eClassroom

INTERNATIONAL A LEVEL MATHEMATICS • FURTHER PURE MATHEMATICS 2

Further Pure 2

Chapter 6: Second-Order Differential Equations

Exam Questions by Topic

Pearson Edexcel International Advanced Level (WFM02/01) • Calculator allowed

Multi-step exam questions at the difficulty of the real WFM02 paper, with a full mark scheme

Chapters (tap a chapter to jump to it)		Qs	Marks	Page	My score
1	**Inequalities** Rational and modulus inequalities, critical values, graphs	10	80	separate file	
2	**Series** Method of differences, partial fractions, telescoping sums	10	80	separate file	
3	**Complex Numbers** Exponential form, de Moivre's theorem, trigonometric identities, nth roots	10	80	separate file	
4	**Further Argand Diagrams** Loci, regions and transformations of the complex plane	10	80	separate file	
5	**First-Order Differential Equations** Integrating factors, substitutions, general and particular solutions	10	80	separate file	
6	**Second-Order Differential Equations** Auxiliary equations, complementary functions, particular integrals, substitutions	10	80	2	
7	**Maclaurin and Taylor Series** Higher derivatives, standard and compound series, Taylor series, series solutions	10	80	separate file	
8	**Polar Coordinates** Polar curves, areas, tangents parallel and perpendicular to the initial line	10	80	separate file	

Full mark scheme | Free courses | More worksheets

Name ..... Class ..... Date .....

★★★★★

Join students already acing their exams.

Book your FREE lesson at www.eclassroom.org.uk

CHAPTER 6 • KEY SKILLS

6 Second-Order Differential Equations

Specification FP2: Second-order differential equations • 10 questions • 80 marks • about 96 minutes

1 Auxiliary equation

For $ay'' + by' + cy = 0$ solve $am^2 + bm + c = 0$

2 Complementary function

Distinct α, β: $Ae^{\alpha x} + Be^{\beta x}$. Repeated α: $(A + Bx)e^{\alpha x}$. $p \pm qi$: $e^{px}(A\cos qx + B\sin qx)$

3 Particular integral

Match the right-hand side: $ax + b$; λe^{kx}; $p\cos kx + q\sin kx$ (both terms, even if only one appears).

4 When the PI clashes

If the trial form is in the CF, multiply by x (by x^2 for a repeated root): $\lambda x e^{2x}$, $\lambda x^2 e^{2x}$

5 Finding constants

Differentiate the trial PI twice, substitute, and equate coefficients of each type of term.

6 General solution

$y = \text{CF} + \text{PI}$, written as "$y = \ldots$" with two arbitrary constants.

7 Conditions

Use y and $\dfrac{dy}{dx}$ at the given points; differentiate the whole general solution first.

8 Substitution $x = e^t$

$x\dfrac{dy}{dx} = \dfrac{dy}{dt}$ and $x^2\dfrac{d^2y}{dx^2} = \dfrac{d^2y}{dt^2} - \dfrac{dy}{dt}$, by the chain rule. Return to x at the end: $e^{2t} = x^2$

9 Behaviour

As $x \to \infty$, terms with e^{-x} vanish; for a solution to stay finite, some constants must be 0

Skills map every skill above is tested at least once; interleaved skills come from GCSE or other chapters

Skill	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Marks	6	6	6	6	6	10	10	10	10	10
1 Auxiliary equation	•	•	•	•	•	•	•	•	•	•
2 Complementary function	•	•	•	•	•	•	•	•	•	•
3 Particular integral		•	•		•	•	•			•
4 When the PI clashes				•				•		•
5 Finding constants		•	•	•	•	•		•		•
6 General solution		•	•	•	•	•	•	•	•	•
7 Conditions			•				•	•	•	•
8 Substitution $x = e^t$						•			•	
9 Behaviour							•	•		•
Interleaved: differentiation, chain and product rules (P3)				•		•	•	•	•	•
Interleaved: complex roots of quadratics (FP1)	•		•		•		•		•	
Interleaved: simultaneous equations and exponentials (P1, P3)		•	•		•	•	•	•	•	•

Exam difficulty. Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WFM02 paper, where one part leads into the next and skills from different topics are combined.

Must learn. Write the CF first, then choose the PI form, checking it against the CF. Give the final answer as "$y =$" in the original variable, with the exponentials simplified.

★★★★★

Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

1. For each equation below, solve the auxiliary equation and write down y in terms of x and two arbitrary constants.

(a) $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} - 5\dfrac{\mathrm{d}y}{\mathrm{d}x} + 6y = 0$

(2)

(b) $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} + 4\dfrac{\mathrm{d}y}{\mathrm{d}x} + 4y = 0$

(2)

(c) $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} + 2\dfrac{\mathrm{d}y}{\mathrm{d}x} + 5y = 0$

(2)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 1 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

(Total for Question 1 is 6 marks)

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

2. Using a particular integral of the form $ax+b$, solve completely

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 4x$$

(6)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

Question 2 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

(Total for Question 2 is 6 marks)

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

3.

$$\frac{d^2y}{dx^2} + 9y = 10e^x$$

(a) Obtain y in terms of x and two arbitrary constants.

(4)

(b) Hence determine the solution with $y = 2$ and $\frac{dy}{dx} = 4$ at $x = 0$

(2)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

Question 3 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

(Total for Question 3 is 6 marks)

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

4.

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 3e^{2x}$$

(a) Explain why a particular integral of the form λe^{2x} cannot be used.

(1)

(b) Find the general solution of the differential equation.

(5)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 4 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

(Total for Question 4 is 6 marks)

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

5. A trigonometric forcing term produces the equation

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + 2\frac{\mathrm{d}y}{\mathrm{d}x} + 2y = 10\sin x$$

Choosing a suitable trigonometric particular integral, express y in terms of x and two arbitrary constants.

(6)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

Question 5 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

(Total for Question 5 is 6 marks)

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

6.

$$x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = x^3, \qquad x > 0 \qquad \text{(I)}$$

(a) Given that $x = e^t$, show that $x\frac{dy}{dx} = \frac{dy}{dt}$ and $x^2\frac{d^2y}{dx^2} = \frac{d^2y}{dt^2} - \frac{dy}{dt}$

(4)

(b) Hence show that (I) becomes $\frac{d^2y}{dt^2} - 3\frac{dy}{dt} + 2y = e^{3t}$ (II)

(1)

(c) Find the general solution of (II).

(3)

(d) Hence find the general solution of (I), giving y in terms of x.

(2)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 6 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

Question 6 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

(Total for Question 6 is 10 marks)

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

7. The displacement x metres of a particle from a fixed point at time t seconds satisfies

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 5x = 10$$

(a) Find the general solution of the differential equation.

(5)

(b) Given that the particle starts from rest at the fixed point, find x in terms of t.

(4)

(c) Describe the motion of the particle for large values of t.

(1)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 7 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

Question 7 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

(Total for Question 7 is 10 marks)

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

8.

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 6e^{2x}$$

(a) Explain why neither λe^{2x} nor $\lambda x e^{2x}$ can be a particular integral.

(2)

(b) Find the general solution of the differential equation.

(4)

(c) Find the particular solution for which $y = 1$ and $\frac{dy}{dx} = 0$ when $x = 0$

(3)

(d) Show that this particular solution is positive for all values of x.

(1)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 8 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

Question 8 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

(Total for Question 8 is 10 marks)

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

9.

$$x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 4xy = 0, \qquad x > 0 \qquad \text{(I)}$$

(a) By writing $y = \frac{z}{x}$, where z depends on x, show that (I) reduces to $\frac{d^2z}{dx^2} + 4z = 0$

(5)

(b) Solve for z, and deduce y in terms of x and two constants.

(3)

(c) Find the particular solution of (I) for which $y = \frac{4}{\pi}$ when $x = \frac{\pi}{4}$ and $y = -\frac{2}{\pi}$ when $x = \frac{\pi}{2}$

(2)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 9 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

Question 9 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

(Total for Question 9 is 10 marks)

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

10.

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = 4x - 2 + 3e^x$$

(a) Find the complementary function.

(2)

(b) Find a particular integral, explaining your choice of form.

(5)

(c) Find the particular solution for which $y = 1$ when $x = 0$ and y remains finite as $x \to -\infty$

(3)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

Question 10 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

Question 10 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

(Total for Question 10 is 10 marks)

TOTAL FOR CHAPTER 6 IS 80 MARKS

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk