eClassroom

A-LEVEL MATHEMATICS • STATISTICS 2

# Continuous Uniform Distribution

Worked Solutions

Pearson Edexcel International A Level (WST02) • Calculator and statistical tables allowed: show every step

Full worked solutions for the free Continuous Uniform Distribution worksheet, A-Level and International A-Level Maths


A **Examples** & Your turn page 2

B **Fluency** core skills page 2

C **Reasoning** explain it page 4

D **Problem** solving page 5

E **Exam** practice page 6


Tap a section to jump straight to it

**Full worked solutions** for every question on the Statistics 2 Continuous Uniform Distribution worksheet, in the same order and with the same question numbers.

**How to mark your work.** Tick each step you got right, not just the final answer. At A-level most marks are for method, so a wrong final answer can still earn most of the marks. If you lost marks, find the *first* line where your working is different from ours. That is the thing to fix.

In Section E: **M** = method mark, **dM** = method mark that depends on the previous M mark, **A** = accuracy mark (needs the M mark before it), **B** = independent mark.

Question sheet | Exam questions by topic | Statistics 2 exam questions | More worksheets


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# A Your turn answers

Compare each line of your working with the solution.

## CORE

**A1** $f(x) = \frac{1}{10}$ for $-4 \leqslant x \leqslant 6$, 0 otherwise; $\frac{3.5-(-1)}{10}$: **0.45**

**A2** $\frac{-6+4}{2}$, $\frac{10^2}{12}$: $-1, \frac{25}{3}$

**A3** $F(x) = 0$ **for** $x < -2$, $\frac{x+2}{10}$ **for** $-2 \leqslant x \leqslant 8$, **1 for** $x > 8$

## STRETCH

**A4** $(b-a)^2 = 27 \times 12 = 324$, $b-a = 18$, $a+b = 5$: $a = -6.5, b = 11.5$

**A5** $2-a = 0.1L$, $b-10 = 0.5L$; $L-8 = 0.6L$, $L = 20$: $a = 0, b = 20$

**A6** $E(X) = 5$, $\text{Var}(X) = 3$; $5-2(5)$, $(-2)^2(3)$: $-5$, **12**

**A7** $\text{Var}(X) = \frac{36}{12} = 3$, $E(X) = 1$; $3+1^2$: **4**

# B Fluency solutions

One mark for each correct part. The notes in italics explain why each sequence was chosen.

## CORE

**B1** **(a)** $\frac{3}{12}$: **0.25** **(b)** $\frac{3}{12}$: **0.25** **(c)** **0** **(d)** $P(-2 < X < 3) = \frac{5}{12}$: $\frac{5}{12}$

*Trim each interval to* $[-2, 10]$ *before measuring it.*

**B2** **(a)** **6, 12** **(b)** **16, 12** **(c)** **12, 48** **(d)** **0, 48**

*Shifting moves the mean only; doubling the width multiplies the variance by 4.*

**B3** **(a)** $\boxed{\dfrac{x-20}{40}}$ **(b)** $\boxed{40}$ **(c)** $\boxed{30}$ **(d)** $20+0.9(40)$: $\boxed{56}$

*Every percentile is a fraction of the way along the interval.*

**B4** **(a)** $\boxed{a=4,\ b=16}$ **(b)** $(b-3)^2=144$: $\boxed{b=15}$ **(c)** $b=-a,\ (2b)^2=576$: $\boxed{a=-12,\ b=12}$ **(d)** $\dfrac{5}{b}=0.2$: $\boxed{b=25}$

*Each condition gives one equation in $a$ and $b$*

**B5** **(a)** $\dfrac{0.4}{1}$: $\boxed{0.4}$ **(b)** $\boxed{\text{0 and } \dfrac{1}{12}}$ **(c)** U$[-5, 5]$: $\dfrac{2}{10}$: $\boxed{0.2}$

*Rounding to the nearest $u$ gives an error that is uniform on $\left[-\frac{u}{2}, \frac{u}{2}\right]$*

**B6** **(a)** $2(4)+5$: $\boxed{13}$ **(b)** $4(3)$: $\boxed{12}$ **(c)** $Y \sim$ U$[7, 19]$: $\dfrac{3}{12}$: $\boxed{0.25}$ **(d)** $P(Y<12)=\dfrac{5}{12}$: $\boxed{\dfrac{5}{12}}$

*A linear function of a uniform variable is uniform on the image interval.*

## STRETCH

**B7** **(a)** $\boxed{0.4}$ **(b)** $\dfrac{5/20}{8/20}$: $\boxed{\dfrac{5}{8}}$ **(c)** $\dfrac{4/20}{10/20}$: $\boxed{0.4}$ **(d)** $\dfrac{4/20}{10/20}$: $\boxed{0.4}$

*Given an interval, the conditional distribution is uniform on that interval.*

**B8** **(a)** $\displaystyle\int_0^3 \frac{x^2}{3}\,dx$: $\boxed{3}$ **(b)** $\displaystyle\int_0^3 \frac{x^3}{3}\,dx$: $\boxed{\dfrac{27}{4}}$ **(c)** $E(X^4)=\dfrac{81}{5}$; $\dfrac{81}{5}-9$: $\boxed{\dfrac{36}{5}}$ **(d)** $P(X<2)$: $\boxed{\dfrac{2}{3}}$

$E(g(X)) = \int g(x)f(x)\,dx$ *with* $f(x)=\dfrac{1}{3}$

**B9** $p=P(X>4)=0.2$, $Y \sim B(10, 0.2)$

**(a)** $\binom{10}{2}(0.2)^2(0.8)^8$: $\boxed{0.302}$ **(b)** $1-0.8^{10}$: $\boxed{0.893}$ **(c)** $\boxed{2}$ **(d)** $0.9^{10}$: $\boxed{0.349}$

*The uniform probability becomes the $p$ of a binomial.*

**B10** **(a)** $\boxed{2}$ **(b)** $\dfrac{4}{12}+4$: $\boxed{\dfrac{13}{3}}$ **(c)** $\pi E(R^2)$: $\boxed{\dfrac{13\pi}{3}}$ **(d)** $R>2$: $\boxed{0.5}$

$E(\pi R^2)$ *is not* $\pi[E(R)]^2$

# C Reasoning solutions

Your wording can be different. Check that your reason makes the same point.

## CORE

**C1** He has left out the +1: $5(4)+1$: $\boxed{21}$

**C2** The variance scales by $5^2$, and the +1 has no effect: $25\times\frac{4}{3}$: $\boxed{\frac{100}{3}}$

**C3** The variance is $\frac{(b-a)^2}{12}$: $(b-a)^2=144$, $b-a=12$, $a+b=18$: $\boxed{a=3,\ b=15}$

**C4** $X$ cannot be negative, so only $0<X<5$ counts: $\frac{5}{10}$: $\boxed{\frac{1}{2}}$

## STRETCH

**C5** The longer piece is at least 15 cm: $L\sim\text{U}[15,30]$; $\frac{30-20}{15}$: $\boxed{\frac{2}{3}}$

**C6** $X$ is continuous: probabilities are lengths, not counts. $\frac{6-3}{10}$: $\boxed{\frac{3}{10}}$

**C7** $\text{E}(X^2)=\text{Var}(X)+[\text{E}(X)]^2=3+9$: $\boxed{12}$

**C8** **(a)** $\int_a^b \frac{x}{b-a}\,\text{d}x=\frac{b^2-a^2}{2(b-a)}=\frac{a+b}{2}$, as required

**(b)** $\text{E}(X^2)=\frac{b^3-a^3}{3(b-a)}=\frac{a^2+ab+b^2}{3}$; $\frac{a^2+ab+b^2}{3}-\frac{(a+b)^2}{4}=\frac{a^2-2ab+b^2}{12}=\frac{(b-a)^2}{12}$, as required

# D Problem solving solutions

There is often more than one correct method. Any complete, correct method scores full marks.

## CORE

**D1** **(a)** She reaches the stop between 07:45 and 08:05 and waits for the 08:05 bus: the wait is between 0 and 20 minutes; $T = 5 + \text{wait} + 25$, so $T$ runs from 30 to 50, as required

**(b)** $\frac{50-45}{20}$: **0.25**

**D2** **(a)** $(b-a)^2 = 100$, $b-a = 10$; $\frac{6-a}{10} = 0.3$: $a = 3, b = 13$

**(b)** $\frac{k-3}{10} = 0.85$: $k = 11.5$

**D3** **(a)** $5c - 4 = 11$: $c = 3$

**(b)** $9 \times \frac{36}{12}$: **27**

**D4** **(a)** $\frac{0.04}{0.1}$: **0.4**

**(b)** $p = 0.6$ for an error of at least 0.02; B(12, 0.6): at least 10 means at most 2 of the other kind, P($Y \leqslant 2$) for B(12, 0.4): **0.0834**

## STRETCH

**D5** Cut at $X \sim$ U[0, 40]: $X(40-X) > 300$, $X^2 - 40X + 300 < 0$, $(X-10)(X-30) < 0$, $10 < X < 30$

$\frac{20}{40}$: **0.5**

**D6** **(a)** $\mathrm{E}(X) = \frac{5k}{2}$, $\mathrm{Var}(X) = \frac{3k^2}{4}$

**(b)** $\frac{3k^2}{4} + \frac{25k^2}{4}$: $7k^2$

**D7** **(a)** $\frac{5}{20}$: **0.25**

**(b)** $4(0.25)(0.75)^3$: **0.422**

**D8** **(a)** Area $= (10k - 2k) \times \frac{1}{16} = 1$, so $k = 2$, as required

**(b)** $\frac{4+20}{2}$: **12**

**(c)** $E(X^2) = \frac{16^2}{12} + 12^2$: $\frac{496}{3} \approx 165$

## E Exam practice solutions

Mark-scheme notes are shown on the right of each line.

**CORE** *Exam questions for everyone*

**E1** **(a)** $f(t) = \frac{1}{40}$ **for** $0 \leqslant t \leqslant 40$ *B1*

and 0 otherwise *B1*

**(b)** $E(T) = 20$ *B1*

$\frac{40^2}{12}$: $\frac{400}{3}$ *B1*

**(c)** $\frac{10}{40}$: **0.25** *B1*

**(d)** $P(T < 6 \mid T < 10) = \frac{P(T < 6)}{P(T < 10)}$ *M1*

$= \frac{6/40}{10/40}$ *A1*

**0.6** *A1*

**(e)** $W \sim B(5, 0.25)$ *B1*

$1 - 0.75^5 - 5(0.25)(0.75)^4$ *M1*

**0.367** *A1*

**E2** **(a)** $\frac{2-a}{b-a} = 0.2$ and $\frac{b-12}{b-a} = 0.4$ *M1*

the middle 10 units carry 0.4, so $b-a = 25$ *A1*

$\boxed{a = -3,\ b = 22}$ *A1*

**(b)** $\boxed{\mathrm{E}(X) = 9.5}$ *B1*

$\frac{25^2}{12}$: $\boxed{\frac{625}{12} \approx 52.1}$ *B1*

**(c)** $\mathrm{P}(-3 < X < 5)$ *M1*

$\frac{8}{25}$: $\boxed{\mathbf{0.32}}$ *A1*

**(d)** $22 - c = 2(c+3)$ *M1*

$\boxed{c = \frac{16}{3}}$ *A1*

**(e)** $\frac{625}{12} + 9.5^2$ *M1*

$\boxed{\frac{427}{3}}$ *A1*

## STRETCH

**E3** **(a)** $\boxed{S \sim \mathrm{U}[0, 30]}$ *B1*

$\boxed{\mathrm{E}(S) = 15}$ *B1*

$\boxed{\mathrm{Var}(S) = 75}$ *B1*

**(b)** $60 - S > 2S$ *M1*

$S < 20$ *A1*

$\frac{20}{30} = \frac{2}{3}$, as required *A1**

**(c)** $\mathrm{B}\left(100, \frac{2}{3}\right) \approx \mathrm{N}\left(66.\dot{6}, 22.\dot{2}\right)$ *B1*

$\mathrm{P}(Y > 70.5)$ *M1*

$z = \frac{70.5 - 66.67}{4.714} \approx 0.81$ *M1*

$1 - 0.7910$: $\boxed{\mathbf{0.209}}$ *A1*

**(d)** Area $= S(60 - S) = 60S - S^2$ *M1*

$\mathrm{E}(S^2) = 75 + 15^2 = 300$ *M1 A1*

$60(15) - 300$: $\boxed{\mathbf{600\ cm^2}}$ *A1*