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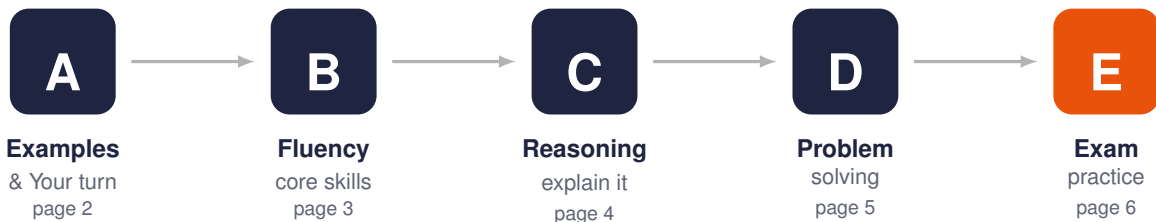
A-LEVEL MATHEMATICS • PURE 2

The Binomial Expansion

Worked Solutions

Pearson Edexcel International A Level (WMA12) & A Level (9MA0) • Calculator allowed: show every step

Full worked solutions for the free The Binomial Expansion worksheet, A-Level and International A-Level Maths



Tap a section to jump straight to it

Full worked solutions for every question on the Pure 2 The Binomial Expansion worksheet, in the same order and with the same question numbers.

How to mark your work. Tick each step you got right, not just the final answer. At A-level most marks are for method, so a wrong final answer can still earn most of the marks. If you lost marks, find the *first* line where your working is different from ours. That is the thing to fix.

In Section E: **M** = method mark, **dM** = method mark that depends on the previous M mark, **A** = accuracy mark (needs the M mark before it), **B** = independent mark.

Question sheet

Exam questions by topic

Sequences & Series revision engine

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A**Your turn answers**

Compare each line of your working with the solution.

CORE

A1 $\frac{7 \times 6 \times 5}{6} = \boxed{35}; \frac{9 \times 8}{2} = \boxed{36}$

A2 $2^5 + 5 \times 2^4(-x) + 10 \times 2^3(-x)^2 = \boxed{32 - 80x + 80x^2}$

A3 ${}^6C_4 \times 3^2 \times (-2)^4 = 15 \times 9 \times 16 = \boxed{2160}$

STRETCH

A4 The x term is ${}^4C_1 \times 3^3 \times kx = 108kx$; $108k = -216$: $\boxed{k = -2}$

A5 $(1+2x)^5 = 1 + 10x + 40x^2 + \dots$; x^2 terms: $3 \times 40x^2$ and $-x \times 10x$: $120 - 10 = \boxed{110}$

A6 $1 + 7(-2x) + 21(-2x)^2 = 1 - 14x + 84x^2$; $1 - 2x = 0.98$ when $x = 0.01$: $1 - 0.14 + 0.0084 = \boxed{0.8684}$

A7 ${}^nC_2 \times 2^2 = 180$, so $2n(n-1) = 180$, $n(n-1) = 90$: $\boxed{n = 10}$



B**Fluency solutions**

One mark for each correct part. The notes in *italics* explain why each sequence was chosen.

CORE

B1 (a) 10 (b) 120 (c) 1 (d) 12

${}^nC_0 = 1$ and ${}^nC_{n-1} = n$: the ends and the second entries of each row.

B2 (a) $1 + 4x + 6x^2 + 4x^3 + x^4$ (b) $1 - 4x + 6x^2 - 4x^3 + x^4$

(c) $1 + 8x + 24x^2 + 32x^3 + 16x^4$ (d) $16 + 32x + 24x^2 + 8x^3 + x^4$

The coefficients 1, 4, 6, 4, 1 are multiplied by the powers of each term in the bracket.

B3 (a) $1 + 8x + 28x^2$ (b) $1 - 24x + 252x^2$ (c) $256 + 1024x + 1792x^2$ (d) $256 - 512x + 448x^2$

Every term carries a power of a as well as a power of bx

B4 (a) 84 (b) -84 (c) $84 \times 8 = 672$ (d) $84 \times 3^6 = 61236$

${}^9C_3 = 84$ every time; the bracket supplies the sign and the extra factors.

B5 (a) $x = 0.01$: ≈ 1.1045 (b) $x = -0.01$: ≈ 0.9045 (c) $x = 0.02$: ≈ 1.218

Choose x so that $1+x$ is the number being raised to the power.

B6 (a) $35x^3$ (b) 32 (c) ${}^6C_2x^4(-1)^2 = 15x^4$ (d) x^4

The r th term contains x^{r-1} : the 4th term is the x^3 term.

STRETCH

B7 (a) $6k = 18$: $k = 3$ (b) $15k^2 = 135$: $k = \pm 3$ (c) $k = 3$ (d) ${}^5C_3 \times 2^2k^3 = 40k^3 = -320$: $k = -2$

A square gives two signs; a second condition or a cube decides the sign.

B8 (a) 15 (b) $10 + 3 \times (-5) = -5$ (c) $2 \times 24 + (-1) \times 8 = 40$ (d) $15 + 1 \times 1 = 16$

Pair each term of the first bracket with the term of the expansion that makes x^2

B9 (a) $\frac{n(n-1)}{2} = 15$: $n = 6$ (b) $3n = 27$: $n = 9$ (c) $n(n-1) = 90$: $n = 10$ (d) $9 \times \frac{n(n-1)}{2} = 135$: $n = 6$

Write nC_2 as $\frac{n(n-1)}{2}$, then reject the negative root.



B10 (a) $x = 0.2$ (b) $1 + 0.4 + 0.07 = 1.47$ (c) $\frac{1.4775 - 1.47}{1.4775} \times 100 \approx 0.505\%$

(d) For 2^8 , $x = 4$: then $x^2, x^3, \dots$ are *larger*, not smaller, so **the neglected terms are not small**

An expansion gives good estimates only when x is small.

C

Reasoning solutions

Your wording can be different. Check that your reason makes the same point.

CORE

C1 Sam lost the minus sign: the term is $4 \times 2^3 \times (-3x)$, with $-3x$ kept in a bracket. $-96x$

C2 Lily squared only the x ; the whole $2x$ is squared: $10 \times (2x)^2 = 40x^2$

C3 Kai has missed the x^2 made by $-x$ times the x term. $(1 + 3x)^5 = 1 + 15x + 90x^2 + \dots$
 $2 \times 90 + (-1) \times 15 = 165$

C4 The 1st term has x^0 , so the 3rd term is the x^2 term: ${}^6C_2(2x)^2 = 60x^2$

STRETCH

C5 The question asks for an exact answer; 1.30 is rounded. ${}^7C_3 \times \left(\frac{1}{3}\right)^3 = \frac{35}{27}$: $\frac{35}{27}$

C6 $x = 0.1$ is not small enough for the x^3 and later terms to be negligible: the next term alone is $120x^3 = 0.12$.
use more terms, or a smaller value of x

C7 Each row of Pascal's triangle is symmetrical, because ${}^nC_r = {}^nC_{n-r}$: ${}^7C_3 = {}^7C_4 = 35$

C8 (a) $10 + 10 = 20 = {}^6C_3$ **as required**

(b) $\frac{n(n-1)}{2} + \frac{n(n-1)(n-2)}{6} = \frac{n(n-1)(3+n-2)}{6} = \frac{(n+1)n(n-1)}{6} = {}^{n+1}C_3$ **as required**

(c) **two neighbouring entries in one row add to give the entry between them in the next row**



D**Problem solving solutions**

There is often more than one correct method. Any complete, correct method scores full marks.

CORE

D1 ${}^8C_4 \times 3^4 \times \left(-\frac{x}{3}\right)^4 = 70 \times 81 \times \frac{x^4}{81} = \boxed{70}$

D2 (a) $1 + 8\left(\frac{x}{2}\right) + 28\left(\frac{x}{2}\right)^2 + 56\left(\frac{x}{2}\right)^3 = \boxed{1 + 4x + 7x^2 + 7x^3}$

(b) $2 \times 7 + (-1) \times 7 = \boxed{7}$

D3 $(1 - 3x)^4 = 1 - 12x + 54x^2 + \dots$; x^2 terms: $2 \times 54x^2$ and $x \times (-12x)$: $108 - 12 = \boxed{96}$

D4 $(1 + 2x)^7 = 1 + 14x + 84x^2 + \dots$; coefficient of x^2 : $3 \times 84 - 14k = 210$, so $14k = 42$: $\boxed{k = 3}$

STRETCH

D5 (a) $\boxed{64 + 192ax + 240a^2x^2}$

(b) x terms: $1 \times 192ax$ and $\frac{1}{x} \times 240a^2x^2$: $192a + 240a^2 = 432$, so $5a^2 + 4a - 9 = 0$, $(5a + 9)(a - 1) = 0$, $a > 0$:

$\boxed{a = 1}$

D6 (a) $\boxed{1 - 2x + \frac{9}{5}x^2}$

(b) $x = 0.1$: $1 - 0.2 + 0.018 = 0.818$; $0.98^{10} = 0.81707\dots$; error $\frac{0.818 - 0.81707}{0.81707} \times 100 \approx \boxed{0.113\%}$

D7 (a) ${}^8C_2k^2 = {}^8C_3k^3$, so $28k^2 = 56k^3$; $k \neq 0$: $\boxed{k = \frac{1}{2}}$

(b) $70 \times \left(\frac{1}{2}\right)^4 = \boxed{\frac{35}{8}}$

D8 (a) $(1 + x)^4 = 1 + 4x + 6x^2 + 4x^3 + \dots$ and $(1 - x)^5 = 1 - 5x + 10x^2 - 10x^3 + \dots$

x : $-5 + 4 = -1$; x^2 : $10 - 20 + 6 = -4$; x^3 : $-10 + 40 - 30 + 4 = 4$: $\boxed{1 - x - 4x^2 + 4x^3}$

(b) $x = 0.1$: $1 - 0.1 - 0.04 + 0.004 = \boxed{0.864}$



E

Exam practice solutions

Mark-scheme notes are shown on the right of each line.

CORE

Exam questions for everyone

E1 (a) $3^6 + 6 \times 3^5(2x) + 15 \times 3^4(2x)^2 + 20 \times 3^3(2x)^3$

M1: coefficients; M1: powers

$$729 + 2916x + 4860x^2 + 4320x^3$$

A1 A1

(b) $3 + 2x = 3.02$: $x = 0.01$

B1

E2 (a) $A = 32$

B1

(b) $B = 5 \times 2^4 p = 80p$ and $C = 10 \times 2^3 p^2 = 80p^2$

M1 A1

$80p + 80p^2 = 0, p \neq 0$: $p = -1$

A1

(c) $D = 10 \times 2^2 p^3 = 40p^3$

M1

$D = -40$

A1

STRETCH

E3 (a) $1 - 5x + \frac{45}{4}x^2 - 15x^3$

M1 A1 A1

(b) x : $-5a + b = -7$; x^2 : $\frac{45}{4}a - 5b = \frac{15}{2}$

M1: both equations

$b = 5a - 7$, so $\frac{45}{4}a - 25a + 35 = \frac{15}{2}$, $\frac{55}{4}a = \frac{55}{2}$

dM1

$a = 2$, $b = 3$

A1

(c) $x = 0.1$: $1 - 0.5 + 0.1125 - 0.015$

M1

0.5975

A1

