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INTERNATIONAL A LEVEL MATHEMATICS • MECHANICS 2 (WME02)

Mechanics 2 Practice Paper 2

Worked Solutions

AIMING FOR A

Written in the style of the Pearson Edexcel International Advanced Level paper (WME02/01) • 75 marks

Every question solved in full on the paper itself, the way your teacher would write it out

How to read these solutions. Each page is the real question page, with a full answer written on the answer lines.

Red pen

M1 A1

boxed

the working a full-marks answer shows, with a short note on why each step is taken

the mark earned on that line: M = method, A = accuracy, B = independent mark

the final answer to each part

Sit the paper first. These solutions are most useful after you have tried every question yourself under timed conditions. Mark your paper with the mark scheme, then use this booklet to see exactly how each lost mark could have been earned.

Q	Topic	Marks	Page
1	A projectile hitting a wall	9	2
2	Variable acceleration and an impulse	10	4
3	A loaded wire frame on a slope	10	5
4	A slope and then a collision	11	6
5	A hinged rod and a string	11	7
6	Three spheres in a line	12	9
7	Force, power and work from a velocity	12	10

Question paper & mark scheme | Exam questions by topic | More practice papers

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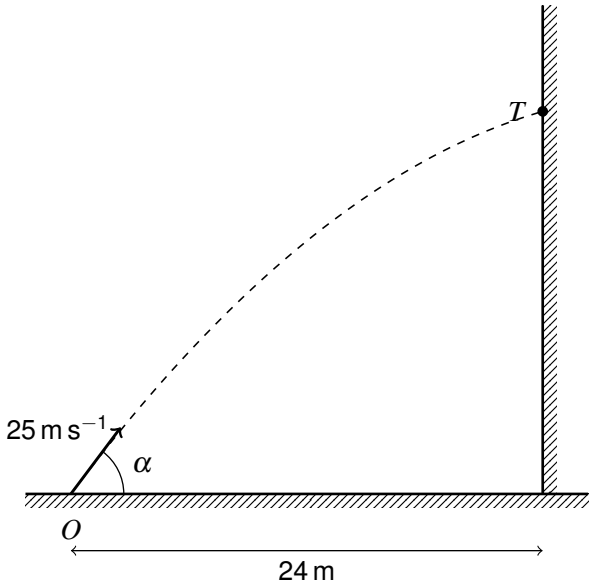

Figure 1

A ball is projected from a point O on horizontal ground with speed 25 m s^{-1} at an angle α above the horizontal, where $\tan\alpha = \frac{4}{3}$. The ball moves freely under gravity in a vertical plane perpendicular to a vertical wall, which is 24 m from O. The ball hits the wall at the point T, as shown in Figure 1.

(a) Find the height of T above the ground.

(3)

(b) Use the principle of conservation of mechanical energy to find the speed of the ball as it hits the wall.

(3)

(c) Find the direction of motion of the ball as it hits the wall.

(3)

(a) $\sin\alpha = 0.8$, $\cos\alpha = 0.6$: horizontal $25 \times 0.6 = 15$; vertical 20

Reaches the wall when $15t = 24 \Rightarrow t = 1.6$ s — M1

$h = 20 \times 1.6 - 4.9 \times 1.6^2 = 32 - 12.544 = 19.456$ — M1

19.5 m — A1

(b) Energy, O to T: $\frac{1}{2}mv^2 = \frac{1}{2}m(25^2) - mg(19.456)$ — M1

$v^2 = 625 - 2 \times 9.8 \times 19.456 = 625 - 381.34 = 243.66$ — A1

$v = 15.60\ldots$ 15.6 m s^{-1} — A1

(c) Vertical velocity at T: $20 - 9.8 \times 1.6 = 4.32$ (positive: still rising) — M1

$\tan\theta = 4.32 \div 15 \Rightarrow \theta = 16.06\ldots$ — M1

16.1° above the horizontal, moving upwards — A1

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Question 1 continued

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(Total for Question 1 is 9 marks)

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2. *In this question,* $\mathbf{i}$ *and* $\mathbf{j}$ *are perpendicular unit vectors in a horizontal plane.*

A particle P of mass 0.5 kg moves on a smooth horizontal plane. At time t seconds, $t \geqslant 0$, the acceleration of P is $(2\mathbf{i} - 6t\mathbf{j})\,\text{m s}^{-2}$. At time $t = 0$, the velocity of P is $(-4\mathbf{i} + 3\mathbf{j})\,\text{m s}^{-1}$

(a) Find the velocity of P at time t seconds.

(3)

(b) Find the speed of P at the instant when it is moving parallel to $\mathbf{i}$

(3)

At time $t = 2$, P receives an impulse. Immediately after the impulse, the velocity of P is $(3\mathbf{i} + 6\mathbf{j})\,\text{m s}^{-1}$

(c) Find the magnitude of the impulse.

(4)

(a) $v = \int (2i - 6t\,j)\,dt = 2t\,i - 3t^2\,j + c$ M1

$t = 0$: $c = -4i + 3j$ M1

$\boxed{v = (2t - 4)i + (3 - 3t^2)j}$ A1

(b) Parallel to i: the j component is zero: $3 - 3t^2 = 0$ M1

$t = 1$ ($t \geqslant 0$, so not -1): $v = (2 - 4)i = -2i$ A1

Speed $\boxed{2\ \text{m s}^{-1}}$ A1

(c) Just before the impulse, $t = 2$: $v = 0i + (3 - 12)j = -9j$ B1

$I = m(v - u) = 0.5((3i + 6j) - (-9j))$ M1

$= 0.5(3i + 15j) = 1.5i + 7.5j$ A1

$|I| = \sqrt{1.5^2 + 7.5^2} = \sqrt{58.5} = 7.648\ldots$ $\boxed{7.65\ \text{N s}}$ A1

(Total for Question 2 is 10 marks)

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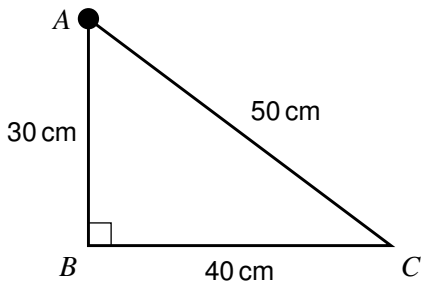

Figure 2

A uniform wire of mass 1.2 kg is bent to form a triangular frame ABC, where $AB = 30$ cm, $BC = 40$ cm, $CA = 50$ cm and angle $ABC = 90°$. A particle of mass 0.6 kg is attached to the frame at A, as shown in Figure 2.

(a) Find the distance of the centre of mass of the loaded frame from **(i)** AB **(ii)** BC

(6)

The loaded frame stands in a vertical plane with BC in contact with a rough plane inclined at an angle α to the horizontal. The edge BC lies along a line of greatest slope, with B below C. The coefficient of friction between the frame and the plane is 0.75. The angle α is gradually increased from zero.

(b) Determine whether the frame first slides or first topples, and find the value of α at which this happens.

(4)

(a) Mass in proportion to length: AB 0.3 kg, BC 0.4 kg, CA 0.5 kg B1

Centres: AB (0, 15), BC (20, 0), CA (20, 15); particle at A (0, 30)

(i) Moments about AB: $0.3 \times 0 + 0.4 \times 20 + 0.5 \times 20 + 0.6 \times 0 = 1.8\bar{x}$ M1

$18 = 1.8\bar{x}$ A1

$\bar{x} = 10$ 10 cm from AB A1

(ii) Moments about BC: $0.3 \times 15 + 0 + 0.5 \times 15 + 0.6 \times 30 = 1.8\bar{y} \Rightarrow 30 = 1.8\bar{y}$ M1

$\bar{y} = 16.66...$ 16.7 cm from BC A1

(b) Topples about B when G is vertically above B: $\tan\alpha = 10 \div 16.67 = 0.6$ M1

Slides when $\tan\alpha = \mu = 0.75$ M1

$0.6 < 0.75$, so the toppling angle comes first: it topples first A1

$\tan\alpha = 0.6 \Rightarrow \alpha = 31.0°$ A1

(Total for Question 3 is 10 marks)

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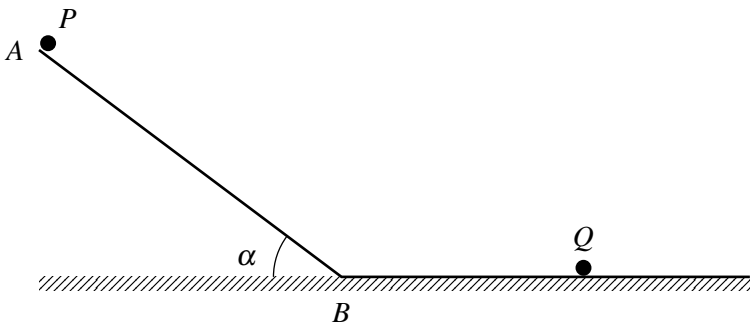

Figure 3

A particle P of mass 2 kg is released from rest at a point A on a rough plane inclined at an angle α to the horizontal, where $\sin\alpha = 0.6$. The coefficient of friction between P and the plane is 0.25. P slides down a line of greatest slope to the point B at the bottom of the plane, where $AB = 6.25$ m. At B, P moves onto a smooth horizontal floor with no change in speed, as shown in Figure 3.

On the floor, P collides directly with a particle Q of mass 3 kg, which is at rest. The coefficient of restitution between P and Q is $\frac{4}{7}$

(a) Use the work-energy principle to find the speed of P at B

(4)

(b) Find the speed of P and the speed of Q immediately after the collision.

(5)

(c) Find the kinetic energy lost in the collision.

(2)

(a) $\sin\alpha = 0.6$, $\cos\alpha = 0.8$. $R = 2g \times 0.8 = 15.68$; friction $= 0.25 \times 15.68 = 3.92$ N B1

Work-energy, A to B: KE gained = PE lost − work done against friction M1

$\frac{1}{2} \times 2 \times v^2 = 2g \times (6.25 \times 0.6) - 3.92 \times 6.25$ A1

$v^2 = 73.5 - 24.5 = 49$ $\boxed{7 \text{ m s}^{-1}}$ A1

(b) Towards Q positive. Momentum: $2 \times 7 + 0 = 2v_P + 3v_Q$ M1

$14 = 2v_P + 3v_Q$ A1

Restitution: $v_Q - v_P = (4/7) \times 7$ M1

$v_Q - v_P = 4 \Rightarrow 14 = 2v_P + 3(v_P + 4) \Rightarrow v_P = 0.4$ A1

$\boxed{P: 0.4 \text{ m s}^{-1}}$ (still forwards), $\boxed{Q: 4.4 \text{ m s}^{-1}}$ A1

(c) Before: 49 J. After: $\frac{1}{2} \times 2 \times 0.4^2 + \frac{1}{2} \times 3 \times 4.4^2 = 0.16 + 29.04 = 29.2$ J M1

Lost $= 49 - 29.2$ $\boxed{19.8 \text{ J}}$ A1

(Total for Question 4 is 11 marks)

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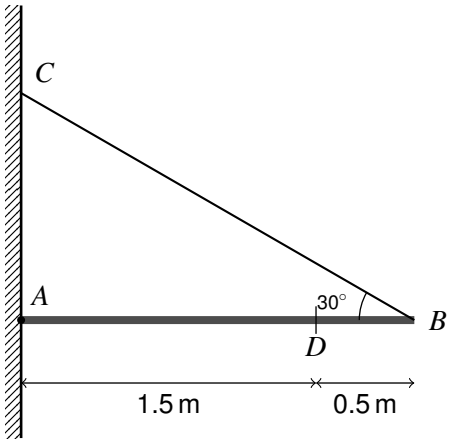

Figure 4

A rod AB of length 2 m is made of two uniform sections joined at D: section AD has length 1.5 m and mass 3 kg, and section DB has length 0.5 m and mass 3 kg. The end A is freely hinged to a fixed point on a vertical wall. The rod is held in equilibrium in a horizontal position, perpendicular to the wall, by a light inextensible string. One end of the string is attached to the rod at B and the other end is attached to the wall at the point C, vertically above A. Angle $ABC = 30°$, as shown in Figure 4.

(a) Find the distance of the centre of mass of the rod from A

(2)

(b) Find the tension in the string.

(3)

(c) Find the magnitude and direction of the force exerted on the rod by the hinge.

(6)

(a) AD: 3 kg at 0.75 m from A. DB: 3 kg at 1.5 + 0.25 = 1.75 m. $3 \times 0.75 + 3 \times 1.75 = 6\bar{x}$ M1

$\bar{x} = 7.5 \div 6$ 1.25 m A1

(b) Moments about A (the hinge force has no moment): $T \sin 30° \times 2 = 6g \times 1.25$ M1

$T = 7.5g$ A1

$T = 73.5$ N A1

(c) The string pulls B towards the wall, so the hinge pushes the rod away from the wall:

$H = T \cos 30° = 73.5 \times 0.866$ M1

$H = 63.65$ N, away from the wall A1

Vertical: $V + T \sin 30° = 6g \Rightarrow V = 58.8 - 36.75$ M1

$V = 22.05$ N, upwards A1

$R = \sqrt{63.65^2 + 22.05^2} = 67.36...$ 67.4 N A1

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Question 5 continued

$\tan\theta = 22.05 \div 63.65 \Rightarrow \theta = 19.1$

19.1° above the rod, away from the wall

A1

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(Total for Question 5 is 11 marks)

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6. Three small smooth spheres A, B and C, of equal radius and of masses m, $2m$ and $3m$ respectively, lie in a straight line on a smooth horizontal surface, with B between A and C. Spheres B and C are at rest. Sphere A is projected towards B with speed u and collides directly with B. The coefficient of restitution between each pair of spheres is e

(a) Find, in terms of u and e, the velocity of A and the velocity of B immediately after the collision.

(4)

B then collides directly with C

(b) Find, in terms of u and e, the velocity of B immediately after it collides with C

(3)

Given that A and B then collide again,

(c) find the range of possible values of e

(3)

(d) Given instead that $e = \frac{1}{4}$, find, in terms of m and u, the kinetic energy lost in the collision between A and B

(2)

(a) Towards B positive. Momentum: $mu = mv_A + 2mv_B$; restitution: $v_B - v_A = eu$ M1

$u = v_A + 2v_B$ and $v_B = v_A + eu \Rightarrow u = 3v_A + 2eu$ A1

$\boxed{v_A = u(1 - 2e)/3}$ A1

$\boxed{v_B = u(1 + e)/3}$ A1

(b) B (2m) at v_B hits C (3m) at rest: $2mv_B = 2mw + 3mc$, $c - w = ev_B$ M1

$2v_B = 2w + 3(w + ev_B) \Rightarrow 5w = v_B(2 - 3e)$ A1

$\boxed{w = u(1 + e)(2 - 3e)/15}$ A1

(c) A catches B if B is now slower: $u(1 + e)(2 - 3e)/15 < u(1 - 2e)/3$ M1

$2 - e - 3e^2 < 5 - 10e \Rightarrow 3e^2 - 9e + 3 > 0 \Rightarrow e^2 - 3e + 1 > 0$ A1

Roots $(3 \pm \sqrt{5})/2$; $e \leqslant 1$, so $\boxed{0 \leqslant e < (3 - \sqrt{5})/2 = 0.382}$ A1

(d) $e = 1/4$: $v_A = u/6$, $v_B = 5u/12$. Loss $= \frac{1}{2}mu^2 - \frac{1}{2}m(u/6)^2 - \frac{1}{2}(2m)(5u/12)^2$ M1

$= mu^2(72 - 1 - 25)/144 = 45mu^2/144$ $\boxed{5mu^2/16}$ A1

(Total for Question 6 is 12 marks)

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7. A particle P of mass 3 kg moves along a straight line under the action of a single force. At time t seconds, $t \geqslant 0$, the velocity of P is $(t^2 - 6t + 13)\,\text{m s}^{-1}$

(a) Find an expression, in terms of t, for the force acting on P at time t seconds.

(2)

(b) Find the least speed of P

(2)

(c) Find the rate at which the force is working when $t = 5$

(2)

(d) Find the work done by the force in the interval $0 \leqslant t \leqslant 5$

(3)

(e) Find the distance travelled by P in the interval $0 \leqslant t \leqslant 5$

(3)

(a) $a = dv/dt = 2t - 6$; $F = ma = 3(2t - 6)$ M1

$F = (6t - 18)$ N A1

(b) $v = t^2 - 6t + 13 = (t - 3)^2 + 4$, least when $t = 3$ M1

least speed 4 m s^{-1} A1

(c) $t = 5$: $F = 30 - 18 = 12$ N, $v = 25 - 30 + 13 = 8$. $P = Fv = 12 \times 8$ M1

96 W A1

(d) Work done = change in KE. $v(0) = 13$, $v(5) = 8$ M1

$W = \frac{1}{2} \times 3 \times 8^2 - \frac{1}{2} \times 3 \times 13^2 = 96 - 253.5$ A1

$W = -157.5$ J (the force mostly slows P down) A1

(e) $v \geqslant 4 > 0$, so P never turns back: distance = $\int$ from 0 to 5 of $(t^2 - 6t + 13)\,dt$ M1

$= [t^3/3 - 3t^2 + 13t]$ from 0 to 5 $= 125/3 - 75 + 65$ A1

$= 95/3 = 31.66\ldots$ 31.7 m A1

(Total for Question 7 is 12 marks)

TOTAL FOR PAPER IS 75 MARKS

END

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