 8\int\left(\frac{3}{2} + 2\cos\theta + \frac{1}{2}\cos 2\theta\right)\text{d}\theta$	M1
	$= 8\left[\frac{3}{2}\theta + 2\sin\theta + \frac{1}{4}\sin 2\theta\right]_{-\pi/3}^{\pi/3} = 8\pi + 18\sqrt{3}$	M1 A1
	Triangle OPQ: $P(3, 3\sqrt{3})$, $Q(3, -3\sqrt{3})$, area $\frac{1}{2}\times 6\sqrt{3}\times 3 = 9\sqrt{3}$	M1
	$\boxed{8\pi + 9\sqrt{3}}$	A1
	The area of R is the polar area from $-\frac{\pi}{3}$ to $\frac{\pi}{3}$ minus the triangle OPQ.	
		(5)
		Total for Question 8: 10 marks

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Question 9

12 marks

Part	Scheme	Marks
(a)	$z = \cos\theta + i\sin\theta$: $z^n + \frac{1}{z^n} = 2\cos n\theta$	B1
	$\left(z + \frac{1}{z}\right)^5 = z^5 + 5z^3 + 10z + \frac{10}{z} + \frac{5}{z^3} + \frac{1}{z^5}$	M1 A1
	$= \left(z^5 + \frac{1}{z^5}\right) + 5\left(z^3 + \frac{1}{z^3}\right) + 10\left(z + \frac{1}{z}\right)$	M1
	$32\cos^5\theta = 2\cos 5\theta + 10\cos 3\theta + 20\cos\theta$	A1
	$\cos^5\theta = \frac{1}{16}(\cos 5\theta + 5\cos 3\theta + 10\cos\theta)$	A1*
		(6)
(b)	$\frac{1}{16}\left[\frac{1}{5}\sin 5\theta + \frac{5}{3}\sin 3\theta + 10\sin\theta\right]_0^{\pi/3}$	M1 A1
	$= \frac{1}{16}\left(-\frac{\sqrt{3}}{10} + 0 + 5\sqrt{3}\right)$	M1
	$\boxed{\frac{49\sqrt{3}}{160}}$	A1
		(4)
(c)	$16\cos^5\theta = 0$, so $\cos\theta = 0$	M1
	$\boxed{\theta = \frac{\pi}{2}}$	A1
		(2)
		Total for Question 9: 12 marks

TOTAL FOR PAPER: 75 MARKS