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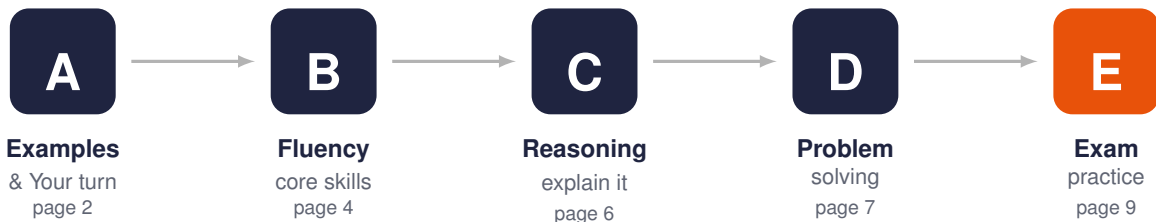
A-LEVEL MATHEMATICS • STATISTICS 1

Discrete Random Variables

Worksheet

Pearson Edexcel International A Level (WST01) & A Level (9MA0) • Calculator and statistical tables allowed: show every step

Free Discrete Random Variables worksheet for A-Level and International A-Level Maths, with full worked solutions



Tap a section to jump straight to it

By the end of this worksheet you can: find unknowns in probability distributions • use the cumulative distribution function • calculate $E(X)$ and $\text{Var}(X)$ • use $E(aX + b)$ and $\text{Var}(aX + b)$ • *Stretch:* the discrete uniform distribution, functions of X , and probabilities for two observations.

Which questions are for me?

CORE The methods every A-level student must secure. Start here.

STRETCH Unknown constants, cumulative functions with formulae, coded variables, and two observations at once.

Every section has both, clearly labelled. Questions get harder as you go, and every skill in Sections A to D is needed in Section E.

Track your progress

Section	B	C	D	E
Core marks	23	10	13	21
Stretch marks	16	14	17	12
My score				

Full worked solutions

Exam questions by topic

Statistics 1 exam questions

Formula sheet builder

Name

Class

Date



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A

Worked examples & Your turn

Read each example carefully. Cover it up, then answer the question beside it using the same method.

CORE

Secure the method

Worked example A1: a probability function

$P(X = x) = kx$ for $x = 1, 2, 3, 4$. Find k

The probabilities add to 1

$$k(1 + 2 + 3 + 4) = 10k = 1$$

$$k = \frac{1}{10}$$

Your turn A1

$P(X = x) = k(x + 2)$ for $x = 1, 2, 3$. Find k

Worked example A2: the cumulative distribution function

$F(1) = 0.2$, $F(2) = 0.5$, $F(3) = 0.85$, $F(4) = 1$. Find $P(X = 3)$ and $P(X \geq 2)$

$$P(X = 3) = F(3) - F(2); \quad P(X \geq 2) = 1 - F(1)$$

$$0.35, \quad 0.8$$

Your turn A2

$F(1) = 0.15$, $F(2) = 0.4$, $F(3) = 0.75$, $F(4) = 1$. Find $P(X = 2)$ and $P(X \geq 3)$

Worked example A3: expectation and variance

Find $E(X)$ and $\text{Var}(X)$

x	1	2	3
$P(X = x)$	0.2	0.5	0.3

$$E(X) = 0.2 + 1 + 0.9 = 2.1; \quad E(X^2) = 0.2 + 2 + 2.7 = 4.9$$

$$\text{Var}(X) = E(X^2) - [E(X)]^2$$

$$2.1, \quad 0.49$$

Your turn A3

Find $E(X)$ and $\text{Var}(X)$

x	0	1	2
$P(X = x)$	0.3	0.4	0.3



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STRETCH

Worked example A4: linear functions

$E(X) = 3$ and $\text{Var}(X) = 2$. Find $E(5 - 2X)$ and $\text{Var}(5 - 2X)$

$$E(aX + b) = aE(X) + b; \text{Var}(aX + b) = a^2\text{Var}(X)$$

$$5 - 6; (-2)^2 \times 2$$

$$-1, 8$$

Your turn A4

$E(X) = 4$ and $\text{Var}(X) = 1.5$. Find $E(3X + 2)$ and $\text{Var}(3X + 2)$

Worked example A5: the discrete uniform distribution

X is the score on a fair six-sided die. Find $E(X)$ and $\text{Var}(X)$

$$E(X) = \frac{n+1}{2}, \text{Var}(X) = \frac{n^2-1}{12}, \text{ with } n = 6$$

$$3.5, \frac{35}{12}$$

Your turn A5

X is the score on a fair ten-sided spinner numbered 1 to 10. Find $E(X)$ and $\text{Var}(X)$

Worked example A6: two unknowns

$P(X = x)$ is $a, b, 0.3, 0.2$ for $x = 1, 2, 3, 4$, and $E(X) = 2.6$. Find a and b

Sum: $a + b = 0.5$; mean: $a + 2b + 0.9 + 0.8 = 2.6$, so $a + 2b = 0.9$

$$a = 0.1, b = 0.4$$

Your turn A6

$P(X = x)$ is $0.1, a, b, 0.2$ for $x = 0, 1, 2, 3$, and $E(X) = 1.6$. Find a and b

Worked example A7: two observations

$P(X = x)$ is $0.5, 0.3, 0.2$ for $x = 1, 2, 3$. Two independent observations are taken. Find $P(\text{their sum is } 4)$.

List the pairs: $(1, 3), (3, 1), (2, 2)$

$$2(0.5)(0.2) + 0.3^2$$

$$0.29$$

Your turn A7

$P(X = x)$ is $0.2, 0.5, 0.3$ for $x = 0, 1, 2$. Two independent observations are taken. Find $P(\text{their sum is } 2)$.



B**Fluency**

Each question is a designed sequence. From one part to the next, ask yourself: what changed, and what did it do to the answer?

CORE

Secure the method

B1 $P(X = x) = k(5 - x)$ for $x = 1, 2, 3, 4$

(a) Find k (b) Find $P(X = 2)$ (c) Find $P(X \leq 2)$

(d) State the mode.

[4 marks]

B2

x	1	2	3	4	5
$F(x)$	0.1	0.25	0.55	0.8	1

(a) Find $P(X = 3)$ (b) Find $P(X > 2)$ (c) Find $F(3.6)$ (d) Find $P(2 \leq X \leq 4)$

[4 marks]

B3

x	-1	0	2	5
$P(X = x)$	0.2	0.3	0.4	0.1

(a) Find $E(X)$ (b) Find $E(X^2)$ (c) Find $\text{Var}(X)$

(d) Find the standard deviation.

[4 marks]

B4 $E(X) = 2.5$ and $\text{Var}(X) = 1.2$. Find

(a) $E(4X - 1)$ (b) $\text{Var}(4X - 1)$ (c) $\text{Var}(3 - X)$ (d) $E(X^2)$

[4 marks]



CORE

continued: uniform and inequalities

B5 X is the score on a fair eight-sided die numbered 1 to 8(a) Find $E(X)$ (b) Find $\text{Var}(X)$ (c) Find $P(X > 6)$

[3 marks]

B6

x	1	2	3	4	5
$P(X = x)$	0.1	0.2	0.3	0.25	0.15

(a) Find $P(2X - 1 > 5)$ (b) Find $P(X^2 < 10)$ (c) Find $P(X \geq 2 | X \leq 4)$ (d) Find $E(X)$

[4 marks]

STRETCH

B7

x	1	2	3	4
$P(X = x)$	a	0.3	b	0.2

$$E(X) = 2.5$$

(a) Find a and b (b) Find $E(X^2)$ (c) Find $\text{Var}(X)$ (d) Find $\text{Var}(10 - 2X)$

[4 marks]

B8 $E(X) = 3$ and $\text{Var}(X) = 4$. $Y = aX + b$, where $a > 0$, $E(Y) = 20$ and $\text{Var}(Y) = 36$ (a) Find a (b) Find b (c) Find $E(X^2)$ (d) Find $E(Y^2)$

[4 marks]

B9 $P(X = x)$ is 0.4, 0.35, 0.25 for $x = 1, 2, 3$. Two independent observations X_1 and X_2 are taken. Find(a) $P(X_1 + X_2 = 4)$ (b) $P(X_1 = X_2)$ (c) $P(X_1 > X_2)$ (d) $P(X_1 + X_2 \geq 5)$

[4 marks]

B10 $F(x) = \frac{x^2}{16}$ for $x = 1, 2, 3, 4$ (a) Find $P(X = 3)$ (b) Find $P(X \geq 2)$ (c) Find $E(X)$

(d) State the mode.

[4 marks]



C**Reasoning**

Show your working and explain your thinking. A correct answer without a reason does not score full marks.

CORE*Secure the method*

- C1** $\text{Var}(X) = 3$. Ali writes $\text{Var}(5 - 2X) = -2 \times 3 = -6$
Explain his error, and find $\text{Var}(5 - 2X)$ [2 marks]
- C2** $E(X) = 2$ and $E(X^2) = 5.5$. Bea says " $E(X^2)$ should be $2^2 = 4$ ".
Explain why she is wrong, and find $\text{Var}(X)$ [2 marks]
- C3** X takes the values 1 to 4 with $F(3) = 0.65$. Cal says " $F(3.5) = 0$ because X cannot be 3.5".
Explain his error, and give $F(3.5)$ [3 marks]
- C4** For $x = 1, 2, 3, 4$ with probabilities 0.1, 0.3, 0.4, 0.2, Dee finds $\sum xP(X = x) = 2.7$ and writes $E(X) = \frac{2.7}{4}$
Explain her error, and state $E(X)$ [3 marks]

STRETCH

- C5** $\text{Var}(X) = 2.5$ and $Y = aX - 1$ with $\text{Var}(Y) = 40$. Eli writes $2.5a = 40$, so $a = 16$
Explain his error, and find a , given that $a > 0$ [3 marks]
- C6** Two players each spin a spinner; the higher score wins, and equal scores are a draw. Faye finds $P(\text{first player wins})$ as $1 - P(\text{second player wins})$
Explain her error. [3 marks]
- C7** X is equally likely to be 2, 4, 6, 8 or 10. Gus writes $E(X) = \frac{n+1}{2} = 3$
Explain his error, and find $E(X)$ and $\text{Var}(X)$ [3 marks]
- C8** (a) Show that $E(aX + b) = aE(X) + b$ (2)
- (b) Hence show that $\text{Var}(aX + b) = a^2\text{Var}(X)$ (3)
- [5 marks]



D**Problem solving**

You decide the method. Read the whole question first, and show every step.

CORE

Secure the method

D1 The discrete random variable X has probability distribution

x	1	2	3	4
$P(X = x)$	0.15	0.4	0.25	0.2

(a) Find $F(2.5)$

(1)

(b) Find $P(4X - 2 > X + 5)$

(2)

[3 marks]

D2 The discrete random variable X has probability distribution

x	1	3	6
$P(X = x)$	0.2	0.5	0.3

(a) Given that $E(X) = 3.5$, show that $\text{Var}(X) = 3.25$

(2)

The random variable $Y = aX - b$, where a and b are positive constants, has $E(Y) = 9$ and $\text{Var}(Y) = 29.25$

(b) Find the value of a and the value of b

(2)

[4 marks]

D3 The discrete random variable X has probability distribution

x	1	2	3
$P(X = x)$	a	0.5	$0.5 - a$

where a is a constant.

(a) Find $E(X)$ in terms of a

(1)

(b) Find the range of possible values of $E(X)$

(2)

[3 marks]

D4 The cumulative distribution function of a discrete random variable X is

x	1	2	3	4
$F(x)$	0.2	$2k$	$5k$	$8k$

where k is a constant. Find the value of k , and the probability distribution of X

[3 marks]



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STRETCH

- D5** In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.

The discrete random variable X has probability distribution

x	1	2	3	4	5
$P(X = x)$	a	a	b	b	0.1

where a and b are constants, and $E(X) = 3.05$

- (a) Form and solve two equations in a and b to show that $a = 0.15$

(3)

- (b) Find $E(X^2)$

(1)

[4 marks]

- D6** Spinner A is equally likely to land on 1, 2 or 3. Spinner B is equally likely to land on 2, 4 or 6. Each spinner is spun once, and the scores are A and B

- (a) Find $P(A + B \leq 5)$

(2)

- (b) Find $P(A^2 \geq B)$

(2)

[4 marks]

- D7** A spinner lands on 1, 2 or 4 with probabilities 0.5, 0.3 and 0.2. Rio and Tess each spin it once, independently, getting X_1 and X_2 . Rio's score is $2X_1$ and Tess's score is $X_2 + 2$. The higher score wins, and equal scores are a draw.

Find the probability that Rio wins. You must show all the outcomes you use.

[4 marks]

- D8** The discrete random variable X has probability distribution

x	1	2	4
$P(X = x)$	0.2	0.5	0.3

- (a) Show that $E\left(\frac{4}{X}\right) = 2.1$

(1)

- (b) Find $\text{Var}\left(\frac{4}{X}\right)$

(2)

- (c) Find $P(X < 4 \mid \frac{4}{X} < 3)$

(2)

[5 marks]



E**Exam practice**

Full-size exam questions in the style of WST01. Write your answers in the spaces, and show every step.

CORE

Exam questions for everyone

- E1** The number of goals, X , scored by a hockey team in a randomly chosen match is modelled by the discrete random variable with probability distribution

x	0	1	2	3	4
$P(X = x)$	0.1	0.25	p	0.2	0.15

- (a) Find the value of p (1)
- (b) Find $F(2.4)$ (1)
- (c) Find $E(X)$ (2)
- (d) Show that $\text{Var}(X) = 1.4475$ (3)
- (e) Find $\text{Var}(7 - 3X)$ (1)
- The team plays two matches, and the numbers of goals scored in the two matches are independent.
- (f) Find the probability that the team scores a total of 7 goals in the two matches. (2)

(Total for Question E1 is 10 marks)



E2 A fair five-sided spinner has sides numbered 1, 2, 3, 4 and 5. The random variable R is the score when it is spun once. A fair six-sided die has faces 0, 0, 1, 1, 2 and 2. The random variable B is the score when it is rolled once.

(a) Write down the name of the distribution of R

(1)

(b) Write down the values of $E(R)$ and $\text{Var}(R)$

(2)

(c) Showing your working, find $E(B)$ and $\text{Var}(B)$

(3)

The spinner is spun once and the die is rolled once.

(d) Find $P(R + B \geq 6)$

(2)

(e) Find the probability that the score on the spinner is more than twice the score on the die.

(3)

(Total for Question E2 is 11 marks)



STRETCH

E3 The cumulative distribution function of a discrete random variable X is given by

$$F(x) = \frac{x^2 + kx}{40} \quad \text{for } x = 1, 2, 3, 4, 5$$

where k is a constant.

(a) Show that $k = 3$

(1)

(b) Find the probability distribution of X

(3)

(c) Find $E(X)$

(1)

(d) Find $\text{Var}(X)$

(2)

Two independent observations of X , denoted X_1 and X_2 , are taken.

(e) Find $P(X_1 + X_2 = 9)$

(2)

(f) Given that the sum of the two observations is 7, find the probability that $X_1 = 2$. You must show your working clearly.

(3)

(Total for Question E3 is 12 marks)

Want more exam practice? The **Exam Questions by Topic** booklets have 10 more full exam questions on Statistics 1 Chapter 6, Discrete Random Variables, with a full mark scheme.

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