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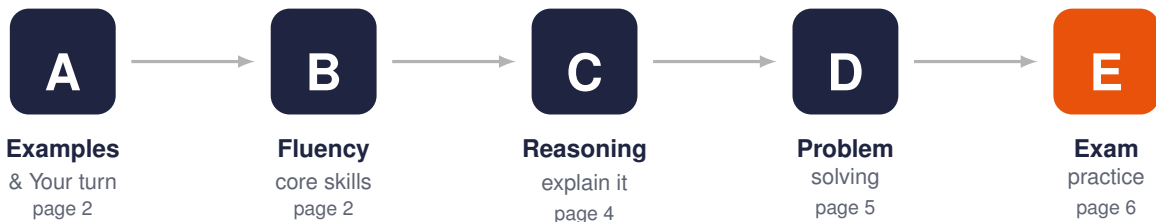
GCSE MATHEMATICS • RATIO & PROPORTION

Direct & Inverse Proportion

Worked Solutions

Pearson Edexcel GCSE (1MA1) & International GCSE (4MA1) • Calculator only where shown

Full worked solutions for the free Direct and Inverse Proportion worksheet, GCSE and iGCSE Maths



Tap a section to jump straight to it

Full worked solutions for every question on the Direct & Inverse Proportion worksheet, in the same order and with the same question numbers.

How to mark your work. Tick each step you got right, not just the final answer. In the exam, method marks are given for correct working even when the final answer is wrong. If you lost marks, find the *first* line where your working is different from ours. That is the thing to fix.

In Section E: **M** = method mark, **P** = process mark, **A** = accuracy mark, **B** = independent mark, **C** = communication mark.

Question sheet

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A**Your turn answers**

Compare each line of your working with the solution.

FOUNDATION

A1 1 m costs $9.45 \div 7 = 1.35$ $1.35 \times 4 = 5.40$ **£5.40**

A2 A: $221 \div 340 = 0.65p$ per gram B: $297 \div 450 = 0.66p$ per gram **Jar A** is better value.

A3 1 cleaner: $5 \times 12 = 60$ hours 3 cleaners: $60 \div 3 =$ **20 hours**
Fewer cleaners take longer. If your answer was less than 12 hours, check which way the proportion goes.

HIGHER

A4 $p = kq^3$ $40 = k \times 8$, so $k = 5$ $p = 5 \times 27 =$ **135**

A5 $F = \frac{k}{d^2}$ $20 = \frac{k}{9}$, so $k = 180$ $F = \frac{180}{36} =$ **5**

A6 $k \left(\frac{x}{2}\right)^3 = \frac{kx^3}{8}$ **y is divided by 8**

A7 $q = mr^2$, so $p = \frac{k}{mr^2} = \frac{K}{r^2}$ $9 = \frac{K}{4}$, so $K = 36$ $p = \frac{36}{9} =$ **4**

B**Fluency solutions**

One mark for each correct part. The notes in italics explain why each sequence was chosen.

FOUNDATION

B1 (a) **£4.80** (b) **£12.00** (c) $2.40 \div 3 =$ **£0.80** (d) $0.80 \times 10 =$ **£8.00**

When the new number is not a multiple of 3, go through 1 pen first.

B2 (a) **600 g** (b) $180 \div 4 \times 6 =$ **270 ml** (c) $60 \div 4 \times 10 =$ **150 g** (d) $300 \div 4 \times 3 =$ **225 g**

Do not add or subtract people: 6 people is 1.5 times the recipe, not "2 more".

- B3** (a) 80p or 75p each: **6 for £4.50** (b) 0.37p or 0.36p per gram: **750 g** (c) 22p or 22.5p each: **12 eggs**

In (c) the bigger pack is **not** better value. Always work it out.

- B4** $6 \times 8 = 48$ worker-days. (a) **4** (b) **16** (c) **12** (d) $48 \div 5 =$ **9.6**

More workers, fewer days. The total work, 48 worker-days, stays the same.

- B5** (a) **direct** (b) **inverse** (c) **neither** (d) **inverse**

In (c) both increase, but doubling a child's age does not double their height, so it is not proportion.

- B6** $y \div x = 3.5$: (a) **35** (b) **63** $x \times y = 60$: (c) **6** (d) **30**

- B7** (a) **20** (b) $35 \div 2.5 =$ **14** (c) **$y = 2.5x$** (d) **250**

The graph stops at $x = 10$. Beyond it, use the formula.

HIGHER

- B8** (a) **$y = 3x$** (b) $50 = 25k$ **$y = 2x^2$** (c) $6 = \frac{k}{4}$ **$y = \frac{24}{x}$** (d) $2 = \frac{k}{9}$ **$y = \frac{18}{x^2}$**

- B9** (a) **48** (b) $x^2 = 25$ **$x = 5$** (c) $36 \div 3 =$ **12** (d) $\sqrt{x} = 9$ **$x = 81$**

- B10** (a) 2^2 : **$\times 4$** (b) 3^3 : **$\times 27$** (c) **$\div 5$** (d) $\sqrt{9} = 3$: **$\div 3$**

Apply the power to the multiplier. For inverse proportion, divide instead of multiply.

- B11** (a) $1.1^2 = 1.21$ **increase of 21%** (b) $1 \div 1.25 = 0.8$ **decrease of 20%**
(c) $0.8^3 = 0.512$ **decrease of 48.8%** (d) $\sqrt{1.44} = 1.2$ **increase of 20%**

A 10% increase in x does not give 20% in y : work with the multiplier.



C

Reasoning solutions

Your wording can be different. Check that your reason makes the same point.

FOUNDATION

- C1** (a) More workers take **less** time. This is inverse proportion, not direct proportion.
 (b) $3 \times 12 = 36$ worker-days $36 \div 6 = \boxed{6 \text{ days}}$
- C2** $6 \div 2 = 3$ and $15 \div 5 = 3$, but $25 \div 8 = 3.125$ **No.** The ratio $y : x$ is not the same for every pair.
- C3** **No.** 6 people is 1.5 times the recipe, so she needs $3 \times 1.5 = \boxed{4.5}$ eggs, not 5. Jo added instead of multiplying.
- C4** Distance = $40 \times \frac{45}{60} = 30$ miles Time = $30 \div 50 = 0.6$ hours = $0.6 \times 60 = \boxed{36}$ minutes as required

HIGHER

- C5** (a) **A** (b) **B** and **C**
 (c) Graph D does not pass through the origin: when $x = 0$, y is not 0
- C6** (a) Ben divided 8 by 4. He should multiply: $8 \times 4 = 32$, so $k = 32$
 (b) $y = \frac{32}{16} = \boxed{2}$
- C7** (a) $y = kx^n$. Doubling x multiplies y by 2^n , and $2^n = 8$, so $\boxed{n = 3}$
 (b) $54 = k \times 27$, so $k = 2$ $y = 2 \times 125 = \boxed{250}$
- C8** $xy = k$ for every point, so $12a = 8(a+3)$ $12a = 8a + 24$ $\boxed{a = 6}$
 $k = 12 \times 6 = 72$ When $x = 16$: $y = 72 \div 16 = \boxed{4.5}$



D**Problem solving solutions**

There is often more than one correct method. Any complete, correct method scores full marks.

FOUNDATION

- D1** 1 kg takes $72 \div 1.6 = 45$ minutes $45 \times 2.2 = 99$ minutes = **1 hour 39 minutes**
- D2** A: $300 \div 4 = 75$ p per tin. B: $435 \div 6 = 72.5$ p per tin. B is cheaper per tin, but the packs are different sizes.
 5 packs of A: £15.00 for 20 tins 4 packs of B: £17.40 for 24 tins
 2 packs of each: $6.00 + 8.70 = 14.70$, for 20 tins
 The least Tom can pay is **£14.70** Check: no other mix of packs giving at least 20 tins is cheaper.
- D3** Total work = $8 \times 15 = 120$ worker-days. After 6 days, $8 \times 6 = 48$ are done, so 72 are left.
 6 workers: $72 \div 6 = 12$ more days Total = $6 + 12 =$ **18 days**
- D4** $520 \div 80 = 6.5$ Fuel = $6.5 \times 6 = 39$ litres $39 \times 1.42 =$ **£55.38**

HIGHER

- D5** $d = kt^2$ $20 = 4k$, so $k = 5$
 First ball: $180 = 5t^2$, $t^2 = 36$, $t = 6$ seconds. The second ball has fallen for 4 seconds: $5 \times 16 = 80$ m
 Height above the ground = $180 - 80 =$ **100 m**
- D6** The brightness falls by 75%, so it is multiplied by $\frac{1}{4}$
 Brightness $\propto \frac{1}{d^2}$, so d^2 is multiplied by 4 and d is multiplied by 2. New distance 4 m, moved **2 m**
- D7** $C = kd^2$ $6 = 100k$, so $k = 0.06$
 (a) $0.06 \times 196 =$ **£11.76**
 (b) $0.06d^2 \leq 20$, so $d^2 \leq 333.3\ldots$ and $d \leq 18.25\ldots$ **18 inches**
 Check: 18 inches costs £19.44, 19 inches costs £21.66
- D8** $T = \frac{k}{r^2}$ $45 = \frac{k}{4}$, so $k = 180$
 Radius 3 cm alone: $T = 180 \div 9 = 20$ minutes. Radius 1 cm alone: $T = 180 \div 1 = 180$ minutes.
 Per minute they fill $\frac{1}{20} + \frac{1}{180} = \frac{9}{180} + \frac{1}{180} = \frac{10}{180} = \frac{1}{18}$ of the tank **18 minutes**
 You cannot add the times. Add the fractions of the tank filled each minute.



E**Exam practice solutions**

Marks are shown on the right of each line.

FOUNDATION*Foundation and Higher crossover*

- E1** $15 \div 6 = 2.5$ M1
1125 g flour 450 g butter 300 g sugar 750 ml milk A2
A1 for any 2 or 3 correct.
-
- E2** Small: $4.20 \div 1.5 = 2.80$ per kg M1
Large: $6.48 \div 2.4 = 2.70$ per kg M1
Large box is better value. C1
-
- E3** (a) $6 \times 10 = 60$ M1
 $60 \div 4 =$ 15 days A1
(b) All the builders work at the same rate. C1
-
- E4** For 1 flapjack: oats 25 g, butter 15 g, syrup 11.25 g P1
Oats: $500 \div 25 = 20$ butter: $270 \div 15 = 18$ syrup: $250 \div 11.25 = 22.2\dots$ P1
The butter runs out first. P1
18 A1
-
- E5** (a) £15 B1
(b) 1 m costs $30 \div 5 = 6$ M1
 $6 \times 40 =$ £240 A1
-
- E6** $3 \times 8 = 24$ decorator-hours for 4 rooms, so 6 for 1 room P1
6 rooms: $6 \times 6 = 36$ decorator-hours P1
 $36 \div 4 =$ 9 hours A1



HIGHER

E7 (a) $y = kx^2$ M1
 $45 = 9k$ M1
 $y = 5x^2$ A1
(b) $5 \times 25 = 125$ B1

E8 (a) $p = \frac{k}{\sqrt{q}}$ M1
 $5 = \frac{k}{4}$, so $k = 20$ M1
 $p = \frac{20}{10} = 2$ A1
(b) $\sqrt{q} = 2$, so $q = 4$ B1

E9 1.2^3 M1
 $= 1.728$ M1
 72.8% A1
Not $3 \times 20 = 60\%$.

E10 (a) $P = kQ^3$: $54 = 216k$, so $k = \frac{1}{4}$ M1
 $Q = \frac{m}{R}$: $6 = \frac{m}{2}$, so $m = 12$ M1
 $P = \frac{1}{4} \left(\frac{12}{R} \right)^3 = \frac{1728}{4R^3}$ M1
 $P = \frac{432}{R^3}$ A1
(b) $\frac{432}{27} = 16$ B1

