



eClassroom

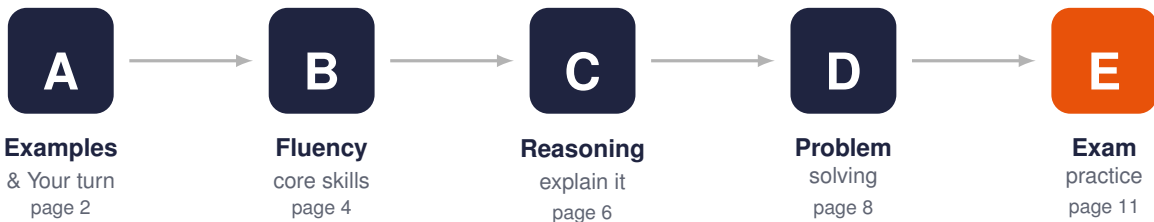
A-LEVEL MATHEMATICS • PURE 3

Exponentials and Logarithms

Worksheet

Pearson Edexcel International A Level (WMA13) & A Level (9MA0) • Calculator allowed: show every step

Free Exponentials and Logarithms worksheet for A-Level and International A-Level Maths, with full worked solutions



Tap a section to jump straight to it

By the end of this worksheet you can: e^x and $\ln x$ • exact equations and models • *Stretch*: hidden quadratics, rates and log graphs.

Which questions are for me?

CORE The methods every A-level student must secure. Start here.

STRETCH Exact equations, hidden quadratics, models with rates, and straight-line log graphs.

Every section has both, clearly labelled. Questions get harder as you go, and every skill in Sections A to D is needed in Section E.

Track your progress

Section	B	C	D	E
Core marks	23	10	13	11
Stretch marks	16	14	17	8
My score				

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A**Worked examples & Your turn**

Read each example carefully. Cover it up, then answer the question beside it using the same method.

CORE*Secure the method***Worked example A1**

Solve $5e^{3x} - 2 = 18$, giving an exact answer.

Make e^{3x} the subject, then take $\ln$ of both sides.

$$e^{3x} = 4, \text{ so } 3x = \ln 4$$

$$x = \frac{1}{3} \ln 4$$

Your turn A1

Solve $2e^{1-x} + 3 = 11$, giving an exact answer.

Worked example A2

Solve $\ln(2x + 1) = 3$, giving an exact answer.

Take e to the power of both sides: $2x + 1 = e^3$

$$x = \frac{1}{2} (e^3 - 1)$$

Your turn A2

Solve $2\ln(x - 3) = 5$, giving an exact answer.

Worked example A3

For $y = 4 - e^{-x}$, state the asymptote and find the exact intercepts.

$e^{-x} \rightarrow 0$ as $x \rightarrow \infty$; $x = 0$ gives y ; $y = 0$ gives $e^{-x} = 4$

$$y = 4; (0, 3); (-\ln 4, 0)$$

Your turn A3

For $y = 2e^x - 6$, state the asymptote and find the exact intercepts.



STRETCH

Worked example A4: a hidden quadraticSolve $e^{2x} - 5e^x + 6 = 0$ Let $u = e^x$: $u^2 - 5u + 6 = 0$, so $u = 2$ or 3

$$x = \ln 2, \quad x = \ln 3$$

Your turn A4Solve $e^x - 8e^{-x} = 2$ **Worked example A5: laws of logarithms**Solve $\ln x + \ln(x+3) = \ln 10$ $x(x+3) = 10$: $x = 2$ or $x = -5$; $\ln(-5)$ is undefined, so reject it.

$$x = 2$$

Your turn A5Solve $2\ln x - \ln(x+6) = 0$ **Worked example A6: a model** $N = 200e^{0.3t}$. Find N when $t = 0$, and t when $N = 1000$

$$e^0 = 1; \text{ then } e^{0.3t} = 5, \quad t = \frac{\ln 5}{0.3}$$

$$200; \quad t = 5.36$$

Your turn A6 $V = 12000e^{-0.15t}$. Find V when $t = 0$, and t when $V = 6000$ **Worked example A7: a log graph** $y = ax^n$. The graph of $\log_{10} y$ against $\log_{10} x$ is a line through $(0, 0.6)$ and $(2, 3.6)$. Find a and n $\log_{10} y = \log_{10} a + n \log_{10} x$: gradient n , intercept $\log_{10} a$

$$n = 1.5, \quad a = 10^{0.6} = 3.98$$

Your turn A7 $y = kb^x$. The graph of $\log_{10} y$ against x is a line through $(0, 1.2)$ and $(4, 2)$. Find k and b 

B**Fluency**

Each question is a designed sequence. From one part to the next, ask yourself: what changed, and what did it do to the answer?

CORE*Secure the method***B1** Simplify

(a) $e^{\ln 5}$

(b) $\ln(e^{3x})$

(c) $e^{2\ln 3}$

(d) $\ln \frac{1}{e^2}$

[4 marks]**B2** Solve, giving exact answers where possible.

(a) $e^x = 7$

(b) $e^{2x} = 7$

(c) $e^{2x+1} = 7$

(d) $e^{2x+1} = -7$

[4 marks]**B3** Solve, giving exact answers.

(a) $\ln x = 2$

(b) $\ln 3x = 2$

(c) $\ln(3x - 1) = 2$

(d) $\ln(3x - 1) = \ln 2 + \ln x$

[4 marks]**B4** State the asymptote and the exact intercepts with the axes.

(a) $y = e^x + 2$

(b) $y = e^{-x} - 3$

(c) $y = \ln(x - 2)$

(d) $y = 3 + \ln 2x$

[4 marks]

CORE *continued: models and log graphs***B5** $N = 50e^{0.4t}$. Find

(a) N when $t = 0$

(b) N when $t = 5$

(c) t when $N = 200$

[3 marks]**B6**

(a) For $y = 3x^2$, give the gradient and intercept of the graph of $\log_{10} y$ against $\log_{10} x$

(b) For $y = 3 \times 2^x$, give the gradient and intercept of the graph of $\log_{10} y$ against x

(c) The graph of $\log_{10} y$ against $\log_{10} x$ is $\log_{10} y = 0.5 + 2\log_{10} x$. Find y in terms of x

(d) The graph of $\log_{10} y$ against x is $\log_{10} y = 1 - 0.3x$. Find y in the form kb^x

[4 marks]**STRETCH****B7** Solve, giving exact answers.

(a) $e^{2x} - 7e^x + 12 = 0$

(b) $e^{2x} + e^x - 6 = 0$

(c) $2e^x + 3e^{-x} = 7$

(d) $e^x + 5e^{-x} = 2$

[4 marks]**B8** Solve, giving exact answers.

(a) $\ln(x+1) + \ln(x-1) = \ln 8$

(b) $\ln(x+4) - \ln x = \ln 3$

(c) $2\ln(x+2) = \ln(x+8)$

(d) $\ln x + \ln 2 = 1$

[4 marks]**B9** $T = 20 + 70e^{-0.05t}$

(a) Find T when $t = 0$

(b) State the value that T approaches in the long term.

(c) Find $\frac{dT}{dt}$ when $t = 10$

(d) Find t when $T = 50$

[4 marks]**B10** $f(x) = e^{2x} - 1$, $x \in \mathbb{R}$, and $g(x) = \ln(x+3)$, $x > -3$

(a) State the range of f

(b) Find $f^{-1}(x)$ and state its domain.

(c) Find $g^{-1}(x)$ and state its domain.

(d) Solve $f(x) = 3$, giving an exact answer.

[4 marks]

C**Reasoning**

Show your working and explain your thinking. A correct answer without a reason does not score full marks.

CORE

Secure the method

- C1** Sam writes $\ln(x+3) = \ln x + \ln 3$
Explain Sam's mistake, and solve $\ln(x+3) = 2$ exactly. [2 marks]
- C2** The temperature of a drink falls at 2.1 degrees per minute when $t = 10$. Lily writes $\frac{dT}{dt} = 2.1$
Explain Lily's mistake. [2 marks]
- C3** Kai finds $k = \frac{1}{10} \ln \frac{4}{3}$ and writes $k = 0.03$ before using it in the next part of the question.
Explain why this may lose marks, and give k to 3 significant figures. [3 marks]
- C4** Asked for an exact answer to $e^x = 2$, Jo writes $x = 0.69$
Explain why Jo would lose the mark, and give the exact answer. [3 marks]



STRETCH

- C5** Max solves $\ln x + \ln(x - 2) = \ln 3$ and gives $x = 3$ and $x = -1$
Explain which answer must be rejected and why. [3 marks]
- C6** A graph of $\log_3 N$ against t is the line $\log_3 N = 2 + 0.5t$. Ali writes $N = 100 \times 10^{0.5t}$
Explain Ali's mistake, and write N in the form ab^t exactly. [3 marks]
- C7** In a model, N is the number of items sold, in thousands. Kai finds $\frac{dN}{dt} = 4.73$ when $t = 7$ months and answers "4.73 items per month".
Explain Kai's mistake, and give the rate correctly. [3 marks]
- C8** (a) Prove that $\ln(e^{2x} - e^x) \equiv x + \ln(e^x - 1)$ for $x > 0$ (2)
- (b) Hence solve $\ln(e^{2x} - e^x) = x + \ln 3$, giving the exact answer. (2)
- (c) Explain why the identity in (a) is only true for $x > 0$ (1)
- [5 marks]



D**Problem solving**

You decide the method. Read the whole question first, and show every step.

CORE

Secure the method

- D1** The intensity, I , of light measured at a distance d metres from a lamp is thought to satisfy $I = ad^n$, where a and n are constants. Figure 1 shows the straight line obtained when $\log_{10} I$ is plotted against $\log_{10} d$. The line passes through the points $(1, 2.3)$ and $(3, 1.1)$

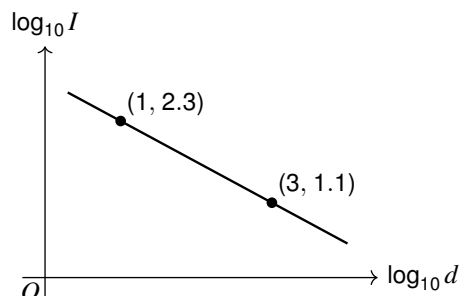


Figure 1

- (a) Find an equation of the line, giving $\log_{10} I$ in terms of $\log_{10} d$

(1)

- (b) Hence find the values of a and n , giving a to 3 significant figures.

(2)

[3 marks]

- D2** The concentration, C mg per litre, of a drug in a patient's blood t hours after an injection is modelled by $C = 15e^{-kt}$, where k is a positive constant. Four hours after the injection the concentration has fallen to 6 mg per litre. Use the model to answer parts (a) to (c).

- (a) State the concentration immediately after the injection.

(1)

- (b) Show that $k = \frac{1}{4} \ln \frac{5}{2}$

(1)

- (c) A second injection is given when the concentration first falls below 1 mg per litre. Find, to the nearest minute, how long after the first injection this happens.

(2)

[4 marks]

- D3** Figure 2 shows the curve with equation $y = 5 - 2e^{\frac{x}{2}}$. The curve meets the y -axis at A and the x -axis at B , and the line $y = 5$ is an asymptote to the curve.

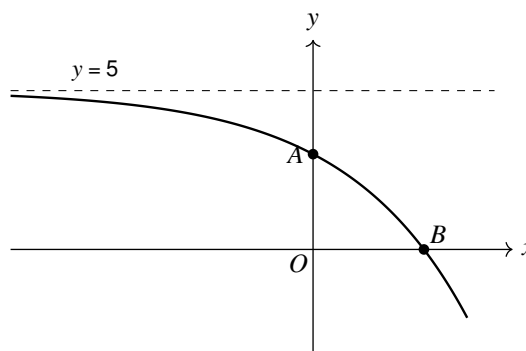


Figure 2

- (a) State the coordinates of A , and explain why the curve approaches $y = 5$ as x decreases.

(1)

- (b) Find the exact x coordinate of B , giving your answer in the form $p \ln q$, where p and q are rational numbers.

(2)

[3 marks]



D4 The value, £ V , of a car t years after it was bought new is modelled by the equation $\ln V = 9.2 - 0.12t$. Use the equation of the model to answer parts (a) and (b).

(a) Write this model in the form $V = ab^t$, giving the exact value of a and the value of b to 3 decimal places, and explain what b tells you about the car's value each year.

(2)

(b) Find the value of the car 5 years after it was bought, to the nearest £10

(1)

[3 marks]

STRETCH

D5 In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.

Figure 3 shows the curves with equations $y = 3e^x$ and $y = 2 + 8e^{-x}$. The curves meet at exactly one point, P

(a) By letting $u = e^x$, show that $3u^2 - 2u - 8 = 0$

(2)

(b) Hence find the exact coordinates of P , explaining why one root of the quadratic is rejected.

(2)

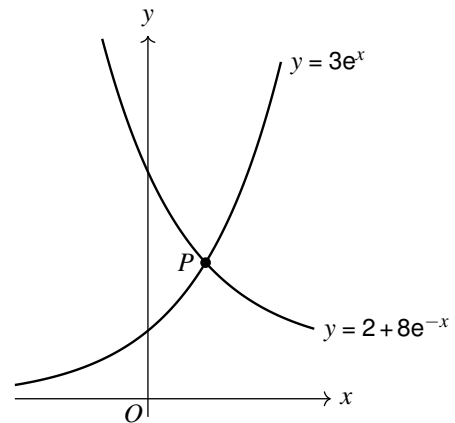


Figure 3

[4 marks]

D6 Figure 4 shows the curves with equations $y = \ln(2x + 1)$ and $y = 2 - \ln x$, for $x > 0$. The curves meet at the point Q

(a) Use the laws of logarithms to show that the x coordinate of Q satisfies $2x^2 + x - e^2 = 0$

(2)

(b) Find the exact x coordinate of Q , and give it to 3 significant figures.

(2)

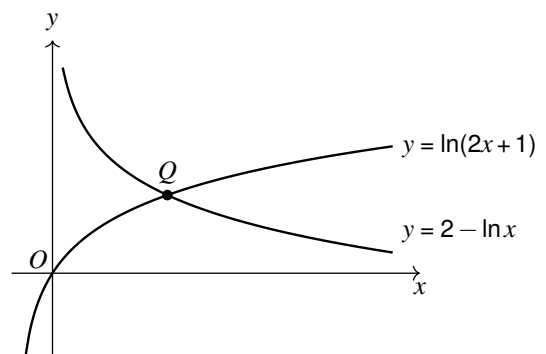


Figure 4

[4 marks]



- D7** The mass, m grams, of a radioactive substance t days after it was first measured is modelled by $m = 80e^{-kt}$. The mass halves every 6 days.

(a) Show that $k = \frac{1}{6} \ln 2$

(2)

- (b) Find the rate at which the mass is decreasing 10 days after it was first measured, stating the units of your answer and explaining how the sign of your derivative shows that the mass is decreasing.

(2)

[4 marks]

- D8** After a meal, the concentration, G units, of glucose in a person's blood t hours later is modelled by

$$G = 40 + 60e^{-0.1t} - 30e^{-0.4t}$$

- (a) State the value of G when $t = 0$, and the value that G approaches in the long term.

(2)

- (b) Show that G is greatest when $t = \frac{10}{3} \ln 2$, explaining how you know this is a maximum.

(2)

- (c) Find the greatest value of G according to the model.

(1)

[5 marks]



E**Exam practice**

Exam-style questions in the style of WMA13. Write your answers in the spaces, and show every step.

CORE

Exam questions for everyone

- E1** The number of followers, P , of a new online account t weeks after it was opened is monitored. The number is modelled by the equation

$$\log_{10} P = 2.1 + 0.04t$$

Use the equation of the model to answer parts (a) and (b).

- (a)** Show that the model can be written in the form $P = ab^t$, where a and b are constants to be found. Give the value of a to one decimal place and the value of b to 3 decimal places.

.....
(3)

- (b)** Find, according to the model, the number of whole weeks after which the account first has more than 1000 followers.

.....
(2)

(Total for Question E1 is 5 marks)



- E2** The mass, M grams, of a crystal growing in a solution t days after it was placed in the solution is modelled by the equation

$$M = 25 - 20e^{-0.08t} \quad t \geq 0$$

Figure 5 shows a sketch of M against t . Use the equation of the model to answer parts (a) to (d).

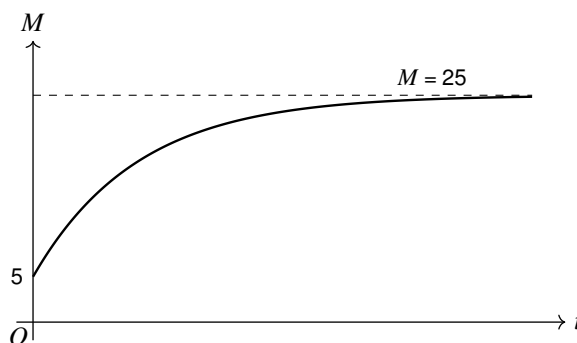


Figure 5

- (a) State the initial mass of the crystal.

.....

(1)

- (b) State the equation of the asymptote shown in Figure 5, and explain what it means for the crystal.

.....

(1)

- (c) Find the value of t for which $M = 20$, giving your answer to 2 decimal places.

.....

(2)

- (d) Show by differentiation that $\frac{dM}{dt} = a + bM$, where a and b are constants to be found.

.....

(2)

(Total for Question E2 is 6 marks)



STRETCH

- E3** In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.

Figure 6 shows a sketch of the curve C with equation

$$y = e^{2x} - 4e^x + 3 \quad x \in \mathbb{R}$$

The curve passes through the origin O and meets the x -axis again at B . The curve has a minimum point P , and the line $y = 3$ is an asymptote to C

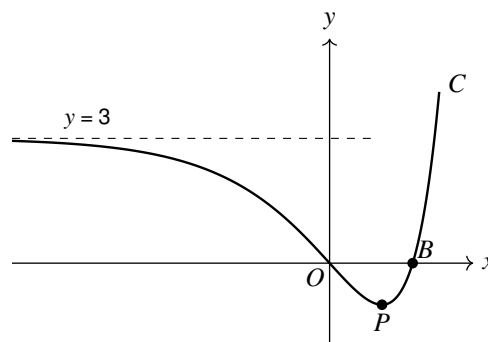


Figure 6

- (a) Find the exact x coordinate of B

.....
(2)

- (b) By writing y in the form $(e^x - p)^2 - q$, find the exact coordinates of P

.....
(3)

- (c) State the range of the function $f(x) = e^{2x} - 4e^x + 3$, $x \in \mathbb{R}$

.....
(1)

- (d) Find the set of values of the constant k for which the equation $e^{2x} - 4e^x + 3 = k$ has exactly two solutions.

.....
(2)

(Total for Question E3 is 8 marks)

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