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INTERNATIONAL A LEVEL MATHEMATICS • PURE 1 (WMA11)

Pure 1 Practice Paper 3

Mark Scheme

AIMING FOR A/A*

Written in the style of the Pearson Edexcel International Advanced Level paper (WMA11/01) • 75 marks

Every question marked line by line the way an examiner marks it, with notes on the common errors

Mark every line, not just the answer. Method marks are given for a correct method even when the final answer is wrong. If you lost marks, find the *first* line where your working differs from the scheme. That is the thing to fix.

Code	Meaning
M1	Method mark: for knowing a correct method and attempting to apply it. Small slips in the working do not lose this mark.
dM1	Dependent method mark: can only be given if the method mark before it has been given.
A1	Accuracy mark: for a correct answer or line of working. It depends on the method mark before it.
B1	Independent mark: for a correct result or statement, whatever method was used.
*	The answer is given in the question. Every step must be shown, and the answer must follow from correct working (cso).
cao / cso	Correct answer only / correct solution only, with no errors anywhere in the working.
ft / oe	Follow through from your own earlier answer / or any correct equivalent form.

"Solutions relying on calculator technology are not acceptable." Questions 1, 2, 4, 5 and 6, parts (a) and (b) of Question 3, parts (c) and (d) of Question 7 and parts (b) and (c) of Question 8 say this. There, a correct final answer with no algebraic working scores **no marks**.

Rough grade guide (out of 75): A **45** • B **38** • C **31**

Based on recent WMA11 grade boundaries. Real boundaries change every series, so treat this as a guide, not a grade.

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What to revise next

Write in your score for each question. Where you dropped marks, tap the chapter to practise ten more exam questions on it.

Q	What it tested	Marks	Practise it	Score
1	Equations that are quadratics in 3^x and in $x^{\frac{2}{3}}$	8	Ch 1 Algebraic Expressions; Ch 2 Quadratics	
2	Perpendicular gradients leading to a quadratic; area	9	Ch 5 Straight-line Graphs; Ch 2 Quadratics	
3	Cosine rule giving two positions; exact area; an obtuse angle	9	Ch 6 Trigonometric Ratios; Ch 2 Quadratics	
4	Finding a constant, then a normal meeting the curve again	10	Ch 8 Differentiation; Ch 3 Equations and Inequalities; Ch 5 Straight-line Graphs	
5	Integrating twice, with a condition given by a normal	9	Ch 9 Integration; Ch 8 Differentiation	
6	A radius from a chord condition; exact arc, perimeter and area	10	Ch 7 Radians; Ch 6 Trigonometric Ratios; Ch 1 Algebraic Expressions	
7	Sketching and solving with a cubic in terms of a	10	Ch 4 Graphs and Transformations; Ch 3 Equations and Inequalities	
8	Reciprocal graph and a line; an inequality solved from the sketch	10	Ch 4 Graphs and Transformations; Ch 3 Equations and Inequalities; Ch 1 Algebraic Expressions	
	Total	**75**		

Where did the marks go? At this level the method is rarely the problem. The marks are lost on completeness: the negative solutions in Question 1, the second position in Questions 2 and 3, the rejected root in Question 6 and the negative branch in Question 8. Before you write a final answer, ask yourself whether there could be another one.

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Question 1

8 marks

Part	Scheme	Marks
(a)	Writes $3^{2x+1} = 3(3^x)^2$ and forms a quadratic in $u = 3^x$: $3u^2 - 28u + 9 = 0$	M1
	Solves: $(3u - 1)(u - 9) = 0$	M1
	$3^x = \frac{1}{3}$ or $3^x = 9$	A1
	$\boxed{x = -1,\ x = 2}$	A1
		(4)
(b)	Lets $u = x^{\frac{2}{3}}$: $u^2 - 10u + 9 = 0$	M1
	$u = 1$ or $u = 9$	A1
	Solves $x^{\frac{2}{3}} = 9$: $\left(x^{\frac{1}{3}}\right)^2 = 9$, so $x^{\frac{1}{3}} = \pm 3$ and $x = \pm 27$	M1
	$\boxed{x = -27,\ -1,\ 1,\ 27}$	A1
	All four values are needed for the final mark. $x^{\frac{2}{3}} = \left(\sqrt[3]{x}\right)^2$ is defined for negative x, so $x = -1$ and $x = -27$ are genuine solutions: check $(-27)^{\frac{2}{3}} = (-3)^2 = 9$.	(4)
		Total for Question 1: 8 marks

Question 2

9 marks

Part	Scheme	Marks
(a)	Writes C as $(t, 2t + 3)$	M1
	Uses perpendicular gradients: $\frac{2t+3}{t-1} \times \frac{2t-3}{t-9} = -1$	M1
	$(2t + 3)(2t - 3) + (t - 1)(t - 9) = 0$	M1
	$5t^2 - 10t = 0$	A1
	Solves: $t = 0$ or $t = 2$	dM1
	$\boxed{C(0, 3)}$ and $\boxed{C(2, 7)}$	A1
	Pythagoras, $AC^2 + BC^2 = AB^2$, gives the same quadratic and is equally good.	(6)
(b)	Uses the right angle: area $= \frac{1}{2} \times AC \times BC$, e.g. for $C(2, 7)$: $AC = BC = \sqrt{50}$	M1
	Finds both areas, 15 and 25, or justifies which is larger	M1
	$\boxed{\text{Area} = 25}$	A1
		(3)
		Total for Question 2: 9 marks

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Question 3

9 marks

Part	Scheme	Marks
(a)	Cosine rule in triangle HLP with $HP = d$: $7^2 = 8^2 + d^2 - 2 \times 8 \times d \cos 60^\circ$	**M1**
	$d^2 - 8d + 15 = 0$	**A1**
	Solves: $(d-3)(d-5) = 0$	**M1**
	$\boxed{HP = 3\,\text{km},\ HQ = 5\,\text{km}}$	**A1**
		(4)
(b)	$PQ = 2\,\text{km}$	**B1**
	Height of L above the course $= 8\sin 60^\circ = 4\sqrt{3}$ (or area $HLQ -$ area HLP)	**M1**
	$\boxed{\text{Area} = 4\sqrt{3}\,\text{km}^2}$	**A1**
		(3)
(c)	$\cos HPL = \dfrac{3^2 + 7^2 - 8^2}{2 \times 3 \times 7} = -\dfrac{1}{7}$	**M1**
	$\boxed{98.2^\circ}$	**A1**
	The angle is obtuse. Using the sine rule here gives 81.8°*, the wrong one of the two angles with the same sine.*	
		(2)
		Total for Question 3: 9 marks

Question 4

10 marks

Part	Scheme	Marks
(a)	$\frac{dy}{dx} = 1 - \frac{k}{x^2}$, and $1 - k = -3$	M1
	$\boxed{k = 4}$	A1
		(2)
(b)	$P(1, 5)$	B1
	Normal gradient $\frac{1}{3}$: $y - 5 = \frac{1}{3}(x - 1)$	M1
	Sets $x + \frac{4}{x} = 5 + \frac{x-1}{3}$ and multiplies by $3x$	M1
	$2x^2 - 14x + 12 = 0$, i.e. $(x - 1)(x - 6) = 0$	A1
	Uses the root other than $x = 1$: $x = 6$	dM1
	$\boxed{Q\left(6, \frac{20}{3}\right)}$	A1
	Knowing that $x = 1$ must be a root is a useful check on the quadratic.	
		(6)
(c)	$PQ^2 = 5^2 + \left(\frac{5}{3}\right)^2 = \frac{250}{9}$	M1
	$\boxed{PQ = \frac{5\sqrt{10}}{3}}$	A1
		(2)
		Total for Question 4: 10 marks

Question 5

9 marks

Part	Scheme	Marks
(a)	Integrates $3x^{-\frac{1}{2}} + 2$: $f'(x) = 6x^{\frac{1}{2}} + 2x + A$	M1 A1
	Normal gradient $\frac{1-2}{5-1} = -\frac{1}{4}$, so the gradient of C at P is 4	B1
	$f'(1) = 6 + 2 + A = 4$, so $A = -4$	M1
	Integrates again: $f(x) = 4x^{\frac{3}{2}} + x^2 - 4x + B$	M1 A1
	Uses $P(1, 2)$: $B = 1$, so $\boxed{f(x) = 4x^{\frac{3}{2}} + x^2 - 4x + 1}$	A1
	The common error is to take the gradient of the normal, $-\frac{1}{4}$, as $f'(1)$. That gives $A = -\frac{33}{4}$ and loses the B1 and both final A marks.	
		(7)
(b)	$6\sqrt{x} + 2x - 4 = 4 \Rightarrow x + 3\sqrt{x} - 4 = 0 \Rightarrow \left(\sqrt{x} + 4\right)\left(\sqrt{x} - 1\right) = 0$	M1
	$\sqrt{x} = -4$ is impossible, so $\sqrt{x} = 1$ and $x = 1$ only: P is the only such point (cso)	A1
		(2)
		Total for Question 5: 9 marks

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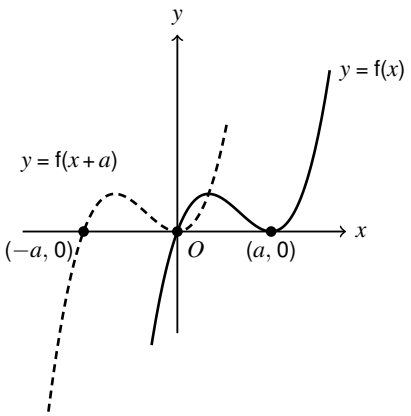
Question 6

10 marks

Part	Scheme	Marks
(a)	Cosine rule: $(r+2)^2 = r^2 + r^2 - 2r^2\cos\frac{2\pi}{3} = 3r^2$	**M1**
	$r^2 - 2r - 2 = 0$ (or $r+2 = \sqrt{3}r$)	**A1**
	Solves: $r = 1 \pm \sqrt{3}$, or $r = \dfrac{2}{\sqrt{3}-1}$ and rationalises	**M1**
	$\boxed{r = 1+\sqrt{3}}$, rejecting $1-\sqrt{3} < 0$	**A1**
		(4)
(b)	Arc $AB = \frac{2\pi}{3}\left(1+\sqrt{3}\right)$	**M1**
	Adds the chord $r+2 = 3+\sqrt{3}$	**M1**
	$\boxed{\text{Perimeter} = \dfrac{2\pi}{3}\left(1+\sqrt{3}\right)+3+\sqrt{3}}$	**A1**
		(3)
(c)	Area $= \frac{1}{2}r^2\left(\frac{2\pi}{3} - \sin\frac{2\pi}{3}\right)$ with $r^2 = 4+2\sqrt{3}$	**M1**
	$= \left(2+\sqrt{3}\right)\left(\frac{2\pi}{3} - \frac{\sqrt{3}}{2}\right)$, expanded	**M1**
	$\boxed{\left(\frac{4}{3}+\frac{2}{3}\sqrt{3}\right)\pi - \frac{3}{2} - \sqrt{3}}$	**A1**
	Equivalent exact forms are accepted. A decimal answer, 4.58*, scores M1 M1 A0.*	
		(3)
		Total for Question 6: 10 marks

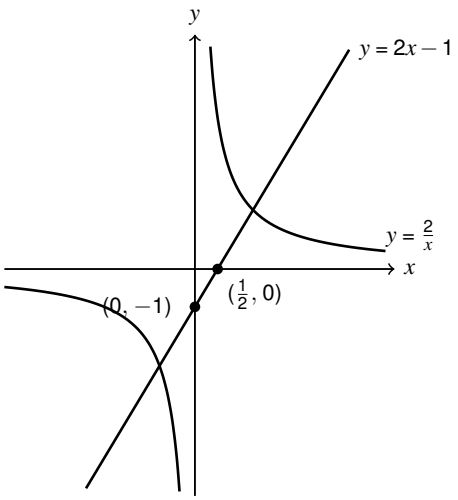
Question 7

10 marks

Part	Scheme	Marks
(a)	 A positive cubic through O that touches the x-axis at a minimum	B1
	$(0, 0)$ and $(a, 0)$ labelled	B1
		(2)
(b)	$y = f(x+a) = x^2(x+a)$: the same shape, translated a to the left, touching the axis at O	B1
	Crosses the axis at $(-a, 0)$, with both curves drawn on the same axes	B1
		(2)
(c)	$x(x-a)^2 = x^2(x+a)$	M1
	$x\left[(x-a)^2 - x(x+a)\right] = 0$, i.e. $ax(a-3x) = 0$	M1
	$\boxed{(0, 0)}$ and $\boxed{\left(\frac{a}{3}, \frac{4a^3}{27}\right)}$	A1
	Dividing both sides by x without comment loses the origin. The y coordinate must be simplified.	
		(3)
(d)	$\frac{4a^3}{27} = \frac{a}{3}$	M1
	$a^2 = \frac{9}{4}$	M1
	$\boxed{a = \frac{3}{2}}$, since $a > 0$	A1
		(3)
	Total for Question 7: 10 marks	

Question 8

10 marks

Part	Scheme	Marks
(a)	 C: two branches, in the first and third quadrants Asymptotes of C: $x = 0$ and $y = 0$ l drawn with $\left(\frac{1}{2}, 0\right)$ and $(0, -1)$, meeting each branch once	B1 B1 B1 (3)
(b)	$\frac{2}{x} = 2x - 1 \Rightarrow 2x^2 - x - 2 = 0$ $x = \frac{1 \pm \sqrt{17}}{4}$ $\left(\frac{1+\sqrt{17}}{4}, \frac{-1+\sqrt{17}}{2}\right)$ and $\left(\frac{1-\sqrt{17}}{4}, \frac{-1-\sqrt{17}}{2}\right)$	M1 M1 A1 A1 (4)
(c)	Uses the sketch: C is above l to the left of the lower intersection and between $x = 0$ and the upper intersection $x < \frac{1-\sqrt{17}}{4}$ $0 < x < \frac{1+\sqrt{17}}{4}$ *Multiplying both sides by x is not valid, because x can be negative. It leads to the wrong answer $\frac{1-\sqrt{17}}{4} < x < \frac{1+\sqrt{17}}{4}$, which scores M0. Multiplying by x^2, which is positive, is a valid alternative.*	M1 A1 A1 (3)
		Total for Question 8: 10 marks

TOTAL FOR PAPER: 75 MARKS

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