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INTERNATIONAL A LEVEL MATHEMATICS • PURE 3 (WMA13)

Pure 3 Practice Paper 3

Mark Scheme

AIMING FOR A/A*

Written in the style of the Pearson Edexcel International Advanced Level paper (WMA13/01) • 75 marks

Every question marked line by line the way an examiner marks it, with notes on the common errors

Mark every line, not just the answer. Method marks are given for a correct method even when the final answer is wrong. If you lost marks, find the *first* line where your working differs from the scheme. That is the thing to fix.

Code	Meaning
M1	Method mark: for knowing a correct method and attempting to apply it. Small slips in the working do not lose this mark.
dM1	Dependent method mark: can only be given if the method mark before it has been given.
A1	Accuracy mark: for a correct answer or line of working. It depends on the method mark before it.
B1	Independent mark: for a correct result or statement, whatever method was used.
*	The answer is given in the question. Every step must be shown, and the answer must follow from correct working (cso).
cao / cso	Correct answer only / correct solution only, with no errors anywhere in the working.
ft / oe	Follow through from your own earlier answer / or any correct equivalent form.

"Solutions relying on calculator technology are not acceptable." Every question in this paper says this. There, a correct final answer with no algebraic working scores **no marks**.

Rough grade guide (out of 75): A* **56** • A **49** • B **43** • C **37**

Based on recent WMA13 grade boundaries. Real boundaries change every series, so treat this as a guide, not a grade.

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What to revise next

Write in your score for each question. Where you dropped marks, tap the chapter to practise ten more exam questions on it.

Q	What it tested	Marks	Practise it	Score
1	$\tan(A+B)$ with inverse tangents; rejecting a root by its angle; a proof with a range argument	9	Ch 3 Trigonometric Functions Ch 4 Trig Addition Formulae	
2	Product rule; telling maxima from minima; a geometric series of heights	10	Ch 6 Differentiation Ch 3 Trigonometric Functions Ch 5 Exponentials and Logarithms	
3	Sketching a quadratic in e^x; restricting a domain for an inverse; four modulus roots	10	Ch 2 Functions and Graphs Ch 5 Exponentials and Logarithms Ch 1 Algebraic Methods	
4	Deriving $\sin 3x$; area between curves that swap order	9	Ch 4 Trig Addition Formulae Ch 7 Integration Ch 3 Trigonometric Functions	
5	$\frac{dy}{dx}$ from $\frac{dx}{dy}$ with an identity; solving a quadratic in e^y	10	Ch 6 Differentiation Ch 5 Exponentials and Logarithms	
6	$R\sin(x+\alpha)$ and a sketch; double angles from a given sum; matching signs to solutions	9	Ch 4 Trig Addition Formulae Ch 3 Trigonometric Functions	
7	A minimum and two roots; an iteration that fails; choosing one that works	9	Ch 8 Numerical Methods Ch 6 Differentiation Ch 5 Exponentials and Logarithms	
8	Algebraic division; integrating $\frac{2x}{x^2+1}$; proving a function is increasing	9	Ch 1 Algebraic Methods Ch 7 Integration Ch 6 Differentiation	
	Total	**75**		

Where did the marks go? If you lost the first marks of a question, the method needs work: go back to that chapter's key skills page. If you lost the final marks, the method is fine and the gap is the trap: a root that has to be rejected, a value that has to be kept, a sign to justify, or a second region or second case. At grade A* the method is rarely the problem; checking every answer against the question is.

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Question 1

9 marks

Part	Scheme	Marks
(a)	Takes tan of both sides and uses $\tan(A+B)$: $\dfrac{x+2x}{1-x\cdot 2x} = \tan\dfrac{\pi}{4}$	M1
	$\dfrac{3x}{1-2x^2} = 1$	A1
	$2x^2+3x-1=0$, solved by the formula	M1
	$x = \dfrac{-3\pm\sqrt{17}}{4}$	A1
	Rejects $x = \dfrac{-3-\sqrt{17}}{4}$: both inverse tangents are then negative, so their sum is $-\dfrac{3\pi}{4}$, not $\dfrac{\pi}{4}$	B1
	$\boxed{x = \dfrac{\sqrt{17}-3}{4}}$ only	A1
	Taking tan loses information: $\tan\theta = 1$ *also when* $\theta = -\frac{3\pi}{4}$*. The B1 needs a reason, not just a rejection. Leaving both roots scores at most 4 marks.*	(6)
(b)	$\tan\left(2\arctan\frac{1}{3}\right) = \dfrac{\frac{2}{3}}{1-\frac{1}{9}} = \dfrac{3}{4}$	M1
	$\tan(\text{sum}) = \dfrac{\frac{3}{4}+\frac{1}{7}}{1-\frac{3}{4}\times\frac{1}{7}} = \dfrac{25/28}{25/28} = 1$	M1
	$0 < \arctan\frac{1}{3} < \frac{\pi}{4}$ and $0 < \arctan\frac{1}{7} < \frac{\pi}{4}$, so the sum lies in $\left(0, \frac{3\pi}{4}\right)$, where tan = 1 only at $\frac{\pi}{4}$ *	A1*
	Without the range argument, $\tan(\text{sum}) = 1$ *only shows the sum is* $\frac{\pi}{4}+k\pi$*. The A1 needs that argument.*	(3)
		Total for Question 1: 9 marks

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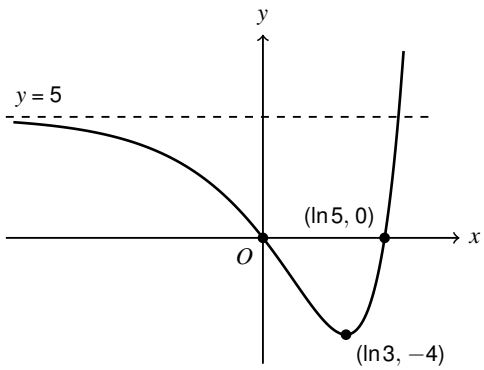
Question 2

10 marks

Part	Scheme	Marks
(a)	Product rule: $\frac{dy}{dx} = -\frac{1}{2}e^{-\frac{1}{2}x}\sin x + e^{-\frac{1}{2}x}\cos x$	M1
	Fully correct	A1
	$e^{-\frac{1}{2}x} \neq 0$, so $\cos x = \frac{1}{2}\sin x$, giving $\tan x = 2$ *	A1*
		(3)
(b)	Stationary points at $x = \alpha + k\pi$, where $\alpha = \arctan 2$	M1
	Identifies the maxima as $x = \alpha + 2n\pi$ (where $\sin x > 0$; the others are minima)	M1
	$y_n = e^{-\frac{1}{2}(\alpha+2n\pi)}\sin\alpha = e^{-\frac{1}{2}\alpha}\sin\alpha \times \left(e^{-\pi}\right)^n$	A1
	Each term is the one before multiplied by the constant $\boxed{e^{-\pi}}$, so the sequence is geometric	A1
	Taking all the points where $\tan x = 2$ as maxima gives ratio $e^{-\frac{1}{2}\pi}$ and an alternating sign: M1 M0 A0 A0.	
		(4)
(c)	$\tan\alpha = 2$, α acute, so $\sin\alpha = \frac{2}{\sqrt{5}}$	M1
	$S_\infty = \frac{a}{1-r}$ with $a = \frac{2}{\sqrt{5}}e^{-\frac{1}{2}\arctan 2}$ and $r = e^{-\pi}$ $(\lvert r\rvert < 1)$	M1
	$\boxed{\frac{2e^{-\frac{1}{2}\arctan 2}}{\sqrt{5}(1-e^{-\pi})}}$ oe $(= 0.537\ldots)$	A1
		(3)
	Total for Question 2: 10 marks	

Question 3

10 marks

Part	Scheme	Marks
(a)	 Shape: falling from the asymptote on the left, through O, to a minimum, then rising steeply O, (ln 5, 0) and minimum (ln 3, −4) Asymptote $\boxed{y = 5}$ $f(x) = (e^x - 1)(e^x - 5)$ *gives the intercepts;* $f'(x) = 2e^{2x} - 6e^x = 0$ *gives* $e^x = 3$.	B1 B1 B1 (3)
(b)	f is many-to-one, for example f(0) = f(ln 5) = 0, so it has no inverse $\boxed{k = \ln 3}$ $y = (e^x - 3)^2 - 4 \Rightarrow e^x = 3 + \sqrt{y+4}$ (positive root, as $e^x \geqslant 3$) $\boxed{g^{-1}(x) = \ln\left(3 + \sqrt{x+4}\right),\ x \geqslant -4}$	B1 B1 M1 A1 (4)
(c)	$f(x) = 3$ or $f(x) = -3$: $e^{2x} - 6e^x + 2 = 0$ or $e^{2x} - 6e^x + 8 = 0$, solved as quadratics in e^x $e^x = 2$ or 4: $\boxed{x = \ln 2,\ \ln 4}$ $e^x = 3 \pm \sqrt{7}$, both positive: $\boxed{x = \ln\left(3 + \sqrt{7}\right),\ \ln\left(3 - \sqrt{7}\right)}$ $3 - \sqrt{7} = 0.35\ldots > 0$, *so* $\ln\left(3 - \sqrt{7}\right)$ *is a valid (negative) solution: rejecting it is the trap. Four solutions in total.*	M1 A1 A1 (3)
		Total for Question 3: 10 marks

Question 4

9 marks

Part	Scheme	Marks
(a)	$\sin 3x = \sin 2x\cos x + \cos 2x \sin x = 2\sin x\cos^2 x + (1 - 2\sin^2 x)\sin x = 3\sin x - 4\sin^3 x$	M1
	$3\sin x - 4\sin^3 x = \sin x \Rightarrow \sin x(2 - 4\sin^2 x) = 0$; $\sin x \neq 0$ at P, so $\sin^2 x = \frac{1}{2}$	M1
	$\boxed{x = \frac{\pi}{4}}$	A1
		(3)
(b)	$R_1 = \int_0^{\frac{\pi}{4}} (\sin 3x - \sin x)\,dx$ (top curve minus bottom curve)	M1
	Integrates: $-\frac{1}{3}\cos 3x + \cos x$	M1
	$R_1 = \left(\frac{\sqrt{2}}{6} + \frac{\sqrt{2}}{2}\right) - \left(-\frac{1}{3} + 1\right) = \frac{2\sqrt{2}}{3} - \frac{2}{3}$	A1
	$R_2 = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (\sin x - \sin 3x)\,dx$ (the order swaps at P)	M1
	$R_2 = \left[-\cos x + \frac{1}{3}\cos 3x\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}} = 0 - \left(-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{6}\right) = \frac{2\sqrt{2}}{3}$	A1
	Total $\boxed{\frac{4}{3}\sqrt{2} - \frac{2}{3}}$	A1
	One integral from 0 to $\frac{\pi}{2}$ gives $R_1 - R_2 = -\frac{2}{3}$, which is not an area. $\cos\frac{3\pi}{4} = -\frac{\sqrt{2}}{2}$ *is the usual slip.*	
		(6)
		Total for Question 4: 9 marks

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Question 5

10 marks

Part	Scheme	Marks
(a)	$\frac{dx}{dy} = e^{y} + e^{-y}$	M1
	Uses $\left(e^{y} + e^{-y}\right)^2 = \left(e^{y} - e^{-y}\right)^2 + 4$	M1
	$= x^2 + 4$	A1
	$e^{y} + e^{-y} > 0$, so $\frac{dx}{dy} = +\sqrt{x^2+4}$	M1
	$\boxed{\frac{dy}{dx} = \frac{1}{\sqrt{x^2+4}}}$	A1
	The third M needs the sign justified. An answer in terms of y only scores M1 at most. Working from part (b) first is a valid route.	(5)
(b)	Multiplies by e^{y}: $e^{2y} - xe^{y} - 1 = 0$	M1
	$e^{y} = \frac{x \pm \sqrt{x^2+4}}{2}$	M1
	$\sqrt{x^2+4} > \lvert x \rvert$, so the minus sign gives $e^{y} < 0$ and is rejected: $\boxed{y = \ln\left(\frac{x+\sqrt{x^2+4}}{2}\right)}$	A1
		(3)
(c)	$y = \ln 2$: $x = 2 - \frac{1}{2} = \frac{3}{2}$, gradient $\frac{1}{\sqrt{25/4}} = \frac{2}{5}$, normal gradient $-\frac{5}{2}$: $y - \ln 2 = -\frac{5}{2}\left(x - \frac{3}{2}\right)$	M1
	$\boxed{y = -\frac{5}{2}x + \frac{15}{4} + \ln 2}$	A1
		(2)
		Total for Question 5: 10 marks

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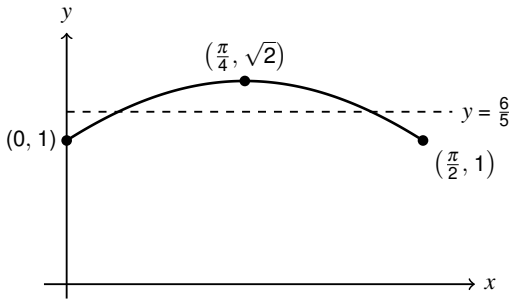
Question 6

9 marks

Part	Scheme	Marks
(a)	$\cos x + \sin x = \sqrt{2}\sin\left(x + \frac{\pi}{4}\right)$ $\left(R = \sqrt{2},\ \alpha = \frac{\pi}{4}\right)$	**M1**
	Both values correct	**A1**
		**B1**
	An arch from (0, 1) up to $\left(\frac{\pi}{4}, \sqrt{2}\right)$ and down to $\left(\frac{\pi}{2}, 1\right)$	
	$1 < \frac{6}{5} < \sqrt{2}$, so the line $y = \frac{6}{5}$ meets the arch exactly twice, once on each side of $x = \frac{\pi}{4}$	**B1**
		(4)
(b)	Squares: $\cos^2 x + 2\sin x\cos x + \sin^2 x = \frac{36}{25}$, so $1 + \sin 2x = \frac{36}{25}$	**M1**
	$\boxed{\sin 2x = \frac{11}{25}}$	**A1**
		(2)
(c)	$\cos^2 2x = 1 - \frac{121}{625} = \frac{504}{625}$	**M1**
	$\cos 2x = \pm\frac{6\sqrt{14}}{25}$	**A1**
	$\boxed{\cos 2x = \frac{6\sqrt{14}}{25}}$ for the solution with $x < \frac{\pi}{4}$ ($2x$ acute) and $\boxed{\cos 2x = -\frac{6\sqrt{14}}{25}}$ for the solution with $x > \frac{\pi}{4}$	**A1**
	Both solutions have the same $\sin 2x$ *but opposite* $\cos 2x$*. Giving only the positive value scores M1 A0 A0.*	
		(3)
		Total for Question 6: 9 marks

Question 7

9 marks

Part	Scheme	Marks
(a)	$\frac{dy}{dx} = e^x - 3 = 0 \Rightarrow x = \ln 3$	M1
	$\boxed{M(\ln 3,\ 3 - 3\ln 3)}$	A1
	$3 - 3\ln 3 < 0$ (as $\ln 3 > 1$), the curve falls then rises, and $y(0) = 1 > 0$, $y(2) = e^2 - 6 > 0$, so there is exactly one root on each side of M	A1
		(3)
(b)	$x_2 = 1.4939$, $x_3 = 1.4848$, $x_4 = 1.4714$	B1
	The values move away from β (they decrease, heading towards α), so the iteration does not converge to β	B1
		(2)
(c)	Uses a rearrangement that converges to β, such as $x_{n+1} = \ln(3x_n)$	M1
	Iterates: $1.5041,\ 1.5068,\ 1.5086,\ \ldots$ (or other valid work towards 1.512)	M1
	Tests the bounds: $f(1.5115) = -0.00097\ldots$ and $f(1.5125) = 0.00056\ldots$	M1
	Change of sign and continuity, so $\boxed{\beta = 1.512}$ to 3 decimal places	A1
	The iterations alone converge slowly and do not justify 3 decimal places. The sign-change test on $[1.5115, 1.5125]$ *is what justifies the answer.*	
		(4)
		Total for Question 7: 9 marks

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Question 8

9 marks

Part	Scheme	Marks
(a)	Divides: $x^3+2x^2+3x+2=(x+2)\left(x^2+1\right)+2x$	M1
	$\mathrm{f}(x)=x+2+\dfrac{2x}{x^2+1}$	A1
	$\displaystyle\int \frac{2x}{x^2+1}\,\mathrm{d}x=\ln\left(x^2+1\right)$	M1
	$\displaystyle\int \mathrm{f}(x)\,\mathrm{d}x=\tfrac{1}{2}x^2+2x+\ln\left(x^2+1\right)$	M1
	Uses the limits 0 and $\sqrt{3}$	dM1
	$\boxed{\dfrac{3}{2}+2\sqrt{3}+\ln 4}$	A1
	Integrating the original fraction as $\ln\left(x^2+1\right)$ *times something, without dividing first, scores M0 M0 M0.*	(6)
(b)	$\mathrm{f}'(x)=1+\dfrac{2\left(x^2+1\right)-2x\cdot 2x}{\left(x^2+1\right)^2}$	M1
	$=\dfrac{x^4+2x^2+1+2-2x^2}{\left(x^2+1\right)^2}=\dfrac{x^4+3}{\left(x^2+1\right)^2}$	A1
	$x^4+3>0$ and $\left(x^2+1\right)^2>0$ for all x, so $\mathrm{f}'(x)>0$ and f is increasing	A1
		(3)
	Total for Question 8: 9 marks	

TOTAL FOR PAPER: 75 MARKS

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