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INTERNATIONAL A LEVEL MATHEMATICS • MECHANICS 2

Mechanics 2

Exam Questions by Topic: Mark Scheme

Pearson Edexcel International Advanced Level (WME02/01) • Calculator allowed • Take $g = 9.8\,\text{m}\,\text{s}^{-2}$

Every question in the booklet, marked line by line the way an examiner marks it

Chapters (tap a chapter to jump to it)		Qs	Marks	Page	My score
1	**Projectiles** Horizontal projection, projection at any angle, the equation of the path	10	84	3	
2	**Variable Acceleration** Calculus for motion in a line and in $\mathbf{i}, \mathbf{j}$ form	10	85	11	
3	**Centres of Mass** Particles, laminas, frameworks, suspended and tilting bodies	10	81	18	
4	**Work and Energy** Work done, kinetic and potential energy, the work–energy principle, power	10	82	25	
5	**Impulses and Collisions** Impulse as a vector, Newton's law of restitution, loss of kinetic energy	10	82	32	
6	**Statics of Rigid Bodies** Rods and ladders in equilibrium, friction, hinge forces	10	81	39	

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How to use this mark scheme

The same marking rules Pearson examiners use for WME02.

Mark every line, not just the answer. Method marks are given for a correct method even when the final answer is wrong. If you lost marks, find the *first* line where your working differs from the scheme. That is the thing to fix.

Code	Meaning
M1	Method mark: for knowing a correct method and attempting to apply it. Small slips in the working do not lose this mark.
dM1	Dependent method mark: can only be given if the method mark before it has been given.
A1	Accuracy mark: for a correct answer or line of working. It can only be given if the method mark it depends on has been given.
B1	Independent mark: for a correct result or statement, whatever method was used.
*	The answer is given in the question. Every step must be shown, and the answer must follow from correct working (cso).
cao	Correct answer only.
cso	Correct solution only: no errors anywhere in the working.
ft	Follow through: the mark can be given for correct work using your own earlier (wrong) answer.
oe	Or equivalent: any correct equivalent form is accepted.

Accuracy after using $g = 9.8$. When $g = 9.8\,\text{m}\,\text{s}^{-2}$ has been substituted, the final answer must be given to 2 or 3 significant figures: more accurate answers (and fractions) lose the final A mark. Exact multiples of g are usually accepted. Using $g = 9.81$ loses accuracy marks.

Marking a sketch. Graph marks are given for the correct shape (straight lines for constant acceleration, no solid vertical lines) and for the key values marked on the axes. An equation of motion or a suvat equation must have the correct number of terms and be dimensionally correct to earn its method mark.

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1 Chapter 1: Projectiles

Mark scheme. Marks are shown on the right of each line.

Question 1

7 marks

Part	Scheme	Marks
(a)	R(↓): $44.1 = \frac{1}{2} \times 9.8t^2$	M1
	$\boxed{3\,\text{s}}$	A1
		(2)
(b)	$14 \times 3 = \boxed{42\,\text{m}}$	B1
		(1)
(c)	Vertical component at Q: $9.8 \times 3 = 29.4\,\text{m s}^{-1}$ (or $v^2 = 2 \times 9.8 \times 44.1$)	M1
	Speed $\sqrt{14^2 + 29.4^2} = \boxed{32.6\,\text{m s}^{-1}}$	A1
	$\tan\theta = \dfrac{29.4}{14}$	M1
	$\boxed{64.5^\circ \text{ below the horizontal}}$	A1
	The angle needs a reference: "below the horizontal" (or 25.5° to the downward vertical).	
		(4)
	Total for Question 1: 7 marks	

Question 2

6 marks

Part	Scheme	Marks
(a)	Vertical component $35 \times \frac{3}{5} = 21$	B1
	$0 = 21^2 - 2 \times 9.8h$	M1
	$\boxed{22.5\,\text{m}}$	A1
		(3)
(b)	$0 = 21t - 4.9t^2$, $t = \frac{21}{4.9} = 4.29\,\text{s}$	M1
	$OA = 28 \times \frac{21}{4.9}$ (horizontal component $35 \times \frac{4}{5} = 28$)	M1
	$\boxed{120\,\text{m}}$	A1
		(3)
	Total for Question 2: 6 marks	

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Question 3

6 marks

Part	Scheme	Marks
(a)	At 45° below the horizontal the vertical component is -9.8 (equal in size to the horizontal component)	B1
	$-9.8 = 14.7 - 9.8t$	M1
	$\boxed{t = 2.5\,\text{s}}$	A1
		(3)
(b)	$x = 9.8 \times 2.5 = 24.5$	B1
	$y = 14.7 \times 2.5 - 4.9 \times 2.5^2$	M1
	$\boxed{(24.5\mathbf{i} + 6.125\mathbf{j})\,\text{m}}$ (6.13)	A1
		(3)
		Total for Question 3: 6 marks

Question 4

6 marks

Part	Scheme	Marks
(a)	Least speed at the top, where only the horizontal component remains: $30\cos 60°$	M1
	$\boxed{15\,\text{m s}^{-1}}$	A1
	Jan 2021 report: the incorrect answer zero was the most common error.	
		(2)
(b)	$v_x = 15$ throughout, so speed $\leqslant 25$ when $v_y^2 \leqslant 25^2 - 15^2 = 400$	M1
	$\lvert v_y \rvert \leqslant 20$	A1
	v_y changes from 20 to -20 at $9.8\,\text{m s}^{-2}$: time $= \dfrac{40}{9.8}$	M1
	$\boxed{4.08\,\text{s}}$	A1
	Jan 2021 report: giving the time for which the speed is greater *than the value is the common misreading.*	
		(4)
		Total for Question 4: 6 marks

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Question 5

7 marks

Part	Scheme	Marks
(a)	R($\rightarrow$): $x = (U\cos\alpha)t$	**B1**
	R($\uparrow$): $y = (U\sin\alpha)t - \frac{1}{2}gt^2$	**B1**
	Eliminates t: $y = x\tan\alpha - \dfrac{gx^2}{2U^2\cos^2\alpha}$	**M1**
	$\dfrac{1}{\cos^2\alpha} = \sec^2\alpha = 1+\tan^2\alpha$, giving the result *	**A1***
	Jan 2023 report: the step $\frac{1}{\cos^2\alpha} = 1+\tan^2\alpha$ must be explained, not just used.	**(4)**
(b)	$10 = 40\tan\alpha - \dfrac{9.8\times 1600}{2\times 784}\left(1+\tan^2\alpha\right) = 40\tan\alpha - 10\left(1+\tan^2\alpha\right)$	**M1**
	$\tan^2\alpha - 4\tan\alpha + 2 = 0$, $\tan\alpha = 2\pm\sqrt{2}$	**A1**
	$\alpha = 73.7^\circ$ or 30.4°	**A1**
		(3)
		Total for Question 5: 7 marks

Question 6

11 marks

Part	Scheme	Marks
(a)	$0 = 7^2 - 2 \times 9.8h$, $h = 2.5$ $\boxed{32.5\,\text{m}}$	**M1** **A1** **(2)**
(b)	R(↑): $-30 = 7t - 4.9t^2$ $4.9t^2 - 7t - 30 = 0$ $\boxed{3.29\,\text{s}}$ (3.3) *Jan 2025 report: one equation for the whole flight is expected; splitting must include both parts.*	**M1** **A1** **A1** **(3)**
(c)	Vertical component at B: $v_y^2 = 7^2 + 2 \times 9.8 \times 30 = 637$; horizontal component $14\cos 30°$, $(14\cos 30°)^2 = 147$ Speed $= \sqrt{147 + 637} = \sqrt{784} = 28\,\text{m s}^{-1}$ *	**M1** **A1*** **(2)**
(d)	At C: $v_x = 12.12$ and $v_y = 7 - 9.8t$; perpendicular to the initial direction (gradient $\tan 30°$) means $\dfrac{v_y}{v_x} = -\dfrac{1}{\tan 30°}$ $7 - 9.8t = -21$, so $t = \frac{20}{7} = 2.86\,\text{s}$ (less than 3.29, so the ball is still in the air) Height above A: $7 \times \frac{20}{7} - 4.9\left(\frac{20}{7}\right)^2 = -20$ $\boxed{10\,\text{m}}$ above the ground *Summer 2023 report: the horizontal component is unchanged, and the displacement must be converted to a height above the ground. A scalar product of the two velocity vectors set to zero is equally valid.*	**M1** **A1** **M1** **A1** **(4)**
		Total for Question 6: 11 marks

Question 7

10 marks

Part	Scheme	Marks
(a)	A falls $\frac{1}{2} \times 9.8 \times 2^2 = 19.6$ m	**M1**
	25.4 m	**A1**
		(2)
(b)	In 2 s, A moves $15 \times 2 = 30$ m and B moves $2V\cos\alpha$ towards the tower; together they cover 60 m	**M1**
	$30 + 2V\cos\alpha = 60$, so $V\cos\alpha = 15$ *	**A1***
		(2)
(c)	B at 25.4 m after 2 s: $2V\sin\alpha - 19.6 = 25.4$	**M1**
	$V\sin\alpha = 22.5$	**A1**
	$V = \sqrt{15^2 + 22.5^2} =$ 27.0	**A1**
	$\tan\alpha = 1.5$: $\alpha = 56.3^\circ$	**A1**
		(4)
(d)	Vertical component of B: $22.5 - 9.8 \times 2 = 2.9$	**M1**
	Positive, so B is still moving upwards	**A1**
		(2)
		Total for Question 7: 10 marks

Question 8

10 marks

Part	Scheme	Marks
(a)	$x = 20t\cos\alpha$ and $y = 20t\sin\alpha - 4.9t^2$	**M1**
	Substitutes $t = \dfrac{x}{20\cos\alpha}$: $y = x\tan\alpha - \dfrac{4.9x^2}{400\cos^2\alpha}$	**A1**
	$\dfrac{4.9}{400} = \dfrac{49}{4000}$ and $\sec^2\alpha = 1 + \tan^2\alpha$ *	**A1***
		(3)
(b)	Over the wall: $y > 2.5$ when $x = 25$	**M1**
	$25\tan\alpha - \dfrac{49\times 625}{4000}\left(1+\tan^2\alpha\right) > 2.5$	**A1**
	$25\tan\alpha - 7.65625 - 7.65625\tan^2\alpha > 2.5$, and multiplies by $\frac{32}{5}$	**M1**
	$49\tan^2\alpha - 160\tan\alpha + 65 < 0$ *	**A1***
		(4)
(c)	Roots $\tan\alpha = \dfrac{160 \pm \sqrt{12\,860}}{98} = 0.4755,\ 2.790$	**M1**
	Between the roots (a positive quadratic is negative between them)	**M1**
	$25.4^\circ < \alpha < 70.3^\circ$	**A1**
		(3)
		Total for Question 8: 10 marks

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Question 9

10 marks

Part	Scheme	Marks
(a)	R(↑): $0 = (U\sin\alpha)T - \frac{1}{2}gT^2$, $T \neq 0$	M1
	$T = \dfrac{2U\sin\alpha}{g}$ *	A1*
		(2)
(b)	$R = U\cos\alpha \times \dfrac{2U\sin\alpha}{g}$	M1
	$2\sin\alpha\cos\alpha = \sin 2\alpha$: $R = \dfrac{U^2\sin 2\alpha}{g}$ *	A1*
		(2)
(c)	$22.5 = \dfrac{441\sin 2\alpha}{9.8}$, $\sin 2\alpha = 0.5$	M1
	$2\alpha = 30^\circ$ or 150°	A1
	$\alpha = 15^\circ$ or 75°	A1
		(3)
(d)	Greatest height $= \dfrac{U^2\sin^2\alpha}{2g}$, so the ratio is $\dfrac{\sin^2 15^\circ}{\sin^2 75^\circ} = \tan^2 15^\circ$ (as $\sin 75^\circ = \cos 15^\circ$)	M1
	$\tan 15^\circ = \tan(45^\circ - 30^\circ) = \dfrac{1 - \frac{1}{\sqrt{3}}}{1 + \frac{1}{\sqrt{3}}} = 2 - \sqrt{3}$	M1
	$\left(2 - \sqrt{3}\right)^2 = 7 - 4\sqrt{3}$ *	A1*
		(3)
		Total for Question 9: 10 marks

Question 10

11 marks

Part	Scheme	Marks
(a)	Q lands when $0 = 4t - 4.9t^2$: $t = \frac{4}{4.9} = 0.816\,\text{s}$ P is then at $\left(3t,\ 12t - 4.9t^2\right) = (2.449,\ 6.531)$ and Q at $(1.633,\ 0)$ $\sqrt{0.816^2 + 6.531^2}$ $\boxed{6.58\,\text{m}}$ *Oct 2023 report: read whether the distance between the particles or only the vertical distance is asked for.*	**M1** **M1** **M1** **A1** **(4)**
(b)	$\mathbf{v}_P = 3\mathbf{i} + (12 - 9.8t)\mathbf{j}$, $\mathbf{v}_Q = 2\mathbf{i} + (4 - 9.8t)\mathbf{j}$ Perpendicular: $\mathbf{v}_P \cdot \mathbf{v}_Q = 0$ (or gradients multiply to -1) $6 + (12 - 9.8t)(4 - 9.8t) = 0$ $(9.8t)^2 - 16(9.8t) + 54 = 0$ *	**B1** **M1** **A1** **A1*** **(4)**
(c)	$9.8t = 8 \pm \sqrt{10}$ $t = 1.14$ or $t = 0.494$; Q lands at $t = 0.816$, so $t = 1.14$ is not valid (the formula for $\mathbf{v}_Q$ no longer applies) $\boxed{t = 0.49\,\text{s}}$ (0.494)	**M1** **A1** **A1** **(3)**
		Total for Question 10: 11 marks

2 Chapter 2: Variable Acceleration

Mark scheme. Marks are shown on the right of each line.

Question 1

7 marks

Part	Scheme	Marks
(a)	$v = \frac{dx}{dt} = 3t^2 - 18t + 24$	M1
	$3(t-2)(t-4) = 0$ (working shown)	A1
	$t = 2$ and $t = 4$	A1
		(3)
(b)	$a = 6t - 18$: $12\,\text{m s}^{-2}$	B1
		(1)
(c)	$x(2) = 20$ and $x(4) = 16$	M1
	$20 + \lvert 16 - 20 \rvert$	M1
	$24\,\text{m}$	A1
	$x(4) = 16$ is the displacement, not the distance: the particle turns at $t = 2$.	
		(3)
	Total for Question 1: 7 marks	

Question 2

6 marks

Part	Scheme	Marks
(a)	$v = \int (6-2t)\,dt = 6t - t^2 + c$, with $c = 5$	M1
	Greatest where $a = 0$: $t = 3$	M1
	$14\,\text{m s}^{-1}$	A1
	Summer 2019 report: forgetting to use the initial velocity to find the constant.	
		(3)
(b)	$x = 5t + 3t^2 - \frac{1}{3}t^3$ (no constant as $x = 0$ at $t = 0$)	M1
	$30 + 108 - 72$	M1
	$66\,\text{m}$	A1
		(3)
	Total for Question 2: 6 marks	

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Question 3

6 marks

Part	Scheme	Marks
(a)	$\mathbf{v} = (3t^2 - 3)\,\mathbf{i} + (4t - 8)\mathbf{j}$	M1
	Parallel to $\mathbf{i}$: $4t - 8 = 0$, $t = 2$	M1
	$9\mathbf{i}\,\text{m s}^{-1}$	A1
	Oct 2023 report: setting the wrong component, or a component of $\mathbf{r}$, to zero.	(3)
(b)	$\mathbf{a} = 6t\mathbf{i} + 4\mathbf{j}$	M1
	$(12\mathbf{i} + 4\mathbf{j})\,\text{m s}^{-2}$	A1
		(2)
(c)	$\mathbf{v} = -3\mathbf{i} - 8\mathbf{j}$: $\sqrt{73} = 8.54\,\text{m s}^{-1}$	B1
		(1)
		Total for Question 3: 6 marks

Question 4

7 marks

Part	Scheme	Marks
(a)	$\mathbf{v} = 3t^2\mathbf{i} - 4t\mathbf{j} + \mathbf{c}$ with $\mathbf{c} = -3\mathbf{i} + 8\mathbf{j}$	M1
	$\mathbf{v} = (3t^2 - 3)\,\mathbf{i} + (8 - 4t)\mathbf{j}$	A1
		(2)
(b)	Due north: $\mathbf{i}$ component zero and $\mathbf{j}$ component positive, $3t^2 - 3 = 0$	M1
	$t = 1$ (then $\mathbf{v} = 4\mathbf{j}$, northwards)	A1
		(2)
(c)	$\mathbf{r} = (t^3 - 3t)\,\mathbf{i} + (8t - 2t^2)\,\mathbf{j} + \mathbf{d}$	M1
	$\mathbf{d} = 2\mathbf{i} + \mathbf{j}$	A1
	$\mathbf{r} = 20\mathbf{i} + 7\mathbf{j}$: $\sqrt{449} = 21.2\,\text{m}$	A1
	Oct 2022 report: the constant of integration must be found, not just added on at the end.	(3)
		Total for Question 4: 7 marks

Question 5

6 marks

Part	Scheme	Marks
(a)	$v = \int a\,\mathrm{d}t = at + c$; $v = u$ when $t = 0$, so $c = u$: $v = u + at$	M1
	Correct	A1
	$s = \int (u + at)\,\mathrm{d}t = ut + \frac{1}{2}at^2 + k$; $s = 0$ when $t = 0$, so $k = 0$	M1
	$s = ut + \frac{1}{2}at^2$ *	A1*
		(4)
(b)	$t = \frac{v-u}{a}$ substituted: $s = \frac{u(v-u)}{a} + \frac{(v-u)^2}{2a}$, so $2as = v^2 - u^2$ *	B1
		(1)
(c)	The derivation assumes a is constant; $3t$ varies with time	B1
		(1)
		Total for Question 5: 6 marks

Question 6

11 marks

Part	Scheme	Marks
(a)	$v = t^2 - 7t + c$	M1
	$c = 10$: $v = t^2 - 7t + 10$ *	A1*
		(2)
(b)	$(t - 2)(t - 5) = 0$	M1
	$t = 2$ and $t = 5$	A1
		(2)
(c)	$x = \frac{1}{3}t^3 - \frac{7}{2}t^2 + 10t$	M1
	$x(2) = \frac{26}{3}$, $x(5) = \frac{25}{6}$, $x(6) = 6$	A1
	Splits at $t = 2$ and $t = 5$	M1
	$\frac{26}{3} + \left(\frac{26}{3} - \frac{25}{6}\right) + \left(6 - \frac{25}{6}\right)$	M1
	15 m	A1
	Summer 2019 report: using the magnitude of each displacement; a single integral from 0 to 6 gives 6 m (the displacement).	
		(5)
(d)	Least velocity where $2t - 7 = 0$: $v = -\frac{9}{4}$	M1
	$2.25\,\mathrm{m\,s^{-1}}$	A1
		(2)
		Total for Question 6: 11 marks

Question 7

11 marks

Part	Scheme	Marks
(a)	$\mathbf{a} = \mathbf{i} + 2t\mathbf{j} = \mathbf{i} + 4\mathbf{j}$	M1
	$\mathbf{F} = 2\mathbf{a} = 2\mathbf{i} + 8\mathbf{j}$	M1
	$\boxed{8.25\,\text{N}}$ $(2\sqrt{17})$	A1
		(3)
(b)	Direction $\mathbf{i}+\mathbf{j}$: components equal, $t + 1 = t^2 - 5$	M1
	$t^2 - t - 6 = 0$, $t = 3$ ($t = -2$ rejected)	A1
	$\mathbf{v} = 4\mathbf{i} + 4\mathbf{j}$	M1
	$\boxed{5.66\,\text{m s}^{-1}}$ $(4\sqrt{2})$	A1
	Jan 2020 report: the velocity was found but not the speed.	
		(4)
(c)	$\mathbf{r} = \left(\frac{1}{2}t^2 + t\right)\mathbf{i} + \left(\frac{1}{3}t^3 - 5t\right)\mathbf{j}$	M1
	$\mathbf{j}$ component zero: $t^2 = 15$, $t = \sqrt{15}$	A1
	$x = \frac{15}{2} + \sqrt{15}$	M1
	$\boxed{11.4\,\text{m}}$	A1
		(4)
	Total for Question 7: 11 marks	

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Question 8

10 marks

Part	Scheme	Marks
(a)	$a = \frac{\mathrm{d}v}{\mathrm{d}t} = 3\mathrm{e}^{-\frac{1}{2}t}$	M1
	$\boxed{3\,\mathrm{m\,s^{-2}}}$	A1
		(2)
(b)	$6\mathrm{e}^{-\frac{1}{2}t} > 0$ for all t, so $v < 6$	B1
		(1)
(c)	$\int_0^4 \left(6 - 6\mathrm{e}^{-\frac{1}{2}t}\right) \mathrm{d}t$	M1
	$\left[6t + 12\mathrm{e}^{-\frac{1}{2}t}\right]_0^4$	A1
	$24 + 12\mathrm{e}^{-2} - 12$	M1
	$\boxed{13.6\,\mathrm{m}}$	A1
		(4)
(d)	$\mathrm{e}^{-\frac{1}{2}t} = \frac{1}{6}$	M1
	$\boxed{t = 2\ln 6}$	A1
		(2)
(e)	e.g. the speed never reaches $6\,\mathrm{m\,s^{-1}}$, or the model ignores changes in effort and gradient	B1
		(1)
		Total for Question 8: 10 marks

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Question 9

10 marks

Part	Scheme	Marks
(a)	$4-2t=0,\ t=2$	M1
	$\boxed{4\,\text{m s}^{-1}}$	A1
		(2)
(b)	At $t=3$: $4-2t=-2$ and $\frac{\mathrm{d}}{\mathrm{d}t}\left(27t^{-2}\right)=-54t^{-3}=-2$	M1
	Both give $-2\,\text{m s}^{-2}$, so the acceleration does not jump *	A1*
		(2)
(c)	$\int_0^3 \left(4t-t^2\right)\mathrm{d}t=9$	M1
	$\int_3^6 27t^{-2}\,\mathrm{d}t=\left[-27t^{-1}\right]_3^6$	M1
	$=4.5$	A1
	$\boxed{13.5\,\text{m}}$	A1
		(4)
(d)	Distance after $t=3$ is $27\left(\frac{1}{3}-\frac{1}{t}\right)<9$ for every t	M1
	So the total is always less than $9+9=18\,\text{m}$ *	A1*
		(2)
		Total for Question 9: 10 marks

Question 10

11 marks

Part	Scheme	Marks
(a)	$\mathbf{r} = \left(t^3 - 4t\right)\mathbf{i} + t^2\mathbf{j} + \mathbf{c}$	M1
	$\mathbf{c} = \mathbf{0}$	A1
	$\boxed{\mathbf{r}_P = \left(t^3 - 4t\right)\mathbf{i} + t^2\mathbf{j}}$	A1
		(3)
(b)	$\mathbf{r}_Q = (-6 + 3t)\mathbf{i} + (2 + t)\mathbf{j}$	B1
	$\mathbf{i}$: $t^3 - 7t + 6 = 0$, $(t-1)(t-2)(t+3) = 0$	M1
	$\mathbf{j}$: $t^2 - t - 2 = 0$, $(t-2)(t+1) = 0$; both hold only at $t = 2$	A1
	$\boxed{4\mathbf{j}\,\text{m}}$	A1
	Jan 2021 report (M1): both components must agree at the same time.	
		(4)
(c)	At $t = 1$: $\mathbf{r}_P = -3\mathbf{i} + \mathbf{j}$, $\mathbf{r}_Q = -3\mathbf{i} + 3\mathbf{j}$	M1
	$\boxed{2\,\text{m}}$	A1
		(2)
(d)	$\mathbf{v}_P = 8\mathbf{i} + 4\mathbf{j}$	M1
	$\boxed{8.94\,\text{m s}^{-1}}$ $(4\sqrt{5})$	A1
		(2)
	Total for Question 10: 11 marks	

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3 Chapter 3: Centres of Mass

Mark scheme. Marks are shown on the right of each line.

Question 1

6 marks

Part	Scheme	Marks
(a)	x: $2(1)+3(4)+5p = 10\times 3$	M1
	$\boxed{p = 3.2}$	A1
	y: $2(2)+3(-1)+5q = 10\times 1$	M1
	$\boxed{q = 1.8}$	A1
		(4)
(b)	$\bar{x} = \dfrac{30}{10+M}$ and $\bar{y} = \dfrac{10}{10+M}$ (the new particle adds nothing to either moment)	M1
	$\bar{x} = 3\bar{y}$ for every M, so the centre of mass lies on $x = 3y$ *	A1*
	Oct 2022 report: the point must satisfy the equation of the line; comparing gradients is not a method.	
		(2)
		Total for Question 1: 6 marks

Question 2

6 marks

Part	Scheme	Marks
(a)	Areas: rectangle $48a^2$, semicircle $2\pi a^2$, lamina $(48-2\pi)a^2$	B1
	Distances from AB: $3a$ and $\dfrac{4(2a)}{3\pi} = \dfrac{8a}{3\pi}$	B1
	Moments about AB: $48a^2(3a) - 2\pi a^2\left(\frac{8a}{3\pi}\right) = (48-2\pi)a^2\bar{y}$	M1
	$144a - \frac{16}{3}a = (48-2\pi)\bar{y}$	A1
	$\bar{y} = \dfrac{416a}{3(48-2\pi)} = \dfrac{208a}{3(24-\pi)}$ *	A1*
	Jan 2020 report: the semicircle's area and $\frac{4r}{3\pi}$ (not $\frac{4\pi r}{3}$) are the common slips.	
		(5)
(b)	By symmetry, $\boxed{4a}$	B1
		(1)
		Total for Question 2: 6 marks

Question 3

6 marks

Part	Scheme	Marks
(a)	Masses in proportion to lengths: AB 3 kg, BC 4 kg, CA 5 kg, at the midpoints	B1
	From AB: $12\bar{x} = 4(4a) + 5(4a)$	M1
	$\boxed{3a}$ from AB	A1
	From BC: $12\bar{y} = 3(3a) + 5(3a)$: $\boxed{2a}$ from BC	A1
	Jan 2021 report: assuming the rods have equal masses is M0.	(4)
(b)	$\dfrac{36a}{12+m} = \dfrac{24a+6ma}{12+m}$	M1
	$\boxed{m = 2}$	A1
		(2)
		Total for Question 3: 6 marks

Question 4

6 marks

Part	Scheme	Marks
(a)	Lamina's centre of mass (3, 2) from A (a third of each perpendicular side)	B1
	Moments: $3M\bar{x} = 2M(3) + M(9)$	M1
	$\boxed{5\text{ cm}}$ from AC	A1
	$3M\bar{y} = 2M(2) + 0$: $\boxed{\frac{4}{3}\text{ cm}}$ from AB	A1
		(4)
(b)	$\tan\theta = \dfrac{5}{6 - \frac{4}{3}}$	M1
	$\boxed{47.0^\circ}$	A1
	Jan 2023 report: check which angle is asked for (47°, not its complement 43°).	(2)
		Total for Question 4: 6 marks

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Question 5

6 marks

Part	Scheme	Marks
(a)	Split into $6a \times 2a$ (area $12a^2$, centre $(3a, a)$) and $2a \times 4a$ (area $8a^2$, centre $(a, 4a)$)	**M1**
	$20\bar{x} = 12(3a) + 8(a)$: $\bar{x} = 2.2a$	**A1**
	By symmetry about the line $y = x$ (or moments), $2.2a$ from both OA and OE	**A1**
		(3)
(b)	About to topple about O: centre of mass vertically above O	**M1**
	$\tan\theta = \dfrac{2.2a}{2.2a}$	**A1**
	$45°$	**A1**
		(3)
	Total for Question 5: 6 marks	

Question 6

10 marks

Part	Scheme	Marks
(a)	Areas: rectangle $24a^2$, triangle $\frac{1}{2}(6a)(2a) = 6a^2$, lamina $18a^2$	**B1**
	Triangle's centre of mass is $\frac{4a+4a+2a}{3} = \frac{10}{3}a$ from AB	**B1**
	$24a^2(2a) - 6a^2\left(\frac{10}{3}a\right) = 18a^2\bar{y}$	**M1**
	$\bar{y} = \frac{28}{18}a = \frac{14}{9}a$ *	**A1***
		(4)
(b)	$\bar{x} = 3a$ by symmetry	**B1**
	$\tan\theta = \dfrac{\frac{14}{9}a}{3a}$	**M1**
	$27°$	**A1**
		(3)
(c)	Moments about A: $F \times 4a = Mg \times 3a$	**M1**
	Correct, with no resolving of the forces	**A1**
	$F = \frac{3}{4}Mg$	**A1**
	Oct 2023 report: the distances $4a$ and $3a$ are already perpendicular to F and Mg.	
		(3)
	Total for Question 6: 10 marks	

Question 7

10 marks

Part	Scheme	Marks
(a)	Mass ratio: square $2 \times 36a^2 = 72$, triangle 9 (area $\frac{1}{2} \times 6a \times 3a$), total 81	**B1**
	Triangle's centre of mass $6a + \frac{1}{3}(3a) = 7a$ from PS	**B1**
	$81\bar{x} = 72(3a) + 9(7a)$	**M1**
	$\bar{x} = \frac{279}{81}a = \frac{31}{9}a$ *	**A1***
	Jan 2025 report: the mass per unit area applies to each part; it is not the total mass.	**(4)**
(b)	$\bar{y} = 3a$ by symmetry	**B1**
	$\tan\theta = \dfrac{3a}{\frac{31}{9}a} = \dfrac{27}{31}$	**M1**
	41°	**A1**
		(3)
(c)	Moments about P: $T_Q \times 6a = Mg \times \frac{31}{9}a$	**M1**
	$T_Q = \frac{31}{54}Mg$	**A1**
	$T_P = \frac{23}{54}Mg$	**A1**
		(3)
	Total for Question 7: 10 marks	

Question 8

10 marks

Part	Scheme	Marks
(a)	Midpoints from B: AB $(2a, 0)$, BC $(0, 1.5a)$, CA $(2a, 1.5a)$	B1
	$6m\bar{x} = 2m(2a) + m(0) + 3m(2a)$	M1
	$\bar{x} = \frac{5}{3}a$ from BC *	A1*
	$6m\bar{y} = 0 + m(1.5a) + 3m(1.5a)$, so $\bar{y} = a$ from AB *	A1*
		(4)
(b)	D is vertically above the centre of mass: $AD = 4a - \frac{5}{3}a$	M1
	$\boxed{\frac{7}{3}a}$	A1
		(2)
(c)	New centre of mass from B: $\left(\frac{10a}{6+k}, \frac{6a+3ka}{6+k}\right)$	M1
	$\tan\theta = \frac{\bar{y}}{4a - \bar{x}} = \frac{6+3k}{14+4k}$	M1
	$5(6+3k) = 3(14+4k)$	A1
	$\boxed{k = 4}$	A1
		(4)
	Total for Question 8: 10 marks	

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Question 9

10 marks

Part	Scheme	Marks
(a)	Areas: triangle rh, semicircle $\frac{1}{2}\pi r^2$	B1
	Distances from O: triangle $+\frac{h}{3}$, semicircle $-\frac{4r}{3\pi}$	B1
	$\left(rh+\frac{1}{2}\pi r^2\right)\bar{y} = rh\cdot\frac{h}{3} - \frac{1}{2}\pi r^2\cdot\frac{4r}{3\pi}$	M1
	$\bar{y} = \dfrac{\frac{1}{3}rh^2 - \frac{2}{3}r^3}{rh+\frac{1}{2}\pi r^2} = \dfrac{2\left(h^2-2r^2\right)}{3(2h+\pi r)}$ *	A1*
		(4)
(b)	$h^2 - 2r^2 = 0$	M1
	$\boxed{h=\sqrt{2}r}$	A1
		(2)
(c)	The reaction at any point of the curved edge acts along a radius, through O, and the weight acts through O: the two forces are always in line	B1
		(1)
(d)	$h=2r$: $\bar{y} = \dfrac{4r}{3(4+\pi)} = 0.187r$	B1
	$\tan\theta = \dfrac{0.187r}{r}$	M1
	$\boxed{10.6^\circ}$	A1
		(3)
	Total for Question 9: 10 marks	

Question 10

11 marks

Part	Scheme	Marks
(a)	Areas: square $16a^2$, hole πa^2, lamina $(16-\pi)a^2$	**B1**
	From AD: $16a^2(2a) - \pi a^2(2.5a) = (16-\pi)a^2\bar{x}$	**M1**
	$\bar{x} = \dfrac{(32-2.5\pi)a}{16-\pi} = \dfrac{(64-5\pi)a}{2(16-\pi)}$ *	**A1***
	From AB: $\bar{y} = \dfrac{(64-3\pi)a}{2(16-\pi)} = \boxed{2.12a}$	**A1**
		(4)
(b)	About to topple about A: the centre of mass is vertically above A	**M1**
	$\tan\theta = \dfrac{\bar{x}}{\bar{y}} = \dfrac{64-5\pi}{64-3\pi} = 0.885$	**A1**
	$\boxed{41.5^\circ}$	**A1**
		(3)
(c)	About to slide when $mg\sin\theta = \mu mg\cos\theta$, i.e. $\tan\theta = 0.9$ ($\theta = 42.0^\circ$)	**M1**
	$0.885 < 0.9$: toppling happens at a smaller angle	**M1**
	The lamina $\boxed{\text{topples first}}$	**A1**
		(3)
(d)	The lamina keeps its shape, so the position of its centre of mass relative to A does not change as θ increases	**B1**
		(1)
		Total for Question 10: 11 marks

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4 Chapter 4: Work and Energy

Mark scheme. Marks are shown on the right of each line.

Question 1

6 marks

Part	Scheme	Marks
(a)	$R = 5g\cos 20^\circ$ Work against friction = $0.3R \times 4$ $\boxed{55.3\,\text{J}}$	B1 M1 A1 (3)
(b)	Constant speed: work = friction work + gain in PE = $55.25 + 5g \times 4\sin 20^\circ$ $\boxed{122\,\text{J}}$ *Jan 2025 report: the gain in potential energy is counted once; adding the weight component as a separate work term as well is double counting.*	M1 A1 (2)
(c)	$\frac{122.3}{8} = \boxed{15.3\,\text{W}}$	B1 (1)
		Total for Question 1: 6 marks

Question 2

6 marks

Part	Scheme	Marks
(a)	At the greatest speed, driving force = 30: $v = \frac{240}{30}$ $\boxed{8\,\text{m s}^{-1}}$	M1 A1 (2)
(b)	$\frac{240}{5} - 30 = 80a$ $\boxed{0.225\,\text{m s}^{-2}}$ *Oct 2022 report: the resistance acts against the motion, so it is subtracted from the driving force.*	M1 A1 (2)
(c)	$F = 30 + 80g \times \frac{1}{40} = 49.6$ and $P = 49.6 \times 4$ $\boxed{198\,\text{W}}$ (198.4)	M1 A1 (2)
		Total for Question 2: 6 marks

Question 3

6 marks

Part	Scheme	Marks
(a)	$\frac{1}{2}m\left(12^2\right)+mg(25)=\frac{1}{2}mv^2$	M1
	$v^2=634$	A1
	$\boxed{25.2\,\text{m s}^{-1}}$	A1
	Summer 2023 report: an exact surd is not accepted after using $g=9.8$.	(3)
(b)	$v^2=144+2g(15)$	M1
	$\boxed{20.9\,\text{m s}^{-1}}$	A1
		(2)
(c)	Energy depends only on the speed and the height, not on the direction of motion	B1
		(1)
		Total for Question 3: 6 marks

Question 4

6 marks

Part	Scheme	Marks
(a)	$R=2g\cos\alpha=1.6g$, friction $=0.25R=0.4g$	B1
	Work–energy: $\frac{1}{2}(2)\left(12^2\right)=0.4g\,d+2g\,d\sin\alpha$	M1
	All terms correct	A1
	$144=15.68\,d$	A1
	$\boxed{9.18\,\text{m}}$	A1
	Jan 2023 report: sine for cosine in R; suvat scores nothing when the work–energy principle is asked for.	(5)
(b)	$0.4g\times 9.18=\boxed{36.0\,\text{J}}$	B1
		(1)
		Total for Question 4: 6 marks

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Question 5

6 marks

Part	Scheme	Marks
(a)	$60\cos 30^\circ \times 20$	M1
	$\boxed{1040\,\text{J}}$ (1039)	A1
		(2)
(b)	$R = 15g - 60\sin 30^\circ = 117$	B1
	Work–energy: $1039.2 - 0.2 \times 117 \times 20 = \frac{1}{2}(15)v^2$	M1
	$\frac{15}{2}v^2 = 571.2$	A1
	$\boxed{8.73\,\text{m s}^{-1}}$	A1
		(4)
		Total for Question 5: 6 marks

Question 6

11 marks

Part	Scheme	Marks
(a)	Level road: $\frac{P}{40} = R$	M1
	Hill: $\frac{P}{25} = R + 1200g \times \frac{1}{20}$	M1
	Both correct ($1200g \times \frac{1}{20} = 588$)	A1
	$40R = 25R + 14\,700$, so $R = 980$ *	A1*
	$\boxed{P = 39\,200}$ (39.2 kW)	A1
		(5)
(b)	Driving force $\frac{39\,200}{28} = 1400$	M1
	$1400 - 980 = 1200a$	M1
	$\boxed{0.35\,\text{m s}^{-2}}$	A1
		(3)
(c)	Loss of KE + loss of PE = work against resistance: $\frac{1}{2}(1200)\left(25^2 - 15^2\right) + 588d = 980d$	M1
	$240\,000 = 392d$	A1
	$\boxed{612\,\text{m}}$	A1
	The weight component helps the car down the slope: sign errors here are common (Jan 2020 report).	
		(3)
		Total for Question 6: 11 marks

Question 7

10 marks

Part	Scheme	Marks
(a)	$0.4 \times 3g \times 1.5$ $\boxed{17.6\,\text{J}}$	**M1** **A1** **(2)**
(b)	Loss of PE of B: $2g(1.5) = 29.4$ $29.4 - 17.64 = \frac{1}{2}(3+2)v^2$ (the KE of both particles) Correct equation $\boxed{2.17\,\text{m s}^{-1}}$ *Summer 2019 report: the kinetic energy of both particles must be included.*	**B1** **M1** **A1** **A1** **(4)**
(c)	String slack: $\frac{1}{2}(3)v^2 = 0.4 \times 3g \times d$ $7.056 = 11.76d$ $\boxed{0.6\,\text{m}}$	**M1** **A1** **A1** **(3)**
(d)	No energy is lost at the pulley, so the tension is the same on both sides (no friction at the pulley)	**B1** **(1)**
		Total for Question 7: 10 marks

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Question 8

10 marks

Part	Scheme	Marks
(a)	Friction $0.2 \times 0.5g\cos 30^\circ$; work = friction $\times 5$	**M1**
	$\boxed{4.24\,\text{J}}$	**A1**
		(2)
(b)	$\frac{1}{2}(0.5)\left(10^2\right) = \frac{1}{2}(0.5)v_B^2 + 0.5g(2.5) + 4.24$	**M1**
	Correct equation	**A1**
	$\boxed{5.83\,\text{m s}^{-1}}$	**A1**
		(3)
(c)	$\frac{1}{2}v^2 = \frac{1}{2}v_B^2 + g(2.5)$	**M1**
	$\boxed{9.11\,\text{m s}^{-1}}$	**A1**
		(2)
(d)	Leaves B at 30° above the horizontal: vertical component $5.833\sin 30^\circ = 2.917$	**B1**
	$-2.5 = 2.917t - 4.9t^2$	**M1**
	$\boxed{1.07\,\text{s}}$	**A1**
	Oct 2022 report: confusion between the speed at the top of the ramp and the speed of projection.	
		(3)
		Total for Question 8: 10 marks

Question 9

10 marks

Part	Scheme	Marks
(a)	At the greatest speed: $\frac{30000}{50} = 50k$	M1
	$\boxed{k = 12}$	A1
		(2)
(b)	$\frac{30000}{20} - 12 \times 20 = 800a$	M1
	$1500 - 240 = 800a$	A1
	$\boxed{1.575\,\text{m s}^{-2}}$ (1.58)	A1
		(3)
(c)	$\frac{30000}{v} = 12v + 800g \times \frac{1}{16}$	M1
	$12v^2 + 490v - 30000 = 0$	A1
	Solves the quadratic (positive root)	M1
	$\boxed{33.6\,\text{m s}^{-1}}$	A1
		(4)
(d)	e.g. air resistance is more likely to depend on v^2; other resistances (friction) do not depend on speed	B1
		(1)
		Total for Question 9: 10 marks

Question 10

11 marks

Part	Scheme	Marks
(a)	Driving force $\frac{45000}{15} = 3000$ Whole system: $3000 - 600 - 2000g \times \frac{1}{14} = 2000a$ $0.5\,\text{m s}^{-2}$ *Jan 2025 report: the vehicles are accelerating, not moving at constant speed.*	**B1** **M1** **A1** **(3)**
(b)	Trailer: $T - 200 - 500g \times \frac{1}{14} = 500 \times 0.5$ 800 N	**M1** **A1** **(2)**
(c)	$\frac{1}{2}(500)\left(15^2\right) = 200d + 500g \times \frac{1}{14} \times d$ $56\,250 = 550d$ 102 m	**M1** **A1** **A1** **(3)**
(d)	Van: $3000 - 400 - 1500g \times \frac{1}{14} = 1500a$ $1.03\,\text{m s}^{-2}$	**M1** **A1** **(2)**
(e)	The power is constant, so as the speed increases the driving force $\frac{P}{v}$ decreases	**B1** **(1)**
		Total for Question 10: 11 marks

5 Chapter 5: Impulses and Collisions

Mark scheme. Marks are shown on the right of each line.

Question 1

6 marks

Part	Scheme	Marks
(a)	$\mathbf{I} = 0.4\left((2\mathbf{i}+4\mathbf{j})-(5\mathbf{i}-3\mathbf{j})\right)$	M1
	$= -1.2\mathbf{i}+2.8\mathbf{j}$	A1
	$\sqrt{1.2^2+2.8^2} = \boxed{3.05\,\text{N s}}$	A1
	Summer 2023 report: stopping at the vector, or finding the magnitude of the change in velocity only.	(3)
(b)	Scalar product (or two angles): $\cos\theta = \dfrac{(-1.2)(5)+(2.8)(-3)}{3.046\times\sqrt{34}}$	M1
	$\cos\theta = -0.811$	A1
	$\boxed{144^\circ}$	A1
		(3)
		Total for Question 1: 6 marks

Question 2

6 marks

Part	Scheme	Marks
(a)	Momentum: $2(6) = 2v_A + 3v_B$	M1
	Impact law: $v_B - v_A = \frac{1}{2}(6)$	M1
	Both correct	A1
	$\boxed{v_A = 0.6\,\text{m s}^{-1},\ v_B = 3.6\,\text{m s}^{-1}}$	A1
		(4)
(b)	$\frac{1}{2}(2)\left(6^2\right) - \left[\frac{1}{2}(2)\left(0.6^2\right) + \frac{1}{2}(3)\left(3.6^2\right)\right]$	M1
	$\boxed{16.2\,\text{J}}$	A1
		(2)
		Total for Question 2: 6 marks

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Question 3

6 marks

Part	Scheme	Marks
(a)	Rebound speed $0.6 \times 8 = 4.8\,\text{m s}^{-1}$	B1
	$I = 0.5\big(4.8 - (-8)\big)$	M1
	$\boxed{6.4\,\text{N s}}$	A1
	Oct 2023 report: the change in direction must be taken into account.	(3)
(b)	$\frac{1}{2}(0.5)\left(8^2 - 4.8^2\right)$	M1
	$\boxed{10.2\,\text{J}}$ (10.24)	A1
		(2)
(c)	$e < 1$, so kinetic energy is lost (it would be kept only if $e = 1$)	B1
		(1)
		Total for Question 3: 6 marks

Question 4

6 marks

Part	Scheme	Marks
(a)	Momentum (direction of P positive): $3m(4u) - m(2u) = 3mv + m(4u)$	M1
	Correct	A1
	$\boxed{v = 2u, \text{ in the same direction as before}}$	A1
		(3)
(b)	Speed of separation $4u - 2u = 2u$; speed of approach $4u + 2u = 6u$	M1
	$2u = e \times 6u$	M1
	$\boxed{e = \frac{1}{3}}$	A1
	Jan 2025 report: write the law as separation = $e\times$ approach to avoid sign errors.	(3)
		Total for Question 4: 6 marks

Question 5

6 marks

Part	Scheme	Marks
(a)	Momentum triangle: $\lvert m\mathbf{u}\rvert = 2$, $\lvert\mathbf{I}\rvert = 2.5$, angle between them in the triangle 60°	M1
	$(mv)^2 = 2^2 + 2.5^2 - 2(2)(2.5)\cos 60^\circ$	M1
	$mv = \sqrt{5.25} = 2.291$	A1
	$\boxed{9.17\,\text{m s}^{-1}}$	A1
	Oct 2023 report: the vector triangle with the cosine rule was very effective; resolving parallel and perpendicular to $\mathbf{u}$ *(*$2 - 1.25$ *and* 2.165*) is equally valid.*	(4)
(b)	$\tan\phi = \dfrac{2.5\sin 120^\circ}{2 + 2.5\cos 120^\circ} = \dfrac{2.165}{0.75}$	M1
	$\boxed{70.9^\circ}$	A1
		(2)
	Total for Question 5: 6 marks	

Question 6

11 marks

Part	Scheme	Marks
(a)	Momentum: $3m(2u) - 5m(u) = 3mv_P + 5mv_Q$	M1
	Correct	A1
	Impact law: $v_Q - v_P = e(2u + u)$	M1
	Correct	A1
	Solves: $\boxed{v_P = \dfrac{(1 - 15e)u}{8}}$	A1
	$\boxed{v_Q = \dfrac{(1 + 9e)u}{8}}$	A1
		(6)
(b)	Reversed when $v_P < 0$: $1 - 15e < 0$	M1
	$e > \frac{1}{15}$ *	A1*
		(2)
(c)	KE before $\frac{1}{2}(3m)(2u)^2 + \frac{1}{2}(5m)u^2 = \frac{17}{2}mu^2$; KE after with (a)	M1
	Loss $= \dfrac{135}{16}\left(1 - e^2\right)mu^2 = \frac{15}{2}mu^2$	A1
	$\boxed{e = \frac{1}{3}}$	A1
	Summer 2023 report: every particle's KE with its own mass, and terms added, not subtracted.	(3)
	Total for Question 6: 11 marks	

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Question 7

10 marks

Part	Scheme	Marks
(a)	Momentum: $m(5u) = mv_A + 3mv_B$	**M1**
	Impact law: $v_B - v_A = \frac{3}{5}(5u)$	**M1**
	Solves	**A1**
	A: u; B: $2u$	**A1**
		(4)
(b)	$v_A = -u$: A moves away from the wall (its direction is reversed)	**B1**
		(1)
(c)	B rebounds with speed $2fu$: $I = 3m(2fu + 2u)$	**M1**
	$6mu(1+f)$	**A1**
	Oct 2023 report: using m instead of $3m$, or ignoring the change in direction.	
		(2)
(d)	After the wall both move away from the wall; B catches A only if its speed is greater	**M1**
	$2fu > u$	**M1**
	$f > \frac{1}{2}$ ($f = \frac{1}{2}$ gives equal speeds and no collision) *	**A1***
	Oct 2023 report: the equal-speed case and the fact that B rebounds must both be addressed.	
		(3)
		Total for Question 7: 10 marks

Question 8

10 marks

Part	Scheme	Marks
(a)	Momentum: $2m(3u) = 2mv_P + mv_Q$	**M1**
	Impact law: $v_Q - v_P = \frac{1}{2}(3u)$	**M1**
	Solves	**A1**
	P: $1.5u$; Q: $3u$	**A1**
		(4)
(b)	Q reaches the wall after $\frac{d}{3u}$; P has moved $\frac{d}{2}$, so is $\frac{d}{2}$ from the wall	**M1**
	Q rebounds at $3fu$; they close at $3fu + 1.5u$, meeting after $t = \dfrac{d/2}{3fu + 1.5u}$	**M1**
	Distance of Q from the wall $= 3fut$	**M1**
	$= \dfrac{3fd}{6f+3} = \dfrac{fd}{2f+1}$ *	**A1***
	Jan 2025 report: a table of times and distances for each sphere is the most successful approach.	**(4)**
(c)	$\dfrac{f}{2f+1} = \dfrac{1}{4}$	**M1**
	$f = \frac{1}{2}$	**A1**
		(2)
		Total for Question 8: 10 marks

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Question 9

10 marks

Part	Scheme	Marks
(a)	$0.5\mathbf{v} = 0.5(4\mathbf{i}+3\mathbf{j}) + \lambda(\mathbf{i}-2\mathbf{j})$	M1
	$\boxed{\mathbf{v} = (4+2\lambda)\mathbf{i} + (3-4\lambda)\mathbf{j}}$	A1
		(2)
(b)	Speed before = 5: $(4+2\lambda)^2 + (3-4\lambda)^2 = 25$	M1
	$20\lambda^2 - 8\lambda = 0$	A1
	$\lambda(20\lambda - 8) = 0$	M1
	$\lambda \neq 0$, so $\lambda = 0.4$ *	A1*
		(4)
(c)	$0.4\sqrt{1^2+2^2} = \boxed{0.894\,\text{N s}}$	B1
		(1)
(d)	$\mathbf{v} = 4.8\mathbf{i} + 1.4\mathbf{j}$	B1
	$\cos\theta = \dfrac{(4)(4.8)+(3)(1.4)}{5\times 5}$ (or difference of two angles)	M1
	$\boxed{20.6^\circ}$	A1
		(3)
		Total for Question 9: 10 marks

Question 10

11 marks

Part	Scheme	Marks
(a)	Momentum: $m(4u) = mv_A + 3mv_B$	**M1**
	Impact law: $v_B - v_A = \frac{1}{2}(4u)$	**M1**
	Solves	**A1**
	$v_A = -\frac{1}{2}u$ (reversed), $v_B = \frac{3}{2}u$	**A1**
		(4)
(b)	Momentum: $3m\left(\frac{3}{2}u\right) = 3mw_B + kmw_C$	**M1**
	Impact law: $w_C - w_B = \frac{1}{2}\left(\frac{3}{2}u\right)$	**M1**
	$w_B = \dfrac{3(6-k)u}{4(k+3)}$	**A1**
	$w_C = \dfrac{27u}{4(k+3)}$	**A1**
		(4)
(c)	C always moves away from B; A moves at $\frac{1}{2}u$ away from B, so they collide again if B moves towards A faster: $w_B < -\frac{1}{2}u$	**M1**
	$3(6-k) < -2(k+3)$	**M1**
	$k > 24$	**A1**
		(3)
		Total for Question 10: 11 marks

6 Chapter 6: Statics of Rigid Bodies

Mark scheme. Marks are shown on the right of each line.

Question 1

6 marks

Part	Scheme	Marks
(a)	M(A): $P \times 2\sin 60^\circ = 50 \times 1\cos 60^\circ$	M1
	Correct	A1
	$\boxed{P = 14.4}$	A1
		(3)
(b)	Hinge: horizontal 14.4, vertical 50; magnitude $\sqrt{14.43^2 + 50^2}$	M1
	$\boxed{52.0\,\text{N}}$	A1
		(2)
(c)	The rod is acted on by more than two forces; the hinge force must balance both P and the weight, so it is not along the rod	B1
		(1)
		Total for Question 1: 6 marks

Question 2

6 marks

Part	Scheme	Marks
(a)	R($\rightarrow$): $T_A\cos 60^\circ = T_B\cos 30^\circ$	M1
	R($\uparrow$): $T_A\sin 60^\circ + T_B\sin 30^\circ = 60$	M1
	$T_A = \sqrt{3}T_B$, so $2T_B = 60$	A1
	$\boxed{T_B = 30\,\text{N},\ T_A = 30\sqrt{3} = 52.0\,\text{N}}$	A1
		(4)
(b)	M(A): $30\sin 30^\circ \times 4 = 60x$	M1
	$\boxed{1\,\text{m}}$	A1
		(2)
		Total for Question 2: 6 marks

Question 3

6 marks

Part	Scheme	Marks
(a)	The peg reaction is perpendicular to the rod; M(A): $R_C \times 4 = 100 \times 3\cos 30^\circ$	M1
	Correct	A1
	65.0 N	A1
		(3)
(b)	R($\uparrow$): $N + R_C \cos 30^\circ = 100$, so $N = 43.75$	M1
	R($\rightarrow$): $F = R_C \sin 30^\circ = 32.5$	M1
	$\mu = \dfrac{32.48}{43.75} = 0.74$	A1
		(3)
		Total for Question 3: 6 marks

Question 4

6 marks

Part	Scheme	Marks
(a)	$\sin\angle ABC = \frac{3}{5}$; M($A$): $T \times \frac{3}{5} \times 2 = 30 \times 1 + 10 \times 2$	M1
	Correct	A1
	41.7 N	A1
		(3)
(b)	R($\rightarrow$): $R = T \times \frac{4}{5} = 33.3$; R($\uparrow$): $F + T \times \frac{3}{5} = 40$, so $F = 15$ (upwards)	M1
	$F \leqslant \mu R$: $\mu \geqslant \dfrac{15}{33.3}$	M1
	0.45	A1
		(3)
		Total for Question 4: 6 marks

Question 5

6 marks

Part	Scheme	Marks
(a)	M(A): $S \times 4\sin 60^\circ = 20g \times 2\cos 60^\circ$	M1
	Correct	A1
	56.6 N	A1
	Summer 2023 report: the wall reaction is perpendicular to the wall, not to the ladder.	(3)
(b)	$R = 20g$ and $F = S$	M1
	$\mu \geqslant \dfrac{56.58}{196}$	M1
	0.29 (0.289)	A1
		(3)
		Total for Question 5: 6 marks

Question 6

10 marks

Part	Scheme	Marks
(a)	Forces: R, F (towards the wall) at A; S, μS (upwards) at B	B1
	R($\rightarrow$): $F = S$; R($\uparrow$): $R + \mu S = W$	M1
	Limiting: $F = \mu R$, so $S = \mu R$ and $R = \dfrac{W}{1+\mu^2}$, $S = \dfrac{\mu W}{1+\mu^2}$	A1
	M(A): $S \times 2a\sin\theta + \mu S \times 2a\cos\theta = W \times a\cos\theta$	M1
	$2\mu(\tan\theta + \mu) = 1 + \mu^2$	A1
	$\tan\theta = \dfrac{1-\mu^2}{2\mu}$ *	A1*
	Summer 2023 report: friction left out at one contact, or drawn in the wrong direction.	(6)
(b)	$\tan\theta = \dfrac{8/9}{2/3} = \frac{4}{3}$	M1
	53.1°	A1
		(2)
(c)	At 45° the ladder is less steep than the limiting angle, so it would need more friction than μR can provide	M1
	$\tan 45^\circ = 1 < \frac{4}{3}$: the ladder slips	A1
		(2)
		Total for Question 6: 10 marks

Question 7

10 marks

Part	Scheme	Marks
(a)	M(A): $T \times 3a = W \times 2a\cos 30° + 2W \times 4a\cos 30°$	**M1**
	All terms dimensionally correct	**A1**
	$3T = 10W\cos 30° = 5\sqrt{3}W$	**A1**
	$T = \dfrac{5\sqrt{3}}{3}W$ *	**A1***
	Jan 2025 report: the working to a given answer must be complete; resolve the forces or the distances.	**(4)**
(b)	R($\rightarrow$): $X = T\sin 30° = \frac{5\sqrt{3}}{6}W$	**M1**
	R($\uparrow$): $Y = 3W - T\cos 30° = \frac{1}{2}W$	**M1**
	Both correct	**A1**
	$\sqrt{\frac{75}{36} + \frac{1}{4}}W = \boxed{1.53W}$ $\left(\sqrt{\frac{7}{3}}W\right)$	**A1**
	Oct 2023 report: overlooking a component of the force at A.	**(4)**
(c)	Angle of the hinge force above the horizontal: $\tan^{-1}\dfrac{0.5}{1.443} = 19.1°$	**M1**
	Angle with the rod $30° - 19.1° = \boxed{10.9°}$	**A1**
	Jan 2025 report: the angle with the rod, not with the horizontal, is required.	**(2)**
		Total for Question 7: 10 marks

Question 8

10 marks

Part	Scheme	Marks
(a)	M(A): $T\sin 30^\circ \times 2a = W \times a + W \times x$	**M1**
	Correct	**A1**
	$T = \dfrac{W(a+x)}{a}$ *	**A1***
		(3)
(b)	Horizontal: $X = T\cos 30^\circ$	**M1**
	$\boxed{X = \dfrac{\sqrt{3}W(a+x)}{2a}}$ (away from the wall)	**A1**
	Vertical: $Y + T\sin 30^\circ = 2W$	**M1**
	$\boxed{Y = \dfrac{W(3a-x)}{2a}}$ (upwards)	**A1**
		(4)
(c)	$x \leqslant 2a < 3a$, so $Y > 0$: the hinge force always has a vertical component	**B1**
		(1)
(d)	45° to the horizontal rod: $Y = X$, so $3a - x = \sqrt{3}(a+x)$	**M1**
	$\boxed{x = \left(2\sqrt{3}-3\right)a}$	**A1**
		(2)
		Total for Question 8: 10 marks

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Question 9

10 marks

Part	Scheme	Marks
(a)	M(A): $T \times 2a = W \times a\cos 60^\circ$	**M1**
	Correct	**A1**
	$\boxed{T = \frac{1}{4}W}$	**A1**
		(3)
(b)	R($\rightarrow$): $F = T\sin 60^\circ = \frac{\sqrt{3}}{8}W$	**M1**
	R($\uparrow$): $R = W - T\cos 60^\circ = \frac{7}{8}W$	**M1**
	Limiting: $F = \mu R$	**M1**
	$\mu = \dfrac{\sqrt{3}/8}{7/8} = \dfrac{\sqrt{3}}{7}$ *	**A1***
		(4)
(c)	M(A): $2aT = Wa\cos 60^\circ + W \times 2a\cos 60^\circ$, $T = \frac{3}{4}W$	**M1**
	$F = \frac{3\sqrt{3}}{8}W$, $R = 2W - \frac{3}{8}W = \frac{13}{8}W$; needs $\frac{F}{R} = \frac{3\sqrt{3}}{13} = 0.400$	**M1**
	$0.400 > \frac{\sqrt{3}}{7} = 0.247$, so the rod $\boxed{\text{cannot remain in equilibrium}}$ (it slips)	**A1**
		(3)
		Total for Question 9: 10 marks

Question 10

11 marks

Part	Scheme	Marks
(a)	Limiting: $F = 0.25R$ with $R = 100g$, so $F = 25g$; R($\rightarrow$): $S = F = 25g$	**M1**
	M(A): $S \times 6\sin\alpha = 20g \times 3\cos\alpha + 80g \times x\cos\alpha$	**M1**
	Correct, with $\tan\alpha = 3$	**A1**
	$450 = 60 + 80x$	**A1**
	$\boxed{4.88\text{ m}}$ (4.875)	**A1**
		(5)
(b)	Man at the top ($x = 6$): M(A): $6S\sin\alpha = 60g\cos\alpha + 480g\cos\alpha$	**M1**
	$S = 30g$	**A1**
	R($\rightarrow$): $S = P + F$ with $F \leqslant 25g$ ($R = 100g$ unchanged)	**M1**
	$P \geqslant 30g - 25g$	**A1**
	$\boxed{49\text{ N}}$	**A1**
		(5)
(c)	e.g. the man's weight acts at one point, but his feet and hands are spread out, so the distance he can climb is approximate	**B1**
		(1)
		Total for Question 10: 11 marks