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INTERNATIONAL A LEVEL MATHEMATICS • STATISTICS 2

Statistics 2

Chapter 1: Binomial Distributions

Exam Questions by Topic

Pearson Edexcel International Advanced Level (WST02/01) • Calculator and Statistical Tables allowed

Multi-step exam questions at the difficulty of the real WST02 paper, with a full mark scheme

Chapters (tap a chapter to jump to it)		Qs	Marks	Page	My score
1	**Binomial Distributions** Conditions, probabilities and tables, mean and variance, unknown n and p	10	80	2	
2	**Poisson Distributions** Probabilities, modelling conditions, scaling the rate, adding Poisson variables	10	80	separate file	
3	**Approximations** Poisson and normal approximations, continuity corrections	10	80	separate file	
4	**Continuous Random Variables** Probability density and cumulative distribution functions, mean, variance, median, mode	10	81	separate file	
5	**Continuous Uniform Distribution** Rectangular distribution, mean and variance, modelling	10	80	separate file	
6	**Sampling and Sampling Distributions** Populations and samples, statistics, sampling distributions	10	80	separate file	
7	**Hypothesis Testing** Critical regions, significance levels, binomial and Poisson tests	10	80	separate file	

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CHAPTER 1 • KEY SKILLS

1 Binomial Distributions

Specification S2: Binomial distribution • 10 questions • 80 marks • about 96 minutes

1 The model

A fixed number of independent trials, each a success or failure, with constant probability p. Write $X \sim \mathrm{B}(n, p)$, with both parameters.

2 The formula

$\mathrm{P}(X = r) = \binom{n}{r} p^r (1-p)^{n-r}$. Use it for a single value, or when n or p is not in the tables.

3 Using the tables

Tables give $\mathrm{P}(X \leqslant x)$.

$\mathrm{P}(X \geqslant 5) = 1 - \mathrm{P}(X \leqslant 4)$;

$\mathrm{P}(2 \leqslant X < 6) = \mathrm{P}(X \leqslant 5) - \mathrm{P}(X \leqslant 1)$

4 When $p > 0.5$

Count failures instead: if $X \sim \mathrm{B}(20, 0.9)$ then $Y = 20 - X \sim \mathrm{B}(20, 0.1)$, and $\mathrm{P}(X \geqslant 18) = \mathrm{P}(Y \leqslant 2)$

5 Mean and variance

$\mathrm{E}(X) = np$ and $\mathrm{Var}(X) = np(1-p)$.

Square a standard deviation first; $\frac{\mathrm{Var}(X)}{\mathrm{E}(X)} = 1 - p$

6 Conditional probability

$\mathrm{P}(A \mid B) = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$, e.g.

$\mathrm{P}(X = 3 \mid X \leqslant 4) = \dfrac{\mathrm{P}(X = 3)}{\mathrm{P}(X \leqslant 4)}$

7 Two-stage models

First find the probability of one "success" (a round won, a bad week), then use a new binomial for how many of these happen.

8 Finding n

$\mathrm{P}(\text{at least one}) = 1 - (1-p)^n$. Solve $(1-p)^n < k$ with logarithms, then round n up.

9 Accuracy

Table values to 4 decimal places; other answers to 3 significant figures. Show the probability statement before the value.

Skills map every skill above is tested at least once; interleaved skills come from GCSE or other chapters

Skill	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Marks	6	6	6	6	6	10	10	10	10	10
1 The model	•	•			•	•	•			
2 The formula	•	•		•		•		•	•	•
3 Using the tables	•		•	•		•	•	•	•	•
4 When $p > 0.5$		•								
5 Mean and variance			•			•	•	•		•
6 Conditional probability				•			•	•		
7 Two-stage models						•	•		•	
8 Finding n					•					
9 Accuracy	•	•	•	•		•	•	•	•	•
Interleaved: probability, tree diagrams, expectation (S1)				•		•	•		•	
Interleaved: logarithms and inequalities (P1, P2)					•					•
Interleaved: algebra and solving equations (P1)			•					•	•	•

Exam difficulty. Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WST02 paper, where one part leads into the next and skills from different topics are combined.

Must learn. Define the random variable and its distribution in full, e.g. "X = number of seeds that grow, $X \sim \mathrm{B}(12, 0.3)$". Write the probability statement, then its rewriting for the tables, then the value.

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1. A spinner lands on red with probability 0.3 each time it is spun. The spinner is spun 12 times, and X is the number of times it lands on red.

(a) Write down the distribution of X, giving any parameters.

(1)

(b) Find P($X = 4$)

(2)

(c) Find P($X \geqslant 5$)

(2)

(d) Find P($2 \leqslant X < 6$)

(1)

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Question 1 continued

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(Total for Question 1 is 6 marks)

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2. The probability that a battery of a particular type lasts longer than its guaranteed life is 0.9. Batteries are sold in packs of 20, and the batteries in a pack may be regarded as independent.

(a) Find the probability that at least 18 batteries in a pack last longer than their guaranteed life.

(3)

(b) Find the probability that all 20 batteries in a pack last longer than their guaranteed life.

(1)

(c) Find the probability that at most 16 batteries in a pack last longer than their guaranteed life.

(2)

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Question 2 continued

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3. The random variable $X \sim \text{B}(n, p)$. Given that $\text{E}(X) = 6$ and $\text{Var}(X) = 4.2$

(a) Find the value of n and the value of p.

(3)

(b) Find $\text{P}(X < 4)$

(1)

(c) Find the probability that X is more than one standard deviation above its mean.

(2)

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Question 3 continued

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4. The random variable $X \sim \text{B}(10, 0.4)$

(a) Find $\text{P}(X \leqslant 4)$

(1)

(b) Find $\text{P}(X = 3)$

(2)

(c) Find $\text{P}(X = 3 \mid X \leqslant 4)$

(3)

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Question 4 continued

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5. A machine produces components, and 5% of its components are defective, independently of each other. A random sample of n components is taken.

(a) Show that the probability that the sample contains at least one defective component is greater than 0.9 when $0.95^n < 0.1$

(2)

(b) Find the smallest value of n for which the probability that the sample contains at least one defective component is greater than 0.9

(3)

(c) State one assumption needed for the binomial model to be valid here.

(1)

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Question 5 continued

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6. In a game, a player throws a ball at a target 5 times. On each throw the probability of hitting the target is 0.4, independently of all other throws. The player wins the round if the target is hit at least 3 times.

(a) Show that the probability of winning a round is 0.3174

(3)

A player plays 8 rounds.

(b) Find the probability that the player wins exactly 3 rounds.

(3)

(c) Find the probability that the player's first win is in the fourth round.

(2)

(d) Find the mean and the variance of the number of rounds won in 8 rounds.

(2)

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Question 6 continued

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7. A factory has two machines, A and B. Machine A makes 60% of the factory's items and 5% of these are defective. Machine B makes the rest of the items and 17.5% of these are defective.

(a) Show that the probability that a randomly chosen item is defective is 0.1

(2)

A random sample of 20 items is taken from the factory's production.

(b) Find the probability that the sample contains more than 3 defective items.

(2)

(c) Given that an item is defective, find the probability that it was made by machine A.

(2)

A batch of 50 items is checked.

(d) Find the mean and the standard deviation of the number of defective items in the batch.

(2)

(e) Find the probability that the number of defective items in the batch is no more than its mean.

(2)

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8. The random variable $X \sim \text{B}(7, p)$, where $0 < p < 1$. Given that $\text{P}(X = 2) = 2\,\text{P}(X = 1)$

(a) Show that $p = 0.4$

(4)

(b) Find $\text{P}(X \geqslant 4)$

(2)

(c) Find $\text{P}(X = 5 \mid X \geqslant 4)$

(2)

(d) Find $\text{E}(X)$ and $\text{Var}(X)$.

(2)

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Question 8 continued

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9. At a fair, a player pays 50p to play a game. A spinner, which lands on red with probability 0.3, is spun 5 times. If it lands on red at least 3 times, the player receives a prize of £k; otherwise the player receives nothing.

(a) Find the probability that a player wins the prize.

(2)

(b) Find the value of k for which the game is fair, so that the expected amount a player receives equals the cost of playing.

(2)

The organiser sets $k = 3$

(c) Find the organiser's expected profit from 200 games.

(3)

(d) Find the probability that the organiser makes a loss over 10 games, and comment on your answer in the light of part (c).

(3)

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Question 9 continued

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10. The random variable $Y \sim \text{B}(n, p)$ has mean 6 and variance 3.6

(a) Find the value of n and the value of p.

(3)

(b) Show that $\dfrac{\text{P}(Y = r+1)}{\text{P}(Y = r)} = \dfrac{2(15-r)}{3(r+1)}$ and hence show that the mode of Y is 6

(4)

(c) Find $\text{P}(Y = 6)$

(1)

(d) Find $\text{P}(|Y-6| \leqslant 2)$

(2)

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TOTAL FOR CHAPTER 1 IS 80 MARKS

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