eClassroom

INTERNATIONAL A LEVEL MATHEMATICS • MECHANICS 2

Mechanics 2

Chapter 2: Variable Acceleration

Exam Questions by Topic

Pearson Edexcel International Advanced Level (WME02/01) • Calculator allowed • Take $g = 9.8\,\text{m}\,\text{s}^{-2}$

Multi-step exam questions at the difficulty of the real WME02 paper, with a full mark scheme

Chapters (tap a chapter to jump to it)		Qs	Marks	Page	My score
1	**Projectiles** Horizontal projection, projection at any angle, the equation of the path	10	84	separate file	
2	**Variable Acceleration** Calculus for motion in a line and in $\mathbf{i}, \mathbf{j}$ form	10	85	2	
3	**Centres of Mass** Particles, laminas, frameworks, suspended and tilting bodies	10	81	separate file	
4	**Work and Energy** Work done, kinetic and potential energy, the work–energy principle, power	10	82	separate file	
5	**Impulses and Collisions** Impulse as a vector, Newton's law of restitution, loss of kinetic energy	10	82	separate file	
6	**Statics of Rigid Bodies** Rods and ladders in equilibrium, friction, hinge forces	10	81	separate file	

Full mark scheme Free courses More worksheets

Name ..... Class ..... Date .....

★★★★★

Join students already acing their exams.

Book your FREE lesson at www.eclassroom.org.uk

CHAPTER 2 • KEY SKILLS

2 Variable Acceleration

Specification M2: Variable acceleration • 10 questions • 85 marks • about 102 minutes

1 Differentiating

$v = \frac{dx}{dt}$ and $a = \frac{dv}{dt}$. Go down from displacement to velocity to acceleration.

2 Integrating

$v = \int a\,dt$ and $x = \int v\,dt$. Find each constant from the given conditions at once.

3 Rest and turning

At rest: $v = 0$. The particle changes direction where v changes sign. Solve the quadratic with working shown.

4 Distance travelled

Split the time at each instant of rest and add the sizes of the displacements: distance is never negative.

5 Greatest speed

Greatest or least velocity where $a = 0$; also check the end points of the interval. Speed is $|v|$.

6 Vectors

Differentiate or integrate each component separately. The constant of integration is a vector, e.g. $\mathbf{c} = 2\mathbf{i} + \mathbf{j}$

7 Direction of motion

The direction of motion is the direction of $\mathbf{v}$, not $\mathbf{r}$. Parallel to $\mathbf{i}$: the $\mathbf{j}$ part of $\mathbf{v}$ is 0.

8 Magnitudes and forces

Speed $= |\mathbf{v}|$; $\mathbf{F} = m\mathbf{a}$. Give a vector when a vector is asked for, and a magnitude when a magnitude is asked for.

9 Constant acceleration

suvat comes from integrating a constant a. It must not be used when the acceleration depends on t.

Skills map every skill above is tested at least once; interleaved skills come from GCSE or other chapters

Skill	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Marks	7	6	6	7	6	11	11	10	10	11
1 Differentiating	•		•				•	•	•	
2 Integrating		•		•	•	•	•	•	•	•
3 Rest and turning	•					•				
4 Distance travelled	•	•				•		•	•	
5 Greatest speed		•				•		•	•	
6 Vectors			•	•			•			•
7 Direction of motion			•				•			
8 Magnitudes and forces			•	•			•			•
9 Constant acceleration					•					
Interleaved: quadratics and cubics, factor theorem (P1, P2)	•					•	•			•
Interleaved: exponentials, logarithms and chain rule (P3)								•	•	
Interleaved: $F = ma$ and relative position (M1)							•			•

Exam difficulty. Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WME02 paper, where one part leads into the next and skills from different topics are combined.

Must learn. When the acceleration depends on t, use calculus, never suvat: suvat earns no marks. Include and find every constant of integration. Read whether a vector or a magnitude is asked for.

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

1. A particle P moves along a straight line. At time t seconds, $t \geqslant 0$, the displacement of P from a fixed point O on the line is x metres, where

$$x = t^3 - 9t^2 + 24t$$

(a) Find the values of t when P is instantaneously at rest.

(3)

(b) Find the acceleration of P when $t = 5$

(1)

(c) Find the total distance travelled by P in the interval $0 \leqslant t \leqslant 4$

(3)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 1 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

(Total for Question 1 is 7 marks)

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

2. A particle P moves along a straight line. At time t seconds, the acceleration of P is $(6-2t)\,\text{m s}^{-2}$, in the direction of increasing x. When $t = 0$, P is at the point O and is moving with velocity $5\,\text{m s}^{-1}$ in the direction of increasing x.

(a) Find the greatest velocity of P in the interval $0 \leqslant t \leqslant 6$

(3)

(b) Find the displacement of P from O when $t = 6$

(3)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

Question 2 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

(Total for Question 2 is 6 marks)

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

3. *[In this question,* $\mathbf{i}$ *and* $\mathbf{j}$ *are perpendicular horizontal unit vectors and position vectors are given relative to a fixed origin* O*.]*

A particle P moves in a horizontal plane. At time t seconds, the position vector of P is $\mathbf{r}$ metres, where

$$\mathbf{r} = \left(t^3 - 3t\right)\mathbf{i} + \left(2t^2 - 8t\right)\mathbf{j}$$

(a) Find the velocity of P at the instant when P is moving parallel to the vector $\mathbf{i}$

(3)

(b) Find the acceleration of P at this instant.

(2)

(c) Find the speed of P when $t = 0$

(1)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 3 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

(Total for Question 3 is 6 marks)

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

4. *[In this question,* $\mathbf{i}$ *and* $\mathbf{j}$ *are horizontal unit vectors directed due east and due north respectively, and position vectors are given relative to a fixed origin* O*.]*

A remote-controlled boat is modelled as a particle B moving on the surface of a lake. At time t seconds after it is switched on, its acceleration is $(6t\mathbf{i} - 4\mathbf{j})\,\text{m s}^{-2}$. When $t = 0$, B has velocity $(-3\mathbf{i} + 8\mathbf{j})\,\text{m s}^{-1}$ and is at the point with position vector $(2\mathbf{i} + \mathbf{j})$ m.

(a) Find an expression for the velocity of B at time t seconds.

(2)

(b) Find the time at which B is moving due north.

(2)

(c) Find the distance of B from O when $t = 3$

(3)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 4 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

(Total for Question 4 is 7 marks)

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

5. A particle moves along a straight line with constant acceleration $a\,\text{m s}^{-2}$. At time $t = 0$ it passes through a fixed point O on the line with velocity $u\,\text{m s}^{-1}$. At time t seconds its velocity is $v\,\text{m s}^{-1}$ and its displacement from O is s metres.

(a) By integration, show that $v = u + at$ and $s = ut + \frac{1}{2}at^2$

(4)

(b) Hence show that $v^2 = u^2 + 2as$

(1)

(c) Explain why these formulae cannot be used for a particle whose acceleration at time t seconds is $3t\,\text{m s}^{-2}$

(1)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

Question 5 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

(Total for Question 5 is 6 marks)

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

6. A particle P moves along a straight line. At time t seconds, $t \geqslant 0$, the acceleration of P is $(2t-7)\,\text{m s}^{-2}$. When $t = 0$, P is at the point O and is moving with velocity $10\,\text{m s}^{-1}$

(a) Show that the velocity of P at time t seconds is $\left(t^2 - 7t + 10\right)\,\text{m s}^{-1}$

(2)

(b) Find the values of t when P is instantaneously at rest.

(2)

(c) Find the total distance travelled by P in the interval $0 \leqslant t \leqslant 6$

(5)

(d) Find the greatest speed of P in the interval between the two times when it is at rest.

(2)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

Question 6 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

★★★★★
Join students already acing their exams.
Book your FREE lesson at **www.eclassroom.org.uk**

Question 6 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

(Total for Question 6 is 11 marks)

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

7. *[In this question, $\mathbf{i}$ and $\mathbf{j}$ are perpendicular horizontal unit vectors and position vectors are given relative to a fixed origin O.]*

A particle P of mass 2 kg moves in a horizontal plane. At time t seconds, $t \geqslant 0$, the velocity of P is $\mathbf{v}\,\text{m s}^{-1}$, where

$$\mathbf{v} = (t+1)\mathbf{i} + \left(t^2 - 5\right)\mathbf{j}$$

(a) Find the magnitude of the resultant force acting on P when $t = 2$

(3)

(b) Find the speed of P at the instant when it is moving in the direction of the vector $(\mathbf{i} + \mathbf{j})$

(4)

When $t = 0$, P is at O.

(c) Find the distance of P from O at the instant, after $t = 0$, when P crosses the line through O parallel to $\mathbf{i}$

(4)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 7 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

Question 7 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

(Total for Question 7 is 11 marks)

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

8. A cyclist starts from rest at the point O and moves along a straight horizontal road. At time t seconds after leaving O, the speed of the cyclist is $v\,\text{m s}^{-1}$, where

$$v = 6 - 6e^{-\frac{1}{2}t}$$

The cyclist is modelled as a particle.

(a) Find the acceleration of the cyclist when $t = 0$

(2)

(b) Explain why the speed of the cyclist is always less than $6\,\text{m s}^{-1}$

(1)

(c) Find the distance travelled by the cyclist in the first 4 seconds.

(4)

(d) Find the exact time at which the speed of the cyclist is $5\,\text{m s}^{-1}$

(2)

(e) State one limitation of this model of the motion of the cyclist.

(1)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 8 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA


★★★★★
Join students already acing their exams.
Book your FREE lesson at **www.eclassroom.org.uk**

Question 8 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

(Total for Question 8 is 10 marks)

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

9. A particle P moves along a straight line, starting from rest at the point O at time $t = 0$. At time t seconds, the velocity of P is $v\,\text{m s}^{-1}$, where

$$v = \begin{cases} 4t - t^2 & 0 \leqslant t \leqslant 3 \\ \dfrac{27}{t^2} & t > 3 \end{cases}$$

(a) Find the greatest speed of P in the interval $0 \leqslant t \leqslant 3$

(2)

(b) Show that the acceleration of P does not change suddenly at $t = 3$

(2)

(c) Find the distance travelled by P from $t = 0$ to $t = 6$

(4)

(d) Show that P never reaches the point which is 18 m from O.

(2)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

★★★★★

Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

Question 9 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

Question 9 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

(Total for Question 9 is 10 marks)

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

10. *[In this question,* $\mathbf{i}$ *and* $\mathbf{j}$ *are perpendicular horizontal unit vectors and position vectors are given relative to a fixed origin* O*.]*

At time t seconds, $t \geqslant 0$, the velocity of a particle P is $\left[\left(3t^2 - 4\right)\mathbf{i} + 2t\mathbf{j}\right]$ m s^{-1}, and P is at O when $t = 0$. A second particle Q moves with constant velocity $(3\mathbf{i} + \mathbf{j})$ m s^{-1} and is at the point with position vector $(-6\mathbf{i} + 2\mathbf{j})$ m when $t = 0$.

(a) Find the position vector of P at time t seconds.

(3)

(b) Show that P and Q collide, and find the position vector of the point of collision.

(4)

(c) Show that P and Q have the same $\mathbf{i}$ component of position when $t = 1$, and find the distance between them at this time.

(2)

(d) Find the speed of P at the instant of the collision.

(2)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

Question 10 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

Question 10 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

(Total for Question 10 is 11 marks)

TOTAL FOR CHAPTER 2 IS 85 MARKS

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk