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INTERNATIONAL A LEVEL MATHEMATICS • MECHANICS 2 (WME02)

Mechanics 2 Practice Paper 3

Worked Solutions

AIMING FOR A/A*

Written in the style of the Pearson Edexcel International Advanced Level paper (WME02/01) • 75 marks

Every question solved in full on the paper itself, the way your teacher would write it out

How to read these solutions. Each page is the real question page, with a full answer written on the answer lines.

Red pen

M1 A1

boxed

the working a full-marks answer shows, with a short note on why each step is taken

the mark earned on that line: M = method, A = accuracy, B = independent mark

the final answer to each part

Sit the paper first. These solutions are most useful after you have tried every question yourself under timed conditions. Mark your paper with the mark scheme, then use this booklet to see exactly how each lost mark could have been earned.

Q	Topic	Marks	Page
1	A projectile over a wall	12	2
2	Acceleration in two stages	12	3
3	A lamina with a hole on a slope	13	4
4	A ladder against a rough wall	13	6
5	A collision and a wall	12	8
6	A car on a hill at constant power	13	9

Question paper & mark scheme | Exam questions by topic | More practice papers

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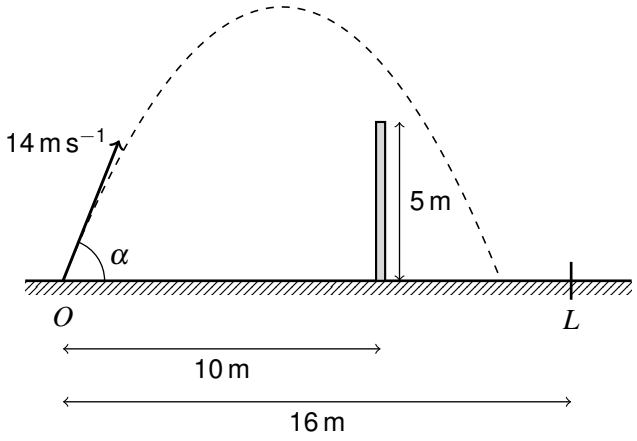

Figure 1

A ball is projected from a point O on horizontal ground with speed 14 m s^{-1} at an angle α above the horizontal. A thin vertical wall of height 5 m stands on the ground at a horizontal distance of 10 m from O. The point L on the ground is 16 m from O, beyond the wall, as shown in Figure 1. The ball moves freely under gravity in a vertical plane perpendicular to the wall.

(a) Find the range of values of $\tan\alpha$ for which the ball passes over the wall.

(6)

(b) Find the range of values of α for which the ball passes over the wall and hits the ground between the wall and L

(6)

(a) Horizontal: $14\cos\alpha \times t = 10 \Rightarrow t = 10/(14\cos\alpha)$ M1

Height at the wall: $y = 14\sin\alpha\, t - 4.9t^2 = 10\tan\alpha - 4.9 \times 100/(196\cos^2\alpha)$ M1

$y = 10\tan\alpha - 2.5\sec^2\alpha$ A1

Over the wall: $y > 5$, $\sec^2\alpha = 1 + \tan^2\alpha$: $10\tan\alpha - 2.5 - 2.5\tan^2\alpha > 5$ M1

$\div(-2.5)$: $\tan^2\alpha - 4\tan\alpha + 3 < 0 \Rightarrow (\tan\alpha - 1)(\tan\alpha - 3) < 0$ A1

$\boxed{1 < \tan\alpha < 3}$ $(45° < \alpha < 71.6°)$ A1

(b) Range on level ground: $R = 14^2\sin 2\alpha / 9.8 = 20\sin 2\alpha$ M1

Lands before L: $20\sin 2\alpha < 16 \Rightarrow \sin 2\alpha < 0.8$ A1

$\sin 2\alpha = 0.8$: $2\alpha = 53.13°$ or $180° - 53.13° = 126.87°$ M1

So $\alpha < 26.57°$ or $\alpha > 63.43°$ A1

$\alpha < 26.6°$ gives $\tan\alpha < 1$: the ball hits the wall. Reject it. M1

$\alpha > 63.4°$ and $\alpha < 71.6°$ $\boxed{63.4° < \alpha < 71.6°}$ A1

(Total for Question 1 is 12 marks)

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2. A particle P moves along a straight line. At time $t = 0$, P is at rest at the point O. At time t seconds, the velocity of P is $v\,\text{m s}^{-1}$, where

$$v = t^2 + 2t, \qquad 0 \leqslant t \leqslant 2$$

For $t \geqslant 2$, the acceleration of P is $(2t - 10)\,\text{m s}^{-2}$. The velocity of P is continuous at $t = 2$

(a) Find v in terms of t for $t \geqslant 2$

(3)

(b) Find the values of t, for $t > 0$, at which P is instantaneously at rest.

(2)

(c) Find the total distance travelled by P in the interval $0 \leqslant t \leqslant 7$

(7)

(a) $t \geqslant 2$: $v = \int (2t - 10)\,dt = t^2 - 10t + c$ M1

Continuous at $t = 2$: $2^2 + 2 \times 2 = 8$, so $4 - 20 + c = 8 \Rightarrow c = 24$ M1

$v = t^2 - 10t + 24$ A1

(b) $t^2 - 10t + 24 = 0 \Rightarrow (t - 4)(t - 6) = 0$ M1

$t = 4$ and $t = 6$ ($t^2 + 2t > 0$ for $0 < t \leqslant 2$) A1

(c) P turns back at $t = 4$ and $t = 6$, so split there. 0 to 2: $\int (t^2 + 2t)\,dt = [t^3/3 + t^2]$ M1

$= 8/3 + 4 = 20/3$ A1

$s = t^3/3 - 5t^2 + 24t$ for the second stage; split at 4 and 6: M1

2 to 4: $(64/3 - 80 + 96) - (8/3 - 20 + 48) = 112/3 - 92/3 = 20/3$ A1

4 to 6: $(72 - 180 + 144) - 112/3 = 36 - 112/3 = -4/3$ (backwards) A1

6 to 7: $(343/3 - 245 + 168) - 36 = 112/3 - 36 = 4/3$ A1

Distance $= 20/3 + 20/3 + 4/3 + 4/3 = 48/3$ 16 m A1

(Total for Question 2 is 12 marks)

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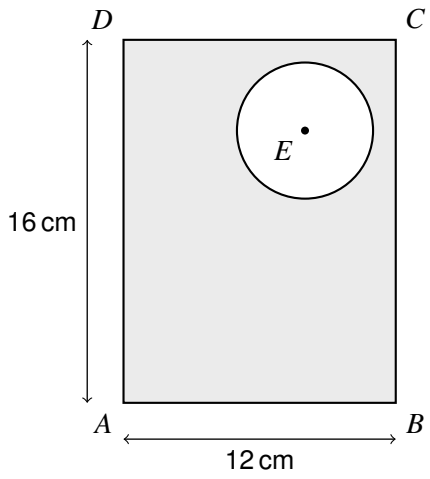

Figure 2

A uniform rectangular lamina $ABCD$ has $AB = 12$ cm and $AD = 16$ cm. A circular hole of radius 3 cm is cut from the lamina. The centre E of the hole is 8 cm from AD and 12 cm from AB, as shown in Figure 2.

(a) Find the distance of the centre of mass of the lamina from **(i)** AD **(ii)** AB

(6)

The lamina stands in a vertical plane with AB in contact with a rough plane inclined at an angle θ to the horizontal. The edge AB lies along a line of greatest slope, with A below B. The coefficient of friction between the lamina and the plane is 0.75. The angle θ is gradually increased from zero.

(b) Determine whether the lamina first slides or first topples, and find the value of θ at which this happens.

(5)

The lamina is now freely suspended from D and hangs in equilibrium.

(c) Find the angle between DA and the vertical.

(2)

(a) Rectangle 192, centre (6, 8). Hole 9π, centre (8, 12). Lamina $192 - 9\pi$ B1

(i) Moments about AD: $192 \times 6 - 9\pi \times 8 = (192 - 9\pi)\bar{x}$ M1

$1152 - 226.19 = 163.73\bar{x}$ A1

$\bar{x} = 5.654...$ 5.65 cm from AD A1

(ii) Moments about AB: $192 \times 8 - 9\pi \times 12 = (192 - 9\pi)\bar{y} \Rightarrow 1196.7 = 163.73\bar{y}$ M1

$\bar{y} = 7.309...$ 7.31 cm from AB A1

(b) Topples about A (the lower corner) when G is vertically above A:

$\tan\theta = 5.654 \div 7.309 = 0.7736 \Rightarrow$ toppling angle 37.7° M1 A1

Slides when $\tan\theta = \mu = 0.75 \Rightarrow$ sliding angle 36.9° M1

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Question 3 continued

$0.75 < 0.774$, so the sliding angle is reached first. M1

It slides first, at $\theta = 36.9°$ (no hole: both at 0.75) A1

(c) Hanging from D, DG is vertical. From D: across 5.654, down $16 - 7.309 = 8.691$ M1

$\tan\phi = 5.654 \div 8.691 \Rightarrow \phi = 33.04...$ $33.0°$ A1

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(Total for Question 3 is 13 marks)

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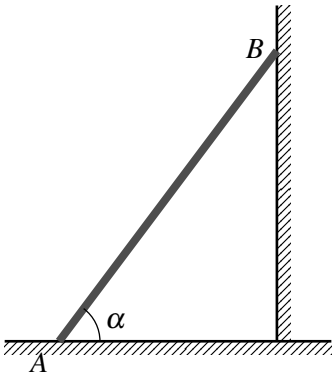

Figure 3

A uniform ladder AB, of mass 30 kg and length 8 m, rests with its end A on rough horizontal ground and its end B against a rough vertical wall. The ladder lies in a vertical plane perpendicular to the wall and makes an angle α with the ground, where $\tan\alpha = \frac{4}{3}$, as shown in Figure 3. The coefficient of friction between the ladder and the ground is 0.4, and the coefficient of friction between the ladder and the wall is 0.5

A man of mass 70 kg climbs up the ladder. The ladder is modelled as a uniform rod and the man as a particle.

(a) Find the greatest distance the man can climb up the ladder before it slips.

(10)

(b) Find the magnitude of the total force exerted on the ladder by the ground when the ladder is about to slip.

(3)

(a) $\sin\alpha = 0.8$, $\cos\alpha = 0.6$. About to slip: A slides out, B slides down.

Ground: R up, $F = 0.4R$ towards the wall — B1

Wall: S away from the wall, friction $0.5S$ acting UP the wall (B moves down) — B1

Horizontal: $F = S \Rightarrow S = 0.4R$ — M1

Vertical: $R + 0.5S = 30g + 70g = 100g$ — M1

$R + 0.2R = 100g \Rightarrow R = 100g/1.2 = 250g/3$; $S = 100g/3$; wall friction $50g/3$ — A1

Moments about A: $S \times 8\sin\alpha + (50g/3) \times 8\cos\alpha = 30g \times 4\cos\alpha + 70g \times x\cos\alpha$ — M1

$(100g/3) \times 6.4 + (50g/3) \times 4.8 = 30g \times 2.4 + 70g \times 0.6x$ — A1

$213.33 + 80 = 72 + 42x$ — A1

$42x = 221.33$ — M1

$x = 5.269\ldots$ 5.27 m up the ladder — A1

(b) Ground force: $R = 250g/3 = 816.7$ N up and $F = 0.4R = 326.7$ N — M1

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Question 4 continued

$\sqrt{816.7^2 + 326.7^2} = 816.7\sqrt{1.16} = 879.5...$ A1

880 N A1

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(Total for Question 4 is 13 marks)

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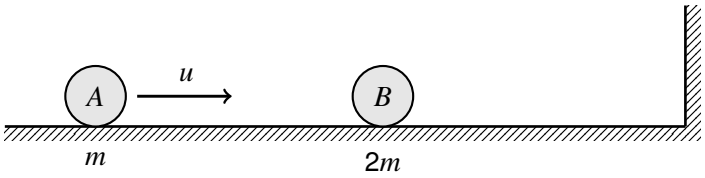

Figure 4

Two small smooth spheres A and B, of equal radius and of masses m and $2m$ respectively, lie on a smooth horizontal floor. Sphere B is at rest between A and a fixed smooth vertical wall. Sphere A is projected towards B with speed u and collides directly with B, as shown in Figure 4. The coefficient of restitution between A and B is e

After the collision, B hits the wall, which is perpendicular to its direction of motion. The coefficient of restitution between B and the wall is $\frac{1}{4}$

(a) Find, in terms of u and e, the velocity of A and the velocity of B immediately after they collide.

(5)

(b) Given that B collides with A again after rebounding from the wall, find the range of possible values of e

(5)

(c) Given instead that $e = \frac{1}{2}$, find, in terms of m and u, the kinetic energy lost when B hits the wall.

(2)

(a) Towards the wall positive. Momentum: $mu = mv_A + 2mv_B$ M1

Restitution: $v_B - v_A = eu$ M1

$u = v_A + 2(v_A + eu) \Rightarrow 3v_A = u - 2eu$ A1

$\boxed{v_A = u(1 - 2e)/3}$ (towards the wall if $e < \frac{1}{2}$, away if $e > \frac{1}{2}$) A1

$\boxed{v_B = u(1 + e)/3}$ A1

(b) B rebounds from the wall at $\frac{1}{4} \times u(1 + e)/3 = u(1 + e)/12$ B1

If $e \leqslant \frac{1}{2}$, A is still moving towards the wall (or at rest), so they must meet. M1

If $e > \frac{1}{2}$, A moves away at $u(2e - 1)/3$: B must be faster: $u(1 + e)/12 > u(2e - 1)/3$ M1

$1 + e > 8e - 4 \Rightarrow 7e < 5$ A1

Together: $\boxed{0 \leqslant e < 5/7}$ A1

(c) $e = \frac{1}{2}$: $v_B = u(1.5)/3 = u/2$, rebounds at $u/8$. Loss $= \frac{1}{2}(2m)(u^2/4 - u^2/64)$ M1

$= m(16u^2 - u^2)/64$ $\boxed{15mu^2/64}$ A1

(Total for Question 5 is 12 marks)

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6. A car of mass 800 kg has an engine working at a constant rate of 20 kW. The resistance to the motion of the car from non-gravitational forces has magnitude kv newtons, where v m s^{-1} is the speed of the car and k is a constant. The maximum speed of the car on a straight horizontal road is 40 m s^{-1}

(a) Find the value of k

(3)

The car now travels on a straight road inclined at an angle α to the horizontal, where $\sin\alpha = \frac{1}{16}$. The engine still works at 20 kW.

(b) Find the maximum speed of the car up the road.

(4)

(c) Find the maximum speed of the car down the road.

(4)

(d) Find the rate at which the engine must work for the car to move up the road at a constant speed of 20 m s^{-1}

(2)

(a) Top speed on the level: driving force = resistance: $20000/40 = 40k$ M1

$500 = 40k$ A1

$\boxed{k = 12.5}$ A1

(b) Up: weight component $800g \times 1/16 = 490$ N acts down the road.

Top speed: $20000/v = 12.5v + 490$ M1

$\times v$: $12.5v^2 + 490v - 20000 = 0$ A1

$v = (-490 + \sqrt{490^2 + 4 \times 12.5 \times 20000}) \div 25 = (-490 + 1113.6) \div 25 \quad (v > 0)$ M1

$v = 24.94\ldots$ $\boxed{24.9 \text{ m s}^{-1}}$ A1

(c) Down: now the 490 N acts WITH the driving force: $20000/v + 490 = 12.5v$ M1

$12.5v^2 - 490v - 20000 = 0$ A1

$v = (490 + 1113.6) \div 25$ (reject the negative root) M1

$v = 64.14\ldots$ $\boxed{64.1 \text{ m s}^{-1}}$ A1

(d) Constant speed 20 up the road: $D = 12.5 \times 20 + 490 = 740$ N; $P = 740 \times 20$ M1

$= 14800$ W $\boxed{14.8 \text{ kW}}$ A1

(Total for Question 6 is 13 marks)

TOTAL FOR PAPER IS 75 MARKS

END

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