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A-LEVEL MATHEMATICS • FURTHER PURE 1

# Numerical Solutions of Equations

Worked Solutions

Pearson Edexcel International A Level (WFM01) • Calculator allowed: show every step

Full worked solutions for the free Numerical Solutions of Equations worksheet, A-Level and International A-Level Maths


A **Examples** & Your turn page 2

B **Fluency** core skills page 2

C **Reasoning** explain it page 4

D **Problem** solving page 5

E **Exam** practice page 6


Tap a section to jump straight to it

**Full worked solutions** for every question on the Further Pure 1 Numerical Solutions of Equations worksheet, in the same order and with the same question numbers.

**How to mark your work.** Tick each step you got right, not just the final answer. At A-level most marks are for method, so a wrong final answer can still earn most of the marks. If you lost marks, find the *first* line where your working is different from ours. That is the thing to fix.

In Section E: **M** = method mark, **dM** = method mark that depends on the previous M mark, **A** = accuracy mark (needs the M mark before it), **B** = independent mark.

Question sheet | Exam questions by topic | Further Pure 1 exam questions | More worksheets


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# A Your turn answers

Compare each line of your working with the solution.

## CORE

**A1** $f(1) = -1 < 0$, $f(2) = 8 > 0$: **sign change and f continuous, so a root lies in $[1, 2]$**

**A2** $f(2.5) = 5.625 > 0$: $[2, 2.5]$; $f(2.25) = 1.891 > 0$: $[2, 2.25]$

**A3** $f(1) = -4$, $f(2) = 5$; $x = 1 + \frac{4}{9}$: **1.444**

## STRETCH

**A4** $f(0.5) = 0.625$, $f'(x) = 3x^2 + 3$, $f'(0.5) = 3.75$; $0.5 - \frac{0.625}{3.75}$: **0.333**

**A5** $f'(x) = -\frac{3}{x^2} - 1$; $f(2.5) = 0.7$, $f'(2.5) = -1.48$; $2.5 + \frac{0.7}{1.48}$: **2.973**

**A6** $f(2.6455) = -0.0013 < 0$, $f(2.6465) = 0.0040 > 0$

**sign change and f continuous, so a root lies in $[2.6455, 2.6465]$, so the root is 2.646 to 3 d.p.**

**A7** $f'(x) = 2x - 4$, $f'(2) = 0$: **the formula would divide by zero**

# B Fluency solutions

One mark for each correct part. The notes in italics explain why each sequence was chosen.

## CORE

**B1** **(a)** 1, $-3$: yes **(b)** $-3$, $-1$: **no sign change** **(c)** $-1$, 13: yes **(d)** $-11$, 3: yes

*No sign change does not prove there is no root, only that there is no single crossing.*

**B2** **(a)** $f(2.5) = 4.125$: $[2, 2.5]$ **(b)** $f(2.25) = 1.141$: $[2, 2.25]$ **(c)** $f(2.125) = -0.029$: $[2.125, 2.25]$

**(d)** $\frac{1}{2^n} < 0.01$: 7

*Each bisection halves the width, whatever the function.*

**B3** **(a)** $\boxed{3.1429}$ **(b)** $\boxed{3.1619}$ **(c)** $\boxed{3.1623}$ **(d)** $\boxed{\text{the narrower the interval, the closer to } \sqrt{10} = 3.1623}$

*On a short interval the curve is close to its chord.*

**B4** **(a)** $\boxed{4x^3 - 3}$ **(b)** $\boxed{x^{-\frac{1}{2}}}$ **(c)** $\boxed{-8x^{-3}}$ **(d)** $\boxed{\frac{3}{2}x^{\frac{1}{2}} + 5x^{-2}}$

*Write every term as a power of $x$ first.*

**B5** **(a)** $2 - \frac{-1}{4}$: $\boxed{2.25}$ **(b)** $2.25 - \frac{0.0625}{4.5}$: $\boxed{2.2361}$ **(c)** $\boxed{\text{equal to } \sqrt{5} = 2.2361 \text{ to 4 d.p.}}$

*Newton–Raphson converges very quickly near a root.*

**B6** **(a)** $\boxed{-0.072}$ **(b)** $\boxed{0.203}$ **(c)** $f(1.2125) = -0.0049 < 0$, $f(1.2135) = 0.0005 > 0$: $\boxed{\text{sign change and } f \text{ continuous, so a root lies in } [1.2125, 1.2135]}$

**(d)** $\boxed{f(1.213) \text{ only shows which side of the root 1.213 lies, not that the root rounds to 1.213}}$

*Test the two values that round to the answer's boundaries.*

## STRETCH

**B7** **(a)** $\boxed{2x - x^{-2}}$ **(b)** $2 - \frac{0.5}{3.75}$: $\boxed{1.8667}$ **(c)** $\boxed{1.8608}$ **(d)** $f(1.855) = -0.0199$, $f(1.865) = 0.0144$: $\boxed{\text{root 1.86 to 2 d.p.}}$

*Negative powers differentiate like any other power.*

**B8** **(a)** $\boxed{f'(1) = 0, \text{ so division by zero}}$ **(b)** $\boxed{\text{an asymptote at } \frac{\pi}{2}, \text{ not a root}}$

**(c)** $\boxed{\text{the chord is far from the curve; the root is } \sqrt{2} = 1.414}$

**(d)** $\boxed{f'(0.9) \text{ is near zero, so the step overshoots}}$

*Every method can fail; the reason is always in the function.*

**B9** **(a)** $f(0) = 2$, $f(1) = -2$: $\boxed{\text{sign change}}$ **(b)** $f(2) = 8$: $\boxed{\text{sign change on } [1, 2]}$ **(c)** $0 + \frac{2}{4}$: $\boxed{0.5}$ **(d)** $0.5 - \frac{-0.4375}{-4.5}$: $\boxed{0.403}$

*Interpolation gives a starting value; Newton–Raphson improves it.*

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**B10** **(a)** $-8, 13$ **(b)** $2 + \frac{8}{21}$: **2.381** **(c)** f(2.5) = 0.625: **[2, 2.5], midpoint 2.25**

**(d)** **interpolation (error 0.089, not 0.22)**

*Interpolation uses the sizes of* f*, bisection only the signs.*

## C Reasoning solutions

Your wording can be different. Check that your reason makes the same point.

### CORE

**C1** He has not given a reason or a conclusion.

$f(1) < 0 < f(2)$**, a sign change, and** f **is continuous, so a root lies in** $[1, 2]$

**C2** The formula subtracts: $x_1 = 2 - \frac{f(2)}{f'(2)} = 2 - \frac{1}{4}$: **1.75**

**C3** $\frac{4}{\sqrt{x}} = 4x^{-\frac{1}{2}}$ differentiates to $-2x^{-\frac{3}{2}}$, so $f'(x) = 1 + 2x^{-\frac{3}{2}}$

$2.5 - \frac{-0.0298}{1.506}$: **2.520**

**C4** One value cannot show a sign change. $f(1.725) = -0.024 < 0$, $f(1.735) = 0.010 > 0$: **sign change and** f **continuous, so a root lies in** $[1.725, 1.735]$**, so 1.73 to 2 d.p.**

### STRETCH

**C5** f is not continuous on the interval: it has an asymptote at $x = 3$, where the sign changes. $1 + \frac{1}{x-3} = 0$: $x = 2$

**C6** $f'(x) = 3x^2 - 12$, so $f'(2) = 0$ and the formula divides by zero; start inside $[3, 4]$: **e.g.** $x_0 = 3.5$

**C7** $f(1) = -1$, $f(3) = 7$; $1 + \frac{2}{8}$: **1.25**; the curve is not straight, so the chord meets the axis away from the root $\sqrt{2} = 1.414$

**C8** **(a)** Tangent: $y - f(x_0) = f'(x_0)(x - x_0)$; at $y = 0$: $x - x_0 = -\frac{f(x_0)}{f'(x_0)}$, so $x = x_0 - \frac{f(x_0)}{f'(x_0)}$, as required

**(b)** If $f'(x_0) = 0$ the tangent is horizontal and never meets the $x$-axis, so there is no $x_1$

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# D Problem solving solutions

There is often more than one correct method. Any complete, correct method scores full marks.

## CORE

**D1** $f(0) = -2$, $f(1) = 3$; $f(0.5) = 0.125 > 0$: $[0, 0.5]$; $f(0.25) = -0.984 < 0$: $\boxed{[0.25, 0.5]}$

**D2** **(a)** $f(2) = -1.24$, $f(3) = 2.80$: **sign change and f continuous, so a root lies in** $[2, 3]$

**(b)** $\boxed{2x - \frac{3}{2}x^{-\frac{1}{2}}}$

**(c)** $f(2.5) = 0.5066$, $f'(2.5) = 4.0513$; $2.5 - \frac{0.5066}{4.0513}$: $\boxed{\mathbf{2.375}}$

**D3** **(a)** $g(0.5) = -0.541$, $g(1) = 0.683$: **sign change, g continuous, so a root lies in** $[0.5, 1]$

**(b)** $0.5 + 0.5 \times \frac{0.541}{1.224}$: $\boxed{\mathbf{0.721}}$

**D4** **(a)** $f(0) = -1$, $f(0.5) = 2.125$: **sign change and f continuous, so a root lies in** $[0, 0.5]$

**(b)** $f(0.2) = 0.568$, $f'(0.2) = 6.72$; $0.2 - \frac{0.568}{6.72}$: $\boxed{\mathbf{0.115}}$

## STRETCH

**D5** **(a)** $\boxed{2x + 5x^{-2}}$

**(b)** $f(2.5) = 1.25$, $f'(2.5) = 5.8$; $2.5 - \frac{1.25}{5.8}$: $\boxed{\mathbf{2.284}}$

**D6** **(a)** $f(1) = -3$, $f(2) = 5$; $f(1.5) = -0.125$: $[1.5, 2]$; $f(1.75) = 2.109$: $\boxed{[1.5, 1.75]}$, as required

**(b)** $1.5 + 0.25 \times \frac{0.125}{2.234}$: $\boxed{\mathbf{1.514}}$

**D7** **(a)** $f = -14, 1, 4, 1, -2, 1$; sign changes: $\boxed{[-3, -2]}$, $[0, 1]$ **and** $[1, 2]$

**(b)** $-3 + \frac{14}{15}$: $\boxed{-2.07}$

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**D8** **(a)** $f(2) = -0.6569$, $f(2.5) = 6.3004$; $2 + 0.5 \times \dfrac{0.6569}{6.9573}$: **2.047**

**(b)** $3x^2 - 2x^{-\frac{1}{2}}$

**(c)** $f'(2) = 10.586$; $2 - \dfrac{-0.6569}{10.586}$: **2.062**

## E Exam practice solutions

Mark-scheme notes are shown on the right of each line.

### CORE *Exam questions for everyone*

**E1** **(a)** $g(3) = -2.809$, $g(4) = 1$ *M1*

**sign change, g continuous, so $\beta \in [3,4]$** *A1*

**(b)** $g(x) = \dfrac{1}{2}x^2 - 4x^{-\frac{1}{2}} - 5$ *M1*

$g'(x) = x + 2x^{-\frac{3}{2}}$ *A1*

**(c)** $g(3.5) = -1.0131$, $g'(3.5) = 3.8054$; $3.5 - \dfrac{-1.0131}{3.8054}$ *M1*

**3.766** *A1*

**E2** **(a)** $f(0) = 3$, $f(1) = -3$ *M1*

**sign change, f continuous, so $\alpha \in [0,1]$** *A1*

**(b)** $f(0.5) = -0.375 < 0$: $\alpha \in [0, 0.5]$ *M1 A1*

$f(0.25) = 1.266 > 0$: $\alpha \in [0.25, 0.5]$ *A1*

**(c)** $\dfrac{x - 0.25}{1.266} = \dfrac{0.5 - x}{0.375}$ *M1*

**0.443** *A1*

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**STRETCH**

**E3** **(a)** $f(x) = 2x - 6x^{\frac{1}{2}} + 3$ *M1*

$f'(x) = 2 - 3x^{-\frac{1}{2}}$ *A1*

**(b)** $f'(2.25) = 2 - \dfrac{3}{1.5} = 0$: **the formula would divide by zero** *B1*

**(c)** $f(5) = -0.416$, $f(6) = 0.303$ *M1*

**sign change, f continuous, so $\alpha \in [5,6]$** *A1*

**(d)** $f(5.5) = -0.0712$, $f'(5.5) = 0.7208$; $5.5 - \dfrac{-0.0712}{0.7208}$ *M1*

**5.599** *A1*

**(e)** $f(5.595) = -0.0023 < 0$, $f(5.605) = 0.0051 > 0$ *M1*

**sign change, so $\alpha \in [5.595, 5.605]$ and $\alpha = 5.60$ to 2 d.p.** *A1*

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