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A-LEVEL MATHEMATICS • FURTHER PURE 1

Complex Numbers

Worked Solutions

Pearson Edexcel International A Level (WFM01) • Calculator allowed: show every step

Full worked solutions for the free Complex Numbers worksheet, A-Level and International A-Level Maths

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B — **Fluency** core skills — page 2

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Tap a section to jump straight to it

Full worked solutions for every question on the Further Pure 1 Complex Numbers worksheet, in the same order and with the same question numbers.

How to mark your work. Tick each step you got right, not just the final answer. At A-level most marks are for method, so a wrong final answer can still earn most of the marks. If you lost marks, find the *first* line where your working is different from ours. That is the thing to fix.

In Section E: **M** = method mark, **dM** = method mark that depends on the previous M mark, **A** = accuracy mark (needs the M mark before it), **B** = independent mark.

Question sheet | Exam questions by topic | Further Pure 1 exam questions | More worksheets

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A Your turn answers

Compare each line of your working with the solution.

CORE

A1 $zw = 15 - 5\text{i} + 6\text{i} - 2\text{i}^2$: $\boxed{17 + \text{i}}$; $2z + w = 10 + 4\text{i} + 3 - \text{i}$: $\boxed{13 + 3\text{i}}$

A2 $\dfrac{(5 + 15\text{i})(2 - \text{i})}{5} = \dfrac{25 + 25\text{i}}{5}$: $\boxed{5 + 5\text{i}}$

A3 Second quadrant: $|z| = \sqrt{3 + 1}$, $\arg z = \pi - \dfrac{\pi}{6}$: $\boxed{2, \dfrac{5\pi}{6}}$

STRETCH

A4 $z = \dfrac{-2 \pm \sqrt{4 - 40}}{2}$: $\boxed{-1 \pm 3\text{i}}$

A5 $3(a + b\text{i}) - (a - b\text{i}) = 2a + 4b\text{i} = 4 + 12\text{i}$: $\boxed{z = 2 + 3\text{i}}$

A6 $(z + 1 - 2\text{i})(z + 1 + 2\text{i}) = z^2 + 2z + 5$; $z^3 + 5z^2 + 11z + 15 = (z^2 + 2z + 5)(z + 3)$: $\boxed{-1 - 2\text{i}, -3}$

A7 $\left|\dfrac{z}{w}\right| = 2$, $\arg\left(\dfrac{z}{w}\right) = \dfrac{\pi}{2}$: $\boxed{2\text{i}}$

B Fluency solutions

One mark for each correct part. The notes in italics explain why each sequence was chosen.

CORE

B1 **(a)** $\boxed{-7 + 24\text{i}}$ **(b)** $\boxed{25}$ **(c)** $\boxed{6}$ **(d)** $\boxed{-25}$

zz^* and $z + z^*$ are always real.

B2 **(a)** $\dfrac{10(1 - 3\text{i})}{10}$: $\boxed{1 - 3\text{i}}$ **(b)** $\boxed{3 + \text{i}}$ **(c)** $\dfrac{(1 + 3\text{i})^2}{10}$: $\boxed{-\dfrac{4}{5} + \dfrac{3}{5}\text{i}}$ **(d)** $\dfrac{(3 + 4\text{i})^2}{25}$: $\boxed{-\dfrac{7}{25} + \dfrac{24}{25}\text{i}}$

Always multiply by the conjugate of the denominator.

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B3 **(a)** $\boxed{\sqrt{2}, \frac{\pi}{4}}$ **(b)** $\boxed{\sqrt{2}, \frac{3\pi}{4}}$ **(c)** $\boxed{\sqrt{2}, -\frac{3\pi}{4}}$ **(d)** $\boxed{\sqrt{2}, -\frac{\pi}{4}}$

Same modulus; the quadrant decides the argument.

B4 **(a)** $\boxed{\mathbf{5,\ 0.927}}$ **(b)** $\pi - 0.927$: $\boxed{\mathbf{5,\ 2.21}}$ **(c)** $-\pi + 0.927$: $\boxed{\mathbf{5,}\ -2.21}$ **(d)** $\boxed{\mathbf{4,}\ \frac{\pi}{2}}$

Sketch first; $\tan^{-1}$ *alone gives only the first-quadrant angle.*

B5 **(a)** $\boxed{\pm 3\mathrm{i}}$ **(b)** $\frac{6 \pm \sqrt{-4}}{2}$: $\boxed{3 \pm \mathrm{i}}$ **(c)** $\frac{-2 \pm \sqrt{-36}}{4}$: $\boxed{-\frac{1}{2} \pm \frac{3}{2}\mathrm{i}}$

The roots of a real quadratic come as a conjugate pair.

B6 **(a)** $\boxed{4\mathrm{i}}$ **(b)** $\boxed{1 + \sqrt{3}\mathrm{i}}$ **(c)** $\boxed{3\sqrt{3} - 3\mathrm{i}}$ **(d)** $\boxed{-1 + \mathrm{i}}$

$a = r\cos\theta$, $b = r\sin\theta$

STRETCH

B7 **(a)** $4a - 2b\mathrm{i} = 8 + 6\mathrm{i}$: $\boxed{2 - 3\mathrm{i}}$ **(b)** $3a + b\mathrm{i}$: $\boxed{2 + \mathrm{i}}$ **(c)** $(2a - b) + (a - 2b)\mathrm{i} = 1 - 4\mathrm{i}$: $\boxed{2 + 3\mathrm{i}}$ **(d)** $a^2 - b^2 = 3$, $2ab = 4$: $\boxed{\pm(2 + \mathrm{i})}$

Two real equations from one complex equation.

B8 **(a)** $8 - 12 + 8 - 4 = 0$, as required **(b)** $\boxed{z^2 - z + 2}$ **(c)** $\boxed{\frac{1}{2} \pm \frac{\sqrt{7}}{2}\mathrm{i}}$ **(d)** $\boxed{3}$

The sum of the roots is minus the z^2 *coefficient.*

B9 **(a)** $\boxed{z^2 - 6z + 13}$ **(b)** $\boxed{z^2 - 2z + 17}$ **(c)** $\boxed{z^3 - 4z^2 + 6z - 4}$ **(d)** $\boxed{z^4 - 4z^3 + 6z^2 - 4z + 5}$

Each conjugate pair $p \pm q\mathrm{i}$ *gives* $z^2 - 2pz + p^2 + q^2$

B10 **(a)** $\boxed{6}$ **(b)** $\frac{\pi}{6} - \frac{\pi}{3}$: $\boxed{-\frac{\pi}{6}}$ **(c)** $\boxed{\frac{3}{2}}$ **(d)** $\frac{\pi}{6} + \frac{\pi}{3}$: $\boxed{\frac{\pi}{2}}$

Moduli multiply and divide; arguments add and subtract.

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C Reasoning solutions

Your wording can be different. Check that your reason makes the same point.

CORE

C1 $-4+4\text{i}$ is in the second quadrant, so the argument is $\pi - \frac{\pi}{4}$: $\boxed{\frac{3\pi}{4}}$

C2 $(3\text{i})^2 = -9$ and she missed the middle term $2(2)(3\text{i})$: $\boxed{-5+12\text{i}}$

C3 Complex roots of a real polynomial come in conjugate pairs, so -2i is a root: $\boxed{-2\text{i}}$ $(z^3 - 2z^2 + 4z - 8 = (z-2)(z^2+4))$

C4 -2 is real, so it lies on the real axis: $\boxed{(-2,0)\text{, } (0,3) \textbf{ and } (0,-3)}$

STRETCH

C5 Arguments subtract: $\arg\left(\frac{z}{w}\right) = \frac{\pi}{2} - \frac{\pi}{4}$: $\boxed{\frac{\pi}{4}}$ (indeed $\frac{z}{w} = 2+2\text{i}$)

C6 The coefficients are not all real, so roots need not be conjugate. Sum of roots $= 3+\text{i}$: $\boxed{2}$

C7 The form needs $+\text{i}\sin$: $\cos\frac{\pi}{3} - \text{i}\sin\frac{\pi}{3} = \cos\left(-\frac{\pi}{3}\right) + \text{i}\sin\left(-\frac{\pi}{3}\right)$

$\boxed{z = 3\left(\cos\left(-\frac{\pi}{3}\right) + \text{i}\sin\left(-\frac{\pi}{3}\right)\right)}$, argument $-\frac{\pi}{3}$

C8 **(a)** $(a+b\text{i})(a-b\text{i}) = a^2 - b^2\text{i}^2 = a^2 + b^2 = |z|^2$, as required

(b) $\frac{1}{z} = \frac{z^*}{zz^*} = \frac{z^*}{|z|^2}$; $\left|\frac{1}{z}\right| = \frac{|z^*|}{|z|^2} = \frac{|z|}{|z|^2} = \frac{1}{|z|}$, as required

D Problem solving solutions

There is often more than one correct method. Any complete, correct method scores full marks.

CORE

D1 **(a)** $3+4\text{i}+2-\text{i}$: $\boxed{5+3\text{i}}$

(b) $\dfrac{50}{2-\text{i}} = \dfrac{50(2+\text{i})}{5} = 10(2+\text{i})$: $\boxed{k=10}$

D2 **(a)** 4×3: $\boxed{\mathbf{12}}$

(b) $\arg z_1 + \arg z_2 = 0$, so $\arg z_2 = -\dfrac{2\pi}{3}$; $z_2 = 3\left(\cos\left(-\dfrac{2\pi}{3}\right)+\text{i}\sin\left(-\dfrac{2\pi}{3}\right)\right)$: $\boxed{-\dfrac{3}{2}-\dfrac{3\sqrt{3}}{2}\text{i}}$

D3 **(a)** $\lambda-3\text{i}+\lambda\text{i}+3$: $\boxed{(\lambda+3)+(\lambda-3)\text{i}}$

(b) $\lambda+3=0$: $\boxed{\lambda=-3,\ z=-6\text{i}}$

D4 $1-2\text{i}$ is also a root: $(z-1-2\text{i})(z-1+2\text{i}) = z^2-2z+5$; $z^3+z^2-z+15 = (z^2-2z+5)(z+3)$: $\boxed{1-2\text{i}\ \textbf{and}\ -3}$

STRETCH

D5 $3+2\text{i}$ is a root, so $z^2-6z+13$ is a factor; constant term: $13c=26$, $c=2$

$(z^2-6z+13)(z+2) = z^3-4z^2+z+26$: $\boxed{p=-4,\ q=1,\ \textbf{real root}\ -2}$

D6 **(a)** $z = \dfrac{-1+8\text{i}}{1+2\text{i}} = \dfrac{(-1+8\text{i})(1-2\text{i})}{5} = \dfrac{15+10\text{i}}{5}$: $\boxed{3+2\text{i}}$

(b) Second quadrant: $\pi-\tan^{-1}8$: $\boxed{\mathbf{1.70}}$

D7 **(a)** $z^3-5z^2+7z+13 = (z+1)(z^2-6z+13)$: $\boxed{3\pm 2\text{i}}$

(b) Base from $3+2\text{i}$ to $3-2\text{i}$ is 4; height from $x=-1$ to $x=3$ is 4; $\dfrac{1}{2}(4)(4)$: $\boxed{\mathbf{8}}$

D8 **(a)** $z = \dfrac{4\pm\sqrt{-16}}{2}$: $\boxed{2\pm 2\text{i}}$

(b) $R(-2,0)$, $P(2,2)$, $S(6,0)$, $Q(2,-2)$; each side is $\sqrt{4^2+2^2} = 2\sqrt{5}$: $\boxed{8\sqrt{5}}$

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E Exam practice solutions

Mark-scheme notes are shown on the right of each line.

CORE *Exam questions for everyone*

E1 **(a)** $\boxed{1-2\text{i}}$ *B1*

(b) $(z-1-2\text{i})(z-1+2\text{i}) = z^2-2z+5$ *M1*

$\text{f}(z) = (2z-1)(z^2-2z+5) = 2z^3-5z^2+12z-5$ *M1*

$\boxed{A=-5,\ B=12,\ C=-5}$ *A1*

(c) $1\pm2\text{i}$ plotted as a conjugate pair, symmetric in the real axis *B1*

$\frac{1}{2}$ plotted on the real axis *B1*

E2 **(a)** $|z_1||z_2| = \sqrt{5}\times\sqrt{10}$ *M1*

$\boxed{5\sqrt{2}}$ *A1*

(b) $\dfrac{(2+\text{i})(1+3\text{i})}{(1-3\text{i})(1+3\text{i})} = \dfrac{-1+7\text{i}}{10}$ *M1*

$\boxed{-\frac{1}{10}+\frac{7}{10}\text{i}}$ *A1*

(c) Second quadrant: $\pi-\tan^{-1}7$ *M1*

$\boxed{\textbf{1.71}}$ *A1*

(d) $\boxed{\textbf{the second quadrant}}$ *B1*

STRETCH

E3 **(a)** $\boxed{2-3\text{i}}$ *B1*

(b) $(z-2-3\text{i})(z-2+3\text{i})$ *M1*

$\boxed{z^2-4z+13}$ *A1*

(c) $\text{f}(z) = (z^2-4z+13)(z^2+bz+2)$, as $13\times2=26$ *M1*

z^2 term: $2-4b+13=7$, so $b=2$ *M1*

$(z^2-4z+13)(z^2+2z+2) = z^4-2z^3+7z^2+18z+26$: $\boxed{P=-2,\ Q=18}$ *A1*

(d) $z^2+2z+2=0,\ z=\dfrac{-2\pm\sqrt{-4}}{2}$ *M1*

$\boxed{-1\pm\text{i}}$ *A1*

(e) Two conjugate pairs plotted symmetrically about the real axis *B1*

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