eClassroom

A-LEVEL MATHEMATICS • FURTHER PURE 2

Inequalities

Worksheet

Pearson Edexcel International A Level (WFM02) • Calculator allowed: show every step

Free Inequalities worksheet for A-Level and International A-Level Maths, with full worked solutions

A **Examples** & Your turn page 2

B **Fluency** core skills page 4

C **Reasoning** explain it page 6

D **Problem** solving page 7

E **Exam** practice page 10

Tap a section to jump straight to it

By the end of this worksheet you can: solve inequalities with rational expressions • find every critical value • solve modulus inequalities • use graphs • *Stretch:* variables on both sides, the $|x|$ versions, and inequalities with a parameter.

Which questions are for me?

CORE The methods every A-level student must secure. Start here.

STRETCH Rational inequalities with both sides, modulus cases, and solution sets from graphs.

Every section has both, clearly labelled. Questions get harder as you go, and every skill in Sections A to D is needed in Section E.

Track your progress

Section	B	C	D	E
Core marks	23	10	13	13
Stretch marks	16	14	17	9
My score				

Full worked solutions | Exam questions by topic | Further Pure 2 exam questions | Formula sheet builder

Name ..... Class ..... Date .....

A Worked examples & Your turn

Read each example carefully. Cover it up, then answer the question beside it using the same method.

CORE *Secure the method*

Worked example A1: one fraction

Solve $\dfrac{x+3}{x-1} > 2$

Take 2 across, never multiply by $x-1$

$\dfrac{x+3-2(x-1)}{x-1} = \dfrac{5-x}{x-1} > 0$; critical values 1, 5

$1 < x < 5$

Your turn A1

Solve $\dfrac{2x-1}{x+2} \leqslant 1$

Worked example A2: multiplying by a square

Solve $\dfrac{3}{x-2} < x$

Multiply by $(x-2)^2 > 0$

$(x-2)\big(x(x-2)-3\big) > 0$, $(x-2)(x-3)(x+1) > 0$

$-1 < x < 2$ or $x > 3$

Your turn A2

Solve $\dfrac{4}{x+1} \geqslant x-2$

Worked example A3: fractions on both sides

Solve $\dfrac{x}{x+2} \geqslant \dfrac{1}{x-1}$

$\dfrac{x^2-2x-2}{(x+2)(x-1)} \geqslant 0$; critical values -2, 1, $1 \pm \sqrt{3}$

$x < -2,\ 1-\sqrt{3} \leqslant x < 1,\ x \geqslant 1+\sqrt{3}$

Your turn A3

Solve $\dfrac{2x}{x-4} < \dfrac{3}{x+1}$

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STRETCH

Worked example A4: a linear modulus

Solve $|2x-1| < x+4$

$-(x+4) < 2x-1 < x+4$

$x > -1$ and $x < 5$

$-1 < x < 5$

Your turn A4

Solve $|x+2| \geqslant 3x-2$

Worked example A5: a quadratic modulus

Solve $|x^2-5| < 4$

$-4 < x^2-5 < 4$, so $1 < x^2 < 9$

$-3 < x < -1$ or $1 < x < 3$

Your turn A5

Solve $|x^2-2x| > 3$

Worked example A6: a modulus in the denominator

Solve $\dfrac{2}{|x-1|} > 1$

$|x-1| < 2$ with $x \neq 1$

$-1 < x < 3,\ x \neq 1$

Your turn A6

Solve $\dfrac{6}{|x+2|} \geqslant 3$

Worked example A7: reading a graph

The curve $y = \dfrac{3}{x}$ meets $y = x+2$ at $x = 1$ and $x = -3$.

Solve $\dfrac{3}{x} > x+2$

Where is the curve above the line? Remember the asymptote $x = 0$

$x < -3$ or $0 < x < 1$

Your turn A7

Solve $\dfrac{4}{x-1} \leqslant x+2$, using the points where $y = \dfrac{4}{x-1}$ meets $y = x+2$

B Fluency

Each question is a designed sequence. From one part to the next, ask yourself: what changed, and what did it do to the answer?

CORE *Secure the method*

B1 Solve

(a) $\frac{x+4}{x-3} > 0$

(b) $\frac{(x-1)(x+2)}{x-5} \leqslant 0$

(c) $\frac{x^2}{x+3} \geqslant 0$

(d) $\frac{x^2+1}{x-2} < 0$

[4 marks]

B2 Solve

(a) $\frac{x+5}{x+1} < 3$

(b) $\frac{x+5}{x+1} \leqslant 3$

(c) $\frac{x+5}{x+1} \geqslant 3$

(d) $\frac{x+5}{x+1} = 3$

[4 marks]

B3 Solve

(a) $\frac{4}{x+3} > x$

(b) $\frac{4}{x+3} \leqslant x$

(c) $\frac{x}{x-1} > \frac{2}{x}$

(d) $\frac{5}{x-2} < x+2$

[4 marks]

B4 Solve

(a) $|x-4| < 3$

(b) $|x-4| < 2x$

(c) $|x-4| \geqslant 2x$

(d) $|x-4| < |x+2|$

[4 marks]

B5 For $|x^2-9|$, find the exact values of x for which

(a) $|x^2-9| = 7$

(b) $|x^2-9| < 7$

(c) $|x^2-9| > 7$

[3 marks]

B6 Write each solution set in set notation.

(a) $x^2 - x - 6 < 0$

(b) $\frac{x+1}{x-4} \geqslant 0$

(c) $\frac{1}{x} < 1$

(d) $|x| < \frac{4}{x}$

[4 marks]

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STRETCH

B7 Solve

(a) $\frac{1}{x-1} > \frac{1}{x+2}$

(b) $\frac{x}{x-1} \geqslant \frac{x}{x+2}$

(c) $\frac{3}{x+1} \leqslant \frac{2}{x-1}$

(d) $\frac{x+1}{x-2} > \frac{x-3}{x+1}$

[4 marks]

B8 Solve

(a) $|x+1| < \frac{6}{x}$

(b) $|x-3| > \frac{2}{x}$

(c) $|2x-1| \leqslant x+1$

(d) $\frac{|x|}{x+2} > \frac{1}{2}$

[4 marks]

B9 The curve $y = \frac{6}{x+1}$ and the line $y = x$

(a) Find where they meet.

(b) Solve $\frac{6}{x+1} > x$

(c) Solve $\frac{6}{|x|+1} > |x|$

(d) Solve $\frac{6}{x+1} \leqslant x$

[4 marks]

B10 $\frac{x}{x-3} > 2$

(a) Solve it correctly.

(b) Show that multiplying both sides by $x-3$ gives a wrong answer.

(c) Solve $\frac{x}{x-3} \geqslant 2$

(d) Solve $\frac{x}{x-3} \leqslant 2$

[4 marks]

C Reasoning

Show your working and explain your thinking. A correct answer without a reason does not score full marks.

CORE *Secure the method*

C1 To solve $\dfrac{x+1}{x-2} < 2$, Amir writes $x+1 < 2(x-2)$, so $x > 5$. Explain his error, and solve it correctly.

[2 marks]

C2 Bea writes the solution of $\dfrac{x-1}{x+3} \leqslant 0$ as $-3 \leqslant x \leqslant 1$. Explain her error, and correct it. **[2 marks]**

C3 To solve $|x-2| < 2x-1$, Cal uses only $x-2 < 2x-1$ and writes $x > -1$

Explain his error, and solve the inequality. **[3 marks]**

C4 To solve $\dfrac{2x}{x+1} \leqslant x$, Dev divides both sides by x and obtains $x < -1$ or $x \geqslant 1$

Explain his error, and solve the inequality. **[3 marks]**

STRETCH

C5 To solve $|x+1| > x-3$, Eve squares both sides and obtains $x > 1$

Explain why squaring is not valid here, and solve the inequality. **[3 marks]**

C6 Fay gives the solution of $(x+1)(x-2) > 0$ as $\{x : x < -1\} \cap \{x : x > 2\}$

Explain her error, and write the solution correctly. **[3 marks]**

C7 Gus says that $\dfrac{1}{x} > \dfrac{1}{2}$ means $x < 2$

Explain his error, and solve the inequality. **[3 marks]**

C8 $\mathrm{f}(x) = \dfrac{x^2+x+1}{x^2+1}$, $x \in \mathbb{R}$

(a) Show that $\mathrm{f}(x) \leqslant \dfrac{3}{2}$ for all real x, stating when equality holds.

(3)

(b) Show that $\mathrm{f}(x) \geqslant \dfrac{1}{2}$ for all real x, and hence state the range of f

(2)

[5 marks]

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D Problem solving

You decide the method. Read the whole question first, and show every step.

CORE *Secure the method*

D1 *In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.*

Use algebra to determine the set of values of x for which

$$\frac{x^2}{x+1} < 2$$

giving your answer in exact form. **[3 marks]**

D2 *In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.*

Use algebra to determine the values of x for which

$$\frac{x+2}{x-1} \leqslant \frac{3}{x+1}$$

[4 marks]

D3 **(a)** Use algebra to determine the x coordinates of the points of intersection of the curves with equations $y = \frac{x}{x+2}$ and $y = \frac{3}{x+4}$

(1)

(b) Hence determine the set of values of x for which $\frac{x}{x+2} < \frac{3}{x+4}$

(2)

[3 marks]

D4 **(a)** Use algebra to determine the set of values of x for which $x+1 > \frac{6}{x}$

(2)

(b) Hence, or otherwise, determine the set of values of x for which $|x|+1 > \frac{6}{|x|}$

(1)

[3 marks]

STRETCH

D5 *In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.*

Given that

$$\frac{1}{x-2} \leqslant kx$$

where k is a positive constant,

(a) show that $(x-2)(px^2+qx+r) \geqslant 0$, where p, q and r are expressions in terms of k to be determined.

(2)

(b) Hence determine the exact values of x for which $\frac{1}{x-2} \leqslant 3x$

(2)

[4 marks]

D6 Figure 1 shows a sketch of the curve with equation $y = |x^2 - 9|$ and the line with equation $y = x + 3$

(a) Use algebra to determine the x coordinates of the points where the line meets the curve.

(2)

(b) Hence determine the set of values of x for which $|x^2 - 9| < x + 3$

(2)

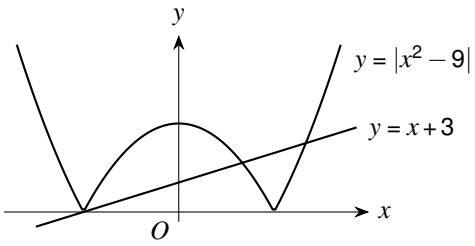

Figure 1

[4 marks]

D7 *In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.*

Figure 2 shows a sketch of the curve with equation $y = \frac{8}{x+2}$ and the line with equation $y = x$

(a) Use algebra to determine the values of x for which $\frac{8}{x+2} < x$

(2)

(b) Hence determine the values of x for which $\frac{8}{|x|+2} < |x|$

(2)

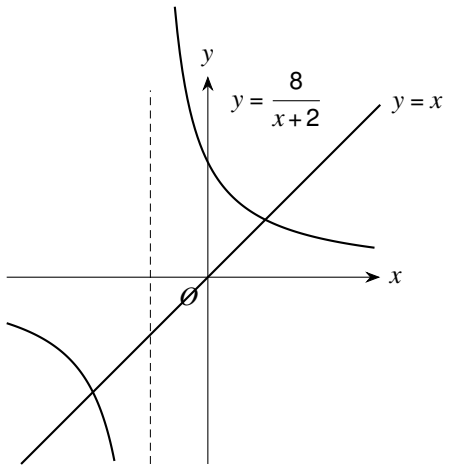

Figure 2

[4 marks]

D8 $S_n = \sum_{r=1}^{n} \frac{1}{r(r+1)(r+2)} = \frac{n(n+3)}{4(n+1)(n+2)}$

(a) Show that $\frac{1}{4} - S_n = \frac{1}{2(n+1)(n+2)}$

(2)

(b) Hence form and solve a quadratic inequality to determine the smallest value of n for which $S_n > 0.249$

(3)

[5 marks]

E Exam practice

Exam-style questions in the style of WFM02. Write your answers in the spaces, and show every step.

CORE *Exam questions for everyone*

E1 *In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.*

Use algebra to determine the set of values of x for which

$$\frac{x^2-1}{x+4} \geqslant 2-x$$

giving your answer in exact form, in set notation, and making clear which critical values are included.

(6)

(Total for Question E1 is 6 marks)

E2 **(a)** Use algebra to determine the exact x coordinates of the points of intersection of the curves with equations

$$y = \frac{2x}{x+1} \quad \text{and} \quad y = \frac{1}{x-2}$$

(3)

Hence

(b) determine the values of x for which $\frac{2x}{x+1} < \frac{1}{x-2}$

(2)

(c) state the values of x for which $\frac{2|x|}{|x|+1} < \frac{1}{|x|-2}$

(2)

(Total for Question E2 is 7 marks)

STRETCH

E3 *In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.*

(a) Use algebra to determine the values of x for which

$$x - 3 > \frac{4x}{x+2}$$

(5)

(b) Hence use algebra to determine the exact values of x for which

$$|x - 3| > \frac{4x}{x+2}$$

(4)

(Total for Question E3 is 9 marks)

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