



eClassroom

INTERNATIONAL A LEVEL MATHEMATICS • PURE 3

Pure Mathematics 3

Exam Questions by Topic: Mark Scheme

Pearson Edexcel International Advanced Level (WMA13/01) • Calculator allowed

Every question in the booklet, marked line by line the way an examiner marks it

Chapters (tap a chapter to jump to it)	Qs	Marks	Page	My score
1 Algebraic Methods Algebraic fractions, improper fractions and algebraic division	10	69	3	
2 Functions and Graphs Modulus, composite and inverse functions, transformations	10	76	9	
3 Trigonometric Functions sec, cosec, cot, identities, inverse trigonometric functions	10	72	16	
4 Trigonometric Addition Formulae Addition and double-angle formulae, $R\cos(\theta \pm \alpha)$	10	75	22	
5 Exponentials and Logarithms e^x , $\ln x$, modelling, logarithmic graphs	10	78	28	
6 Differentiation Chain, product and quotient rules, trigonometric and exponential functions	10	76	35	
7 Integration Standard integrals, $f(ax+b)$, trigonometric identities, reverse chain rule	10	77	42	
8 Numerical Methods Locating roots, iteration	10	77	49	

[Question booklet](#)

[Free courses](#)

[More worksheets](#)



Join students already acing their exams.

Book your FREE lesson at www.eclassroom.org.uk

Page 1 of 55

How to use this mark scheme

The same marking rules Pearson examiners use for WMA13.

Mark every line, not just the answer. Method marks are given for a correct method even when the final answer is wrong. If you lost marks, find the *first* line where your working differs from the scheme. That is the thing to fix.

Code	Meaning
M1	Method mark: for knowing a correct method and attempting to apply it. Small slips in the working do not lose this mark.
dM1	Dependent method mark: can only be given if the method mark before it has been given.
A1	Accuracy mark: for a correct answer or line of working. It can only be given if the method mark it depends on has been given.
B1	Independent mark: for a correct result or statement, whatever method was used.
*	The answer is given in the question. Every step must be shown, and the answer must follow from correct working (cso).
cao	Correct answer only.
cso	Correct solution only: no errors anywhere in the working.
ft	Follow through: the mark can be given for correct work using your own earlier (wrong) answer.
oe	Or equivalent: any correct equivalent form is accepted.

“Solutions relying on calculator technology are not acceptable.” When a question says this, a correct final answer with no algebraic working scores **no marks**. The notes under each scheme line say when this applies.



1

Chapter 1: Algebraic Methods

Mark scheme. Marks are shown on the right of each line.

Question 1

4 marks

Part	Scheme	Marks
	Factorises: $2x^2 + x - 15 = (2x - 5)(x + 3)$ and $3x^2 - 27 = 3(x - 3)(x + 3)$	M1
	Cancels $(x + 3)$: $\frac{3(x - 3)}{2x - 5}$	A1
	Adds with the common denominator $2x - 5$: $\frac{3x - 9 + 4}{2x - 5}$	M1
	$= \frac{3x - 5}{2x - 5}$ *	A1*
	<i>Losing the factor 3 in $3(x - 3)(x + 3)$ is the common error. Every step must be shown for this printed answer.</i>	(4)
Total for Question 1: 4 marks		

Question 2

5 marks

Part	Scheme	Marks
(a)	Cancels the common factor x : $\frac{2x^3 + 7x^2 + 2x + 2}{x + 3}$ Divides by $(x + 3)$: quotient $2x^2 + x - 1$ Remainder 5 $A = 2, B = 1, C = -1, D = 5$, i.e. $f(x) = 2x^2 + x - 1 + \frac{5}{x + 3}$ <i>Dividing the quartic by $x^2 + 3x$ directly gives remainder $5x$, and $\frac{5x}{x(x+3)} = \frac{5}{x+3}$: also full marks.</i>	M1 M1 A1 A1 (4)
(b)	$-1 + \frac{5}{x + 3} = 0 \Rightarrow x = 2$	B1 (1)
Total for Question 2: 5 marks		



Question 3

6 marks

Part	Scheme	Marks
(a)	$x^2 + x - 2 = (x+2)(x-1)$ is the common denominator $\frac{3(x-1) + 2(x+2) - (7x-1)}{(x+2)(x-1)}$ $= \frac{-2x+2}{(x+2)(x-1)} = \frac{-2(x-1)}{(x+2)(x-1)}$ $\frac{-2}{x+2}, \text{ so } a = -2, b = 2$	B1 M1 A1 A1 (4)
(b)	$-2 = (x+1)(x+2) \Rightarrow x^2 + 3x + 4 = 0$ Discriminant $9 - 16 = -7 < 0$, so no real solutions *	M1 A1* (2)
Total for Question 3: 6 marks		

Question 4

5 marks

Part	Scheme	Marks
(a)	Divides by $(2x+3)$: quotient $3x + \dots$ Quotient $3x - 2$, remainder 2 $f(x) = 3x - 2 + \frac{2}{2x+3}$ <i>The final mark needs the expression written out in full.</i>	M1 A1 A1 (3)
(b)	$\frac{2}{2x+3} > 0$ $x > -\frac{3}{2}$	M1 A1 (2)
Total for Question 4: 5 marks		



Question 5

5 marks

Part	Scheme	Marks
	$x^2 - x - 2 = (x - 2)(x + 1)$; multiplies through by it: $3x(x + 1) - 4(x - 2) = 12$ $3x^2 - x - 4 = 0$ $(3x - 4)(x + 1) = 0$ $x = \frac{4}{3}$ or $x = -1$ $x = -1$ makes the denominators $x + 1$ and $x^2 - x - 2$ zero, so it is rejected: $x = \frac{4}{3}$	M1 A1 M1 A1 A1 (5)
Total for Question 5: 5 marks		

Question 6

9 marks

Part	Scheme	Marks
(a)	$1 + 3 - 4$ $= 0$, so $(x - 1)$ is a factor *	M1 A1* (2)
(b)	$x^3 + 3x^2 - 4 = (x - 1)(x^2 + 4x + 4)$ $= (x - 1)(x + 2)^2$ $x^2 + x - 2 = (x - 1)(x + 2)$ $f(x) = \frac{(x - 1)(x + 2)^2}{(x - 1)(x + 2)} = x + 2$ *	M1 A1 B1 A1* (4)
(c)	$y = x + 2$ is a straight line, but f is not defined at $x = 1$ and $x = -2$ Substitutes into $y = x + 2$ $(1, 3)$ and $(-2, 0)$	B1 M1 A1 (3)
Total for Question 6: 9 marks		



Question 7

8 marks

Part	Scheme	Marks
(a)	Divides $x^4 + 2x^3 + 0x^2 + 7x + 7$ by $x^2 + 1$ Quotient $x^2 + 2x - 1$ Remainder $5x + 8$ $B = 2, C = -1, D = 5, E = 8$ $g(x) \equiv x^2 + 2x - 1 + \frac{5x+8}{x^2+1}$ Comparing coefficients in $x^4 + 2x^3 + 7x + 7 \equiv (x^2 + Bx + C)(x^2 + 1) + Dx + E$ is equally valid. A zero x^2 term in the numerator must be allowed for in long division.	M1 A1 A1 A1 A1 (5)
(b)	$x^2 + 1 > 0$ for all x, so the inequality sign is unchanged: $5x + 8 > x^2 + 1$ $x^2 - 5x - 7 < 0$, critical values $\frac{5 \pm \sqrt{53}}{2}$ $\frac{5 - \sqrt{53}}{2} < x < \frac{5 + \sqrt{53}}{2}$	M1 M1 A1 (3)
Total for Question 7: 8 marks		

Question 8

8 marks

Part	Scheme	Marks
(a)	Multiplies by the reciprocal of the second fraction Factorises: $(x-2)(x+2)$, $(x+1)(x+2)$, $(x-2)(x-3)$, $(x-3)(x+1)$ All four correct $\frac{(x-2)(x+2)}{(x+1)(x+2)} \times \frac{(x-3)(x+1)}{(x-2)(x-3)} = 1$ *	M1 M1 A1 A1* (4)
(b)	Denominators zero: $x = -1, -2$ and $x = 3$; the divisor is zero at $x = 2$ and $x = 3$ $x = -2, -1, 2, 3$	M1 A1 (2)
(c)	$1 = x^2 - 3 \Rightarrow x = \pm 2$ Both values are excluded (part (b)), so there are no solutions *	M1 A1* (2)
Total for Question 8: 8 marks		



Question 9

9 marks

Part	Scheme	Marks
(a)	$\frac{(r+1) - r}{r(r+1)}$ $= \frac{1}{r(r+1)} *$	M1 A1* (2)
(b)	Writes the sum as $(1 - \frac{1}{2}) + (\frac{1}{2} - \frac{1}{3}) + \dots + (\frac{1}{n} - \frac{1}{n+1})$ At least the first two and the last term shown Middle terms cancel, leaving $1 - \frac{1}{n+1}$ $= \frac{n+1-1}{n+1} = \frac{n}{n+1} *$	M1 A1 M1 A1* (4)
(c)	$\frac{n}{n+1} > 0.99 \Rightarrow n > 0.99n + 0.99$ $0.01n > 0.99, \text{ so } n > 99$ $n = 100$ <i>Multiplying by $n+1$ is valid because $n+1 > 0$.</i>	M1 A1 A1 (3)
Total for Question 9: 9 marks		



Question 10

10 marks

Part	Scheme	Marks
(a)	$2x^2 + kx - 3 \equiv (2x + 1)(x + 2) + C$ $(2x + 1)(x + 2) = 2x^2 + 5x + 2$ $k = 5$ $C = -5$	M1 A1 A1 A1 (4)
(b)	$2x + 1 - \frac{5}{x+2} = 2x - 1 \Rightarrow \frac{5}{x+2} = 2$ $x = \frac{1}{2}$	M1 A1 (2)
(c)	$f(x) - (2x + 1) = -\frac{5}{x+2} < 0$ because $x + 2 > 0$ for $x > -2$	B1 (1)
(d)	$1 - \frac{5}{x+2} > 0$ Multiplies by $(x + 2)^2$ (positive): $(x + 2)(x - 3) > 0$, critical values -2 and 3 $x < -2$ or $x > 3$ <i>Multiplying by $(x + 2)$ alone loses the region $x < -2$, where $x + 2$ is negative.</i>	M1 M1 A1 (3)
Total for Question 10: 10 marks		



2

Chapter 2: Functions and Graphs

Mark scheme. Marks are shown on the right of each line.

Question 1

6 marks

Part	Scheme	Marks
(a)	Shape: the part below the x -axis (for $x < -4$ and $1 < x < 5$) reflected above it; cusps at $x = -4, 1$ and 5	B1
	Maximum points $(-2, 6)$ and $(3, 4)$	B1
	Meets the axes at $(-4, 0)$, $(1, 0)$, $(5, 0)$ and $(0, 3)$	B1
		(3)
(b)	Shape: the part for $x \geq 0$ and its reflection in the y -axis (symmetrical W shape with a sharp point at $(0, 3)$)	B1
	Minimum points $(3, -4)$ and $(-3, -4)$	B1
	Crosses the x -axis at $(\pm 1, 0)$ and $(\pm 5, 0)$, and meets the y -axis at $(0, 3)$	B1
	<i>The maximum at $(-2, 6)$ is not on $y = f(x)$: only the curve for $x \geq 0$ is used.</i>	
		(3)
Total for Question 1: 6 marks		

Question 2

5 marks

Part	Scheme	Marks
(a)	$3x - 5 = x + 7 \Rightarrow x = 6$	B1
	$-(3x - 5) = x + 7$	M1
	$x = 6$ and $x = -\frac{1}{2}$	A1
		(3)
(b)	Chooses the region between the solutions (the line is above the V)	M1
	$-\frac{1}{2} < x < 6$	A1
		(2)
Total for Question 2: 5 marks		



Question 3

5 marks

Part	Scheme	Marks
(a)	V shape with vertex $\left(-\frac{a}{3}, a\right)$ (above the x -axis)	B1
	Meets the y -axis at $(0, 2a)$	B1
		(2)
(b)	$ 3x + a = 4a$	M1
	$3x + a = 4a$ or $3x + a = -4a$	M1
	$x = a, x = -\frac{5a}{3}$	A1
		(3)
Total for Question 3: 5 marks		

Question 4

5 marks

Part	Scheme	Marks
(a)	$(3, 2)$	B1
		(1)
(b)	$(3, -3)$	B1
	and its reflection $(-3, -3)$	B1
		(2)
(c)	$(2, -8)$	B1
		(1)
(d)	$\left(\frac{3}{2}, 4\right)$	B1
		(1)
Total for Question 4: 5 marks		



Question 5

5 marks

Part	Scheme	Marks
(a)	$(2x+1)^2 - 4(2x+1)$	M1
	$4x^2 - 4x - 3$	A1
		(2)
(b)	gf(x) = $2x^2 - 8x + 1$ and sets equal: $4x^2 - 4x - 3 = 2x^2 - 8x + 1$	M1
	$x^2 + 2x - 2 = 0$	A1
	$x = -1 \pm \sqrt{3}$	A1
		(3)
Total for Question 5: 5 marks		

Question 6

10 marks

Part	Scheme	Marks
(a)	$f(x) = 2 + \frac{1}{x+1}$: decreasing, $f(0) = 3$, and $f(x) \rightarrow 2$	M1
	$2 < f(x) \leq 3$	A1
	Both ends with the correct inequality signs are needed for the A mark.	(2)
(b)	$y(x+1) = 2x+3 \Rightarrow x(y-2) = 3-y$	M1
	Correct rearrangement	A1
	$f^{-1}(x) = \frac{3-x}{x-2}$	A1
	Domain $2 < x \leq 3$	B1ft
		(4)
(c)	$g(-2) = 3$, so $fg(-2) = f(3) = \frac{9}{4}$	B1
		(1)
(d)	$gf(x) = 3 \Rightarrow (f(x))^2 - 1 = 3$	M1
	$f(x) = \pm 2$	A1
	Neither value lies in the range $2 < f(x) \leq 3$, so there are no solutions *	A1*
		(3)
Total for Question 6: 10 marks		



Question 7

10 marks

Part	Scheme	Marks
(a)	(i) $P\left(\frac{b}{2}, a\right)$ (ii) $Q(0, a - b)$ (iii) $a - 2x - b = 0: 2x - b = \pm a$ $\left(\frac{b-a}{2}, 0\right)$ and $\left(\frac{a+b}{2}, 0\right)$	B1 B1 M1 A1 (4)
(b)	At $x = -2$: $y = -2 - 1 = 1$, and $2x - b < 0$, so $1 = a + (-4 - b)$ $a - b = 5$ At $x = 4$: $y = 3$, and $2x - b > 0$ (as $b < 8$), so $3 = a - (8 - b)$ $a + b = 11$ $a = 8, b = 3$ <i>Using the wrong branch of the modulus at either point is the common error: the sign of $2x - b$ must be decided.</i>	M1 A1 M1 A1 A1 (5)
(c)	$k < 8$	B1ft (1)
Total for Question 7: 10 marks		



Question 8

10 marks

Part	Scheme	Marks
(a)	W shape symmetrical in the y -axis Crosses the x -axis at $(\pm 1, 0)$ and $(\pm 5, 0)$ and the y -axis at $(0, 5)$ Minimum points $(\pm 3, -4)$	B1 B1 B1 (3)
(b)	Uses the minimum value -4 and the y intercept 5 $-4 < k < 5$	M1 A1 (2)
(c)	$k = 5$	B1 (1)
(d)	The part between $x = 1$ and $x = 5$ reflected above the x -axis Cusps at $(1, 0)$ and $(5, 0)$, maximum $(3, 4)$ and y intercept $(0, 5)$	B1 B1 (2)
(e)	$x^2 - 6x + 5 = \pm 3$ $x = 3 \pm \sqrt{7}, x = 2, x = 4$	M1 A1 (2)
Total for Question 8: 10 marks		



Question 9

10 marks

Part	Scheme	Marks
(a)	V shape with vertex $(-\frac{3}{2}, 0)$ and y intercept $(0, 3)$	B1
	Parabola $y = x^2$ through the origin	B1
	Two points of intersection, both for $x > -\frac{3}{2}$	B1
		(3)
(b)	$x^2 = 2x + 3 \Rightarrow (x - 3)(x + 1) = 0$	M1
	$x^2 = -(2x + 3)$ has no real roots (discriminant $4 - 12 < 0$)	M1
	$(-1, 1)$ and $(3, 9)$	A1
		(3)
(c)	Parabola below the V between the intersections	M1
	$-1 < x < 3$	A1
		(2)
(d)	$2x + 3 = 3x - 1 \Rightarrow x = 4$; and $-(2x + 3) = 3x - 1 \Rightarrow x = -\frac{2}{5}$	M1
	$x = -\frac{2}{5}$ gives $3x - 1 < 0$, which cannot equal a modulus, so $x = 4$ only	A1
		(2)
Total for Question 9: 10 marks		



Question 10

10 marks

Part	Scheme	Marks
(a)	$f(0) = 2$ and $f(x) \rightarrow 0$ as $x \rightarrow \infty$ (f decreasing) $0 < f(x) \leq 2$	M1 A1 (2)
(b)	$gf(x) = \left(\frac{4}{x+2}\right)^2 - 2\left(\frac{4}{x+2}\right) = \frac{16 - 8(x+2)}{(x+2)^2}$ $= \frac{-8x}{(x+2)^2}$ *	M1 A1* (2)
(c)	$gf(x) = g(t)$ with $t = f(x)$ and $0 < t \leq 2$ $g(t) = (t - 1)^2 - 1$: minimum -1 at $t = 1$; $g(2) = 0$ $-1 \leq gf(x) \leq 0$ <i>Using the range of g over all of $\mathbb{R}$ ($g \geq -1$ only) scores M0: the inputs to g are restricted to $0 < t \leq 2$.</i>	M1 M1 A1 (3)
(d)	$t^2 - 2t + \frac{3}{4} = 0 \Rightarrow t = \frac{1}{2} \text{ or } t = \frac{3}{2}$ $\frac{4}{x+2} = \frac{1}{2} \text{ or } \frac{3}{2}$ $x = 6, x = \frac{2}{3}$	M1 M1 A1 (3)
Total for Question 10: 10 marks		



3

Chapter 3: Trigonometric Functions

Mark scheme. Marks are shown on the right of each line.

Question 1

5 marks

Part	Scheme	Marks
(a)	$\tan^2 \theta = \sec^2 \theta - 1 = 8$ $\tan \theta = \pm 2\sqrt{2}$ Obtuse, so $\tan \theta < 0$: $-2\sqrt{2}$	M1 A1 A1 (3)
(b)	$\cos \theta = -\frac{1}{3}$, so $\sin \theta = \frac{2\sqrt{2}}{3}$ (positive for an obtuse angle) $\operatorname{cosec} \theta = \frac{3}{2\sqrt{2}} = \frac{3}{4}\sqrt{2}$	M1 A1 (2)
Total for Question 1: 5 marks		

Question 2

6 marks

Part	Scheme	Marks
	Uses $\operatorname{cosec}^2 x = 1 + \cot^2 x$: $3\cot^2 x - 4\cot x - 1 = 0$ Correct quadratic $\cot x = \frac{4 \pm \sqrt{28}}{6} = \frac{2 \pm \sqrt{7}}{3}$ $\tan x = 0.6458 \dots$ or $\tan x = -4.645 \dots$ Two correct solutions $x = 0.573, 1.78, 3.71, 4.92$ <i>Working in degrees, or giving only the principal values, loses the last two marks.</i>	M1 A1 M1 A1 A1 A1 (6)
Total for Question 2: 6 marks		



Question 3

5 marks

Part	Scheme	Marks
(a)	U shape between $x = 0$ and $x = \frac{\pi}{2}$, $\cap$ shape between $\frac{\pi}{2}$ and $\frac{3\pi}{2}$, U shape after Asymptotes $x = \frac{\pi}{2}$ and $x = \frac{3\pi}{2}$ Turning points $(0, 1)$, $(\pi, -1)$, $(2\pi, 1)$	B1 B1 B1 (3)
(b)	$\sec x \geq 1$ gives $f(x) \leq 1$; $\sec x \leq -1$ gives $f(x) \geq 5$ $f(x) \leq 1$ or $f(x) \geq 5$	M1 A1 (2)
Total for Question 3: 5 marks		

Question 4

5 marks

Part	Scheme	Marks
(a)	$\frac{1}{\cos x} - \cos x = \frac{1 - \cos^2 x}{\cos x}$ $= \frac{\sin^2 x}{\cos x}$ $= \sin x \times \frac{\sin x}{\cos x} = \sin x \tan x$ *	M1 A1 A1* (3)
(b)	Similarly $\operatorname{cosec} x - \sin x = \frac{\cos^2 x}{\sin x}$ $\frac{\cos^2 x}{\sin x} \times \frac{\sin^2 x}{\cos x} = \sin x \cos x$ *	M1 A1* (2)
Total for Question 4: 5 marks		



Question 5

5 marks

Part	Scheme	Marks
(a)	Decreasing curve from $(-1, \pi)$ to $(1, 0)$ Crosses the y -axis at $(0, \frac{\pi}{2})$	B1 B1 (2)
(b)	$2x - 1 = \frac{1}{2} \therefore x = \frac{3}{4}$	B1 (1)
(c)	Let $\theta = \arccos x$, so $\cos \theta = x$ with $0 \leq \theta \leq \pi$; $\sin^2 \theta = 1 - x^2$ $\sin \theta \geq 0$ on $[0, \pi]$, so $\sin(\arccos x) = \sqrt{1 - x^2}$ *	M1 A1* (2)
Total for Question 5: 5 marks		

Question 6

9 marks

Part	Scheme	Marks
(a)	$\tan^2 \theta = \sec^2 \theta - 1 = (\sec \theta - 1)(\sec \theta + 1)$ Uses the identity and factorises correctly Cancels $(\sec \theta - 1)$: $\sec \theta + 1$ *	M1 A1 A1* (3)
(b)	$\sec 2x + 1 = 3 \Rightarrow \sec 2x = 2$ $\cos 2x = \frac{1}{2}$ $2x = \frac{\pi}{3}, \frac{5\pi}{3}$ (in $0 < 2x < 2\pi$) $x = \frac{\pi}{6}, \frac{5\pi}{6}$	M1 A1 M1 A1 (4)
(c)	$\sec \theta + 1 = 2$ gives $\sec \theta = 1$ but $\sec \theta \neq 1$ (the denominator $\sec \theta - 1$ would be zero), so there are no solutions	M1 A1 (2)
Total for Question 6: 9 marks		



Question 7

9 marks

Part	Scheme	Marks
(a)	$3(1 + \tan^2 x) - 5 \tan x - 5 = 0$ $3 \tan^2 x - 5 \tan x - 2 = 0$ *	M1 A1* (2)
(b)	$(3 \tan x + 1)(\tan x - 2) = 0$ $\tan x = 2: 63.4^\circ, 243.4^\circ$ $\tan x = -\frac{1}{3}: 161.6^\circ, 341.6^\circ$ $63.4^\circ, 161.6^\circ, 243.4^\circ, 341.6^\circ$	M1 A1 A1 A1 (4)
(c)	$\theta > 0$ means $2\theta - 30^\circ > -30^\circ$, so $2\theta - 30^\circ = -18.43^\circ$ is also a solution Smallest solutions: $\theta = 5.8^\circ$, then $\theta = \frac{63.43^\circ + 30^\circ}{2}$ 46.7° <i>Missing the negative value of $2\theta - 30^\circ$ gives 95.8° as the "second smallest" and scores M0 M1 A0.</i>	M1 M1 A1 (3)
Total for Question 7: 9 marks		



Question 8

9 marks

Part	Scheme	Marks
(a)	Increasing curve through the origin Asymptotes $y = \pm \frac{\pi}{2}$	B1 B1 (2)
(b)	$-\frac{\pi}{2} < \arctan 2x < \frac{\pi}{2}$, so adds $\frac{\pi}{4}$ $-\frac{\pi}{4} < h(x) < \frac{3\pi}{4}$	M1 A1 (2)
(c)	$x = \frac{\pi}{4} + \arctan 2y \Rightarrow 2y = \tan\left(x - \frac{\pi}{4}\right)$ $h^{-1}(x) = \frac{1}{2} \tan\left(x - \frac{\pi}{4}\right)$ Domain $-\frac{\pi}{4} < x < \frac{3\pi}{4}$	M1 A1 B1ft (3)
(d)	$\arctan 2x = \frac{\pi}{4}$, so $2x = 1$ $x = \frac{1}{2}$	M1 A1 (2)
Total for Question 8: 9 marks		



Question 9

9 marks

Part	Scheme	Marks
(a)	$\sin y = x$ with $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$, and $\cos\left(\frac{\pi}{2} - y\right) = \sin y = x$ $0 \leq \frac{\pi}{2} - y \leq \pi$, which is the range of $\arccos$ So $\arccos x = \frac{\pi}{2} - y$, giving $\arcsin x + \arccos x = \frac{\pi}{2}$ *	M1 B1 A1* (3)
(b)	$\arcsin x = 2\left(\frac{\pi}{2} - \arcsin x\right)$ $3 \arcsin x = \pi$, so $\arcsin x = \frac{\pi}{3}$ $x = \frac{\sqrt{3}}{2}$	M1 A1 A1 (3)
(c)	$\theta = \arccos \frac{1}{3}$: right-angled triangle with adjacent 1 and hypotenuse 3 Opposite = $\sqrt{9 - 1} = 2\sqrt{2}$ $2\sqrt{2}$	M1 A1 A1 (3)
Total for Question 9: 9 marks		

Question 10

10 marks

Part	Scheme	Marks
(a)	U shape for $0 < x < \pi$ with minimum $\left(\frac{\pi}{2}, 3\right)$; $\cap$ shape for $\pi < x < 2\pi$ with maximum $\left(\frac{3\pi}{2}, 1\right)$ Asymptotes $x = 0, x = \pi, x = 2\pi$ Range $f(x) \leq 1$ or $f(x) \geq 3$	B1 B1 B1 (3)
(b)	$\operatorname{cosec} x = 3$, so $\sin x = \frac{1}{3}$ $x = 0.340$ $x = 0.340, 2.80$	M1 A1 A1 (3)
(c)	Uses the gap in the range between 1 and 3 $1 < k < 3$	M1 A1 (2)
(d)	$f(x) = 2$ is impossible ($\operatorname{cosec} x \neq 0$); $f(x) = -2$ gives $\sin x = -\frac{1}{4}$ 2 solutions, both in $\pi < x < 2\pi$	M1 A1 (2)
Total for Question 10: 10 marks		



4

Chapter 4: Trigonometric Addition Formulae

Mark scheme. Marks are shown on the right of each line.

Question 1

5 marks

Part	Scheme	Marks
(a)	$\sin(45^\circ + 30^\circ) = \frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \times \frac{1}{2}$ $= \frac{\sqrt{6} + \sqrt{2}}{4} *$	M1 A1* (2)
(b)	$\sin A = \frac{3}{5}, \cos A = \frac{4}{5}, \sin B = \frac{5}{13}, \cos B = \frac{12}{13}$ $\sin(A - B) = \frac{3}{5} \times \frac{12}{13} - \frac{4}{5} \times \frac{5}{13}$ $\frac{16}{65}$	B1 M1 A1 (3)
Total for Question 1: 5 marks		

Question 2

5 marks

Part	Scheme	Marks
	$2(1 - 2\sin^2 x) + 5\sin x = 3$ $4\sin^2 x - 5\sin x + 1 = 0$ $(4\sin x - 1)(\sin x - 1) = 0$ $\sin x = 1: x = 90^\circ$ $14.5^\circ, 90^\circ, 165.5^\circ$	M1 A1 M1 A1 A1 (5)
Total for Question 2: 5 marks		



Question 3

5 marks

Part	Scheme	Marks
(a)	$R = \sqrt{5^2 + 12^2} = 13$ $\tan \alpha = \frac{12}{5}$ $13 \sin(x - 67.38^\circ)$	B1 M1 A1 (3)
(b)	Maximum 13 $x - 67.38^\circ = 90^\circ$: $x = 157.38^\circ$	B1ft B1ft (2)
Total for Question 3: 5 marks		

Question 4

5 marks

Part	Scheme	Marks
(a)	$\frac{1 - (1 - 2 \sin^2 \theta)}{2 \sin \theta \cos \theta}$ $= \frac{2 \sin^2 \theta}{2 \sin \theta \cos \theta}$ $= \frac{\sin \theta}{\cos \theta} = \tan \theta \quad *$	M1 A1 A1* (3)
(b)	$\theta = 15^\circ$: $\frac{1 - \frac{\sqrt{3}}{2}}{\frac{1}{2}}$ $2 - \sqrt{3}$	M1 A1 (2)
Total for Question 4: 5 marks		

Question 5

5 marks

Part	Scheme	Marks
	$\frac{\tan x - 1}{1 + \tan x} = 6 \tan x$ (using $\tan \frac{\pi}{4} = 1$) $6 \tan^2 x + 5 \tan x + 1 = 0$ $(3 \tan x + 1)(2 \tan x + 1) = 0$ $\tan x = -\frac{1}{3}$ or $-\frac{1}{2}$ $x = 2.68, 2.82$ <i>Both solutions lie in the second quadrant, as $\tan x < 0$ and $0 \leq x < \pi$.</i>	M1 A1 M1 A1 A1 (5)
Total for Question 5: 5 marks		



Question 6

10 marks

Part	Scheme	Marks
(a)	$R = \sqrt{13}$ $R \cos \alpha = 2, R \sin \alpha = 3$, so $\tan \alpha = \frac{3}{2}$ $\sqrt{13} \cos(x + 0.983)$	B1 M1 A1 (3)
(b)	(i) Maximum when $f(x) = -\sqrt{13}$: $\frac{12}{10 - \sqrt{13}}$ $\frac{12}{10 - \sqrt{13}} \left(= \frac{4(10 + \sqrt{13})}{29} \right)$ (ii) $x + 0.983 = \pi$ $x = 2.159$ <i>Using the maximum of f (giving $\frac{12}{10 + \sqrt{13}}$, the minimum of g) scores M0.</i>	M1 A1 M1 A1 (4)
(c)	$\cos(x + 0.983) = \frac{1}{\sqrt{13}}$, with $0.983 < x + 0.983 < 2\pi + 0.983$ $x + 0.983 = 1.290$ or $2\pi - 1.290$ $x = 0.307, 4.011$	M1 M1 A1 (3)
Total for Question 6: 10 marks		



Question 7

10 marks

Part	Scheme	Marks
(a)	$\cos(2x+x) = \cos 2x \cos x - \sin 2x \sin x$ $= (2\cos^2 x - 1) \cos x - 2\sin^2 x \cos x$ $= 2\cos^3 x - \cos x - 2(1 - \cos^2 x) \cos x$ $= 4\cos^3 x - 3\cos x$ *	M1 M1 A1 A1* (4)
(b)	$4\cos^3 x - \cos x = 0 \Rightarrow \cos x (4\cos^2 x - 1) = 0$ $\cos x = 0$ or $\cos x = \pm \frac{1}{2}$ Two correct solutions $x = \frac{\pi}{3}, \frac{\pi}{2}, \frac{2\pi}{3}$ <i>Dividing by $\cos x$ loses $x = \frac{\pi}{2}$.</i>	M1 A1 A1 A1 (4)
(c)	$4\left(\frac{1}{3}\right)^3 - 3 \times \frac{1}{3}$ $\frac{23}{27}$	M1 A1 (2)
Total for Question 7: 10 marks		



Question 8

10 marks

Part	Scheme	Marks
(a)	$R = 5$ $\tan \alpha = \frac{4}{3}$ $5 \sin(\theta - 0.9273)$	B1 M1 A1 (3)
(b)	Maximum depth $10 + 5 = 15 \text{ m}$ $\frac{\pi t}{6} - 0.9273 = \frac{\pi}{2} \Rightarrow t = 4.771$ $04:46$	B1 M1 A1 (3)
(c)	$5 \sin\left(\frac{\pi t}{6} - 0.9273\right) = 2$, so $\sin(\dots) = 0.4$ $\frac{\pi t}{6} - 0.9273 = 0.4115$ or $\pi - 0.4115$ $t = 2.557$ or $t = 6.985$ $02:33$ and $06:59$	M1 M1 A1 A1 (4)
Total for Question 8: 10 marks		

Question 9

10 marks

Part	Scheme	Marks
(a)	$\frac{1}{\sin 2x} + \frac{\cos 2x}{\sin 2x} = \frac{1 + \cos 2x}{\sin 2x}$ $1 + \cos 2x = 2 \cos^2 x$ and $\sin 2x = 2 \sin x \cos x$ $\frac{2 \cos^2 x}{2 \sin x \cos x}$ $= \frac{\cos x}{\sin x} = \cot x$ *	M1 M1 A1 A1* (4)
(b)	With $x = 2\theta$: $\cot 2\theta = \sqrt{3}$, so $\tan 2\theta = \frac{1}{\sqrt{3}}$ $0 < 2\theta < \pi$ gives $2\theta = \frac{\pi}{6}$ only $\theta = \frac{\pi}{12}$	M1 M1 A1 (3)
(c)	$x = \frac{\pi}{8}$: $\cot \frac{\pi}{8} = \operatorname{cosec} \frac{\pi}{4} + \cot \frac{\pi}{4}$ $= \sqrt{2} + 1$ $1 + \sqrt{2}$, i.e. $a = 1$, $b = 2$	M1 A1 A1 (3)
Total for Question 9: 10 marks		



Question 10

10 marks

Part	Scheme	Marks
(a)	$R = 2$ $\tan \alpha = \frac{1}{\sqrt{3}}$ $2 \cos \left(x - \frac{\pi}{6} \right)$	B1 M1 A1 (3)
(b)	$2 \cos \left(2x - \frac{\pi}{6} \right) = -1$, so $\cos \left(2x - \frac{\pi}{6} \right) = -\frac{1}{2}$ $-\frac{\pi}{6} \leq 2x - \frac{\pi}{6} \leq \frac{11\pi}{6}$: $2x - \frac{\pi}{6} = \frac{2\pi}{3}$ or $\frac{4\pi}{3}$ One correct solution $x = \frac{5\pi}{12}, \frac{3\pi}{4}$	M1 M1 A1 A1 (4)
(c)	$2 \cos \left(x - \frac{\pi}{6} \right)$ takes every value between -2 and 2 twice over one period $k = \pm 2$ gives only one solution (a maximum or minimum), $ k > 2$ gives none $-2 < k < 2$	M1 M1 A1 (3)
Total for Question 10: 10 marks		



5

Chapter 5: Exponentials and Logarithms

Mark scheme. Marks are shown on the right of each line.

Question 1

5 marks

Part	Scheme	Marks
(a)	$e^{2x-1} = 4 \Rightarrow 2x - 1 = \ln 4$	M1
	$x = \frac{1}{2} + \ln 2$	A1
		(2)
(b)	$\ln \frac{2x+3}{x} = 1$	M1
	$2x+3 = ex$	M1
	$x = \frac{3}{e-2}$	A1
		(3)
Total for Question 1: 5 marks		

Question 2

5 marks

Part	Scheme	Marks
	Multiplies by e^x : $2e^{2x} - 7e^x + 3 = 0$	M1
	Correct quadratic in e^x	A1
	$(2e^x - 1)(e^x - 3) = 0$	M1
	$e^x = \frac{1}{2}$ or $e^x = 3$	A1
	$x = -\ln 2, x = \ln 3$	A1
		(5)
Total for Question 2: 5 marks		



Question 3

5 marks

Part	Scheme	Marks
(a)	Increasing curve approaching $y = 3$ from below	B1
	Asymptote $y = 3$ and y intercept $(0, 1)$	B1
	x intercept $(\ln \frac{2}{3}, 0)$	B1
		(3)
(b)	$f(0) = 1$ and $f(x) \rightarrow 3$	M1
	$1 \leq f(x) < 3$	A1
		(2)
Total for Question 3: 5 marks		

Question 4

5 marks

Part	Scheme	Marks
(a)	Gradient $\frac{1.8 - 0.6}{2} = 0.6$	M1
	$\log_{10} y = 0.6 \log_{10} x + 0.6$	A1
		(2)
(b)	$y = 10^{0.6} x^{0.6}$	M1
	$n = 0.6$	A1
	$y = 3.98x^{0.6}$	A1
	The answer must be written as an equation in the form $y = ax^n$ for the final mark.	
		(3)
Total for Question 4: 5 marks		



Question 5

6 marks

Part	Scheme	Marks
(a)	$\log_{10} y = \log_{10} 10^4 - 2 \log_{10} x = 4 - 2 \log_{10} x$ Straight line with negative gradient Meets the axes at $(0, 4)$ and $(2, 0)$	M1 B1 A1 (3)
(b)	Gradient $\frac{1}{2}$: $\log_3 N = \frac{1}{2}t + 2$ $N = 3^{\frac{1}{2}t+2} = 3^2 \left(3^{\frac{1}{2}}\right)^t$ $N = 9(\sqrt{3})^t$, so $a = 9$, $b = \sqrt{3}$ <i>Treating the base as 10 (e.g. $a = 100$) is a misread: maximum M1 M1 A0.</i>	M1 M1 A1 (3)
Total for Question 5: 6 marks		

Question 6

10 marks

Part	Scheme	Marks
(a)	$20 + A = 90$: $A = 70$	B1 (1)
(b)	$55 = 20 + 70e^{-5k}$ $e^{-5k} = \frac{1}{2}$ $-5k = \ln \frac{1}{2}$, so $k = \frac{1}{5} \ln 2$ *	M1 A1 A1* (3)
(c)	$70e^{-kt} = 10$, so $e^{-kt} = \frac{1}{7}$ $t = \frac{\ln 7}{k} = \frac{5 \ln 7}{\ln 2}$ 14.0 minutes	M1 M1 A1 (3)
(d)	$\frac{d\theta}{dt} = -70ke^{-kt}$ At $t = 5$: $-70 \times \frac{\ln 2}{5} \times \frac{1}{2}$ Decreasing at $4.85^\circ\text{C per minute}$ <i>Finding an average rate, e.g. $\frac{55-90}{5}$, scores M0.</i>	M1 M1 A1 (3)
Total for Question 6: 10 marks		



Question 7

10 marks

Part	Scheme	Marks
(a)	$\frac{500}{5} = \boxed{100}$	B1 (1)
(b)	$250 = \frac{500e^{10k}}{4 + e^{10k}} \Rightarrow 1000 + 250e^{10k} = 500e^{10k}$ $e^{10k} = 4$ $k = \frac{1}{10} \ln 4 = \frac{1}{5} \ln 2$ *	M1 A1 A1* (3)
(c)	$P = \frac{500}{4e^{-kt} + 1}$ and $4e^{-kt} \rightarrow 0$ $\boxed{500 \text{ birds}}$	M1 A1 (2)
(d)	$400(4 + e^{kt}) = 500e^{kt} \Rightarrow e^{kt} = 16$ $t = \frac{\ln 16}{\frac{1}{5} \ln 2} = \frac{5 \times 4 \ln 2}{\ln 2}$ $\boxed{20 \text{ years}}$	M1 M1 A1 (3)
(e)	e.g. food or space on the island, disease or predators may change the limit	B1 (1)
Total for Question 7: 10 marks		



Question 8

10 marks

Part	Scheme	Marks
(a)	Gradient $\frac{4.1 - 3.5}{8 - 2} = 0.1$ $\log_{10} N = 0.1t + 3.3$	M1 A1 (2)
(b)	$\log_{10} N = \log_{10} a + t \log_{10} b$ $a = 10^{3.3} = 1995$ $b = 10^{0.1} = 1.26$ <i>The intercept 3.3 is not given on the figure: it must be found from the equation of the line.</i>	M1 A1 A1 (3)
(c)	The number of subscribers is multiplied by about 1.26 each year (an increase of about 26% per year)	B1 (1)
(d)	$10^{0.1 \times 10 + 3.3} = 10^{4.3}$: 20 000	B1 (1)
(e)	$0.1t + 3.3 > \log_{10} 50\,000$ $t = 14.0$ (13.99..., i.e. during the 14th year)	M1 A1 (2)
(f)	e.g. the number would grow without limit, but the number of possible customers is limited	B1 (1)
Total for Question 8: 10 marks		



Question 9

10 marks

Part	Scheme	Marks
(a)	Increasing curve with vertical asymptote $x = 2$ $\ln(2x - 4) = -1 \Rightarrow 2x - 4 = e^{-1}$ $\left(2 + \frac{1}{2e}, 0\right)$	B1 M1 A1 (3)
(b)	$x = \ln(2y - 4) + 1 \Rightarrow 2y - 4 = e^{x-1}$ $f^{-1}(x) = \frac{1}{2}e^{x-1} + 2$ Domain $x \in \mathbb{R}$, range $f^{-1}(x) > 2$	M1 A1 B1 (3)
(c)	$fg(x) = \ln(2e^{2x} - 4) + 1$ $\ln(2e^{2x} - 4) = \ln 4 \Rightarrow 2e^{2x} - 4 = 4$ $e^{2x} = 4$ $x = \ln 2$	B1 M1 M1 A1 (4)
Total for Question 9: 10 marks		



Question 10

12 marks

Part	Scheme	Marks
(a)	$A = 40$ and $B = 120$	B1 (1)
(b)	$40u = 200 - \frac{80}{u}$ $40u^2 - 200u + 80 = 0$ $u^2 - 5u + 2 = 0$ *	M1 A1 A1* (3)
(c)	$u = \frac{5 \pm \sqrt{17}}{2}$ $t \geq 0$ gives $u = e^{0.1t} \geq 1$, so $u = \frac{5 - \sqrt{17}}{2} = 0.438$ is rejected $0.1t = \ln \frac{5 + \sqrt{17}}{2}$ $t = 10 \ln \left(\frac{5 + \sqrt{17}}{2} \right)$	M1 B1 M1 A1 (4)
(d)	$\frac{dA}{dt} = 4e^{0.1t} = 4u$ 18.2 per day	M1 A1 (2)
(e)	$\ln A = \ln 40 + 0.1t$: a straight line of the form $y = mx + c$ in $\ln A$ and t Gradient 0.1, intercept $\ln 40$	M1 A1 (2)
Total for Question 10: 12 marks		



6

Chapter 6: Differentiation

Mark scheme. Marks are shown on the right of each line.

Question 1

5 marks

Part	Scheme	Marks
(a)	Product rule: $3e^{3x} \ln(2x) + e^{3x} \times \frac{1}{x}$ $e^{3x} \left(3 \ln 2x + \frac{1}{x} \right)$	M1 A1 (2)
(b)	Quotient rule: $\frac{2 \cos x - (2x+1)(-\sin x)}{\cos^2 x}$ Correct unsimplified $\frac{2 \cos x + (2x+1) \sin x}{\cos^2 x}$	M1 A1 A1 (3)
Total for Question 1: 5 marks		

Question 2

5 marks

Part	Scheme	Marks
(a)	$\frac{3(x+2) - (3x-1)}{(x+2)^2}$ $= \frac{7}{(x+2)^2}$, so $a = 7$	M1 A1 (2)
(b)	$x = -1: y = -4$ and gradient 7 $y + 4 = 7(x + 1)$ $y = 7x + 3$	B1 M1 A1 (3)
Total for Question 2: 5 marks		



Question 3

5 marks

Part	Scheme	Marks
(a)	$\frac{dx}{dy} = 6 \sec^2 3y$	M1
	$\frac{dy}{dx} = \frac{1}{6 \sec^2 3y}$	A1
		(2)
(b)	$\sec^2 3y = 1 + \tan^2 3y$	M1
	$\tan 3y = \frac{x}{2}$, so $\sec^2 3y = 1 + \frac{x^2}{4}$	M1
	$\frac{1}{6 \left(1 + \frac{x^2}{4}\right)} = \frac{2}{3(x^2 + 4)}$ *	A1*
		(3)
Total for Question 3: 5 marks		

Question 4

5 marks

Part	Scheme	Marks
(a)	Product rule: $e^{-2x} - 2xe^{-2x}$	M1
	$(1 - 2x)e^{-2x}$	A1
	$= 0$ gives $x = \frac{1}{2}$ (as $e^{-2x} \neq 0$)	M1
	$\left(\frac{1}{2}, \frac{1}{2e}\right)$	A1
		(4)
(b)	$\frac{1}{2e}$	B1ft
		(1)
Total for Question 4: 5 marks		



Question 5

5 marks

Part	Scheme	Marks
	$g'(x) = \frac{2}{2x+1} - \frac{8}{(2x+1)^2}$ Both terms correct $= \frac{2(2x+1) - 8}{(2x+1)^2} = \frac{2(2x-3)}{(2x+1)^2}$ $(2x+1)^2 > 0$, so $g'(x) > 0$ when $2x-3 > 0$ $x > \frac{3}{2}$	M1 A1 M1 M1 A1 (5)
Total for Question 5: 5 marks		

Question 6

10 marks

Part	Scheme	Marks
(a)	Product rule: $2(x+1)e^{-2x} - 2(x+1)^2e^{-2x}$ Correct unsimplified $= 2(x+1)e^{-2x}(1 - (x+1)) = -2x(x+1)e^{-2x}$ *	M1 A1 A1* (3)
(b)	$x = 0$ or $x = -1$ $A(0, 1), B(-1, 0)$	M1 A1 (2)
(c)	$3 \times 0 - 2 = -2$	B1 (1)
(d)	The line $y = k$ must lie between the minimum value 0 and the maximum value 1 $0 < k < 1$	M1 A1 (2)
(e)	$f(1) = 4e^{-2}$ and $f'(1) = -4e^{-2}$ $y = 4e^{-2}(2-x)$	M1 A1 (2)
Total for Question 6: 10 marks		



Question 7

10 marks

Part	Scheme	Marks
(a)	Product rule with $\frac{d}{dx} \cos \frac{x}{2} = -\frac{1}{2} \sin \frac{x}{2}$ $f'(x) = 2 \cos \frac{x}{2} - x \sin \frac{x}{2}$	M1 A1 (2)
(b)	$x \sin \frac{x}{2} = 2 \cos \frac{x}{2}$ $x = \frac{2 \cos \frac{x}{2}}{\sin \frac{x}{2}} = 2 \cot \frac{x}{2} *$	M1 A1* (2)
(c)	$f(\pi) = 0$ and $f'(\pi) = -\pi$ Normal gradient $\frac{1}{\pi}$: $y - 0 = \frac{1}{\pi}(x - \pi)$ $y = \frac{x}{\pi} - 1$	B1 M1 A1 (3)
(d)	$P(\pi, 0)$ and $Q(0, -1)$ Area = $\frac{1}{2} \times \pi \times 1$ $\frac{\pi}{2}$	B1 M1 A1 (3)
Total for Question 7: 10 marks		



Question 8

10 marks

Part	Scheme	Marks
(a)	Quotient rule: $\frac{2 \cos 2x \cos 2x - (1 + \sin 2x)(-2 \sin 2x)}{\cos^2 2x}$ Correct unsimplified $= \frac{2(\cos^2 2x + \sin^2 2x) + 2 \sin 2x}{\cos^2 2x} = \frac{2(1 + \sin 2x)}{\cos^2 2x}$ $\cos^2 2x = 1 - \sin^2 2x = (1 - \sin 2x)(1 + \sin 2x)$ Cancels: $\frac{2}{1 - \sin 2x}^*$	M1 A1 M1 M1 A1* (5)
(b)	$x = \frac{\pi}{12}: y = \frac{1 + \frac{1}{2}}{\frac{\sqrt{3}}{2}} = \sqrt{3}$ and $\frac{dy}{dx} = \frac{2}{\frac{1}{2}} = 4$ $y - \sqrt{3} = 4\left(x - \frac{\pi}{12}\right)$ $y = 4x + \sqrt{3} - \frac{\pi}{3}$	B1 M1 A1 (3)
(c)	$\frac{dy}{dx} = \frac{2}{1 - \sin 2x}$ has numerator 2, which is never 0 so $\frac{dy}{dx} \neq 0$ anywhere: no stationary points (it is always positive)	B1 B1 (2)
Total for Question 8: 10 marks		



Question 9

10 marks

Part	Scheme	Marks
(a)	$\frac{x \times \frac{1}{x} - \ln x}{x^2}$	M1
	$f'(x) = \frac{1 - \ln x}{x^2}$	A1
		(2)
(b)	$\ln x = 1 \Rightarrow x = e: \left(e, \frac{1}{e}\right)$	M1
	Correct coordinates	A1
	$f'(x) > 0$ for $x < e$ and $f'(x) < 0$ for $x > e$, so a maximum	A1
		(3)
(c)	$\frac{1}{e}$ is the maximum value of f , and $\pi \neq e$	M1
	so $f(\pi) < f(e)$, i.e. $\frac{\ln \pi}{\pi} < \frac{1}{e}$	A1
		(2)
(d)	$e \ln \pi < \pi$	M1
	$\ln(\pi^e) < \pi = \ln(e^\pi)$	M1
	$\ln$ is increasing, so $\pi^e < e^\pi$ *	A1*
		(3)
Total for Question 9: 10 marks		



Question 10

11 marks

Part	Scheme	Marks
(a)	$\frac{dx}{dy} = e^{2y} + 2ye^{2y}$ $= (1 + 2y)e^{2y}$ $\frac{dy}{dx} = \frac{1}{(1 + 2y)e^{2y}}$	M1 A1 A1 (3)
(b)	$x = \frac{e}{2}$ Gradient $\frac{1}{2e}$ $y - \frac{1}{2} = \frac{1}{2e} \left(x - \frac{e}{2} \right)$ $y = \frac{x}{2e} + \frac{1}{4}$	B1 B1 M1 A1 (4)
(c)	$y = 0: \frac{x}{2e} = -\frac{1}{4}$ $x = -\frac{e}{2}$	M1 A1 (2)
(d)	Tangent parallel to the y-axis when $\frac{dx}{dy} = 0: y = -\frac{1}{2}$ $\left(-\frac{1}{2e}, -\frac{1}{2} \right)$	M1 A1 (2)
Total for Question 10: 11 marks		



7

Chapter 7: Integration

Mark scheme. Marks are shown on the right of each line.

Question 1

5 marks

Part	Scheme	Marks
(a)	$\frac{3}{2}e^{2x}$ or $-4\ln x $	M1
	$\frac{3}{2}e^{2x} - 4\ln x + c$	A1
		(2)
(b)	$k\ln 2x-3 $	M1
	$\frac{5}{2}\ln 2x-3 + c$	A1
		(2)
(c)	$\frac{1}{4}\tan 4x + c$	B1
		(1)
Total for Question 1: 5 marks		

Question 2

5 marks

Part	Scheme	Marks
(a)	$[x^2 + 3\ln x]_1^e$	M1
	$(e^2 + 3) - (1 + 0)$	M1
	$e^2 + 2$	A1
		(3)
(b)	$[\frac{1}{2}e^{2x}]_0^{\ln 2} = \frac{1}{2}(4 - 1)$	M1
	$\frac{3}{2}$	A1
		(2)
Total for Question 2: 5 marks		



Question 3

5 marks

Part	Scheme	Marks
(a)	$\sin^2 x + \cos^2 x + 2 \sin x \cos x = 1 + \sin 2x$	M1
	Integrates: $x - \frac{1}{2} \cos 2x$	M1
	$x - \frac{1}{2} \cos 2x + c$	A1
		(3)
(b)	$\tan^2 x = \sec^2 x - 1$	M1
	$\tan x - x + c$	A1
		(2)
Total for Question 3: 5 marks		

Question 4

5 marks

Part	Scheme	Marks
(a)	Numerator is the derivative of the denominator: $\ln(3x^2 + 1)$	M1
	$\ln(3x^2 + 1) + c$	A1
		(2)
(b)	$[-\ln(2 + \cos x)]_0^{\frac{\pi}{3}}$	M1
	$-\ln \frac{5}{2} + \ln 3$	M1
	$\ln \frac{6}{5}$	A1
		(3)
Total for Question 4: 5 marks		



Question 5

6 marks

Part	Scheme	Marks
(a)	Divides: quotient $2x - 1$, remainder 2 $2x - 1 + \frac{2}{x+3}$	M1 A1 (2)
(b)	$[x^2 - x + 2\ln(x+3)]_0^2$ Correct integral (4 - 2 + 2ln5) - (0 - 0 + 2ln3) $2 + 2\ln\frac{5}{3}$	M1 A1 M1 A1 (4)
Total for Question 5: 6 marks		

Question 6

10 marks

Part	Scheme	Marks
(a)	Divides by $(x+2)$: $(x+2)^2 = x^2 + 4x + 4$ Remainder 3, so $y = x + 2 + \frac{3}{x+2}$ *	M1 A1* (2)
(b)	$\left[\frac{x^2}{2} + 2x + 3\ln(x+2)\right]_0^2$ Correct integral (2 + 4 + 3ln4) - (3ln2) $6 + 3\ln 2$	M1 A1 M1 A1 (4)
(c)	$\frac{dy}{dx} = 1 - \frac{3}{(x+2)^2}$ (x+2) ² = 3, so $x = -2 + \sqrt{3}$ (as $x > -2$) $y = \sqrt{3} + \frac{3}{\sqrt{3}}$ $(-2 + \sqrt{3}, 2\sqrt{3})$	M1 A1 M1 A1 (4)
Total for Question 6: 10 marks		



Question 7

10 marks

Part	Scheme	Marks
(a)	$4 \cos^2 x - 4 \sin x \cos x + \sin^2 x$ Uses $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$, $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$, $2 \sin x \cos x = \sin 2x$ $2 + 2 \cos 2x - 2 \sin 2x + \frac{1}{2} - \frac{1}{2} \cos 2x$ $\frac{5}{2} + \frac{3}{2} \cos 2x - 2 \sin 2x$	M1 M1 A1 A1 (4)
(b)	$\left[\frac{5}{2}x + \frac{3}{4} \sin 2x + \cos 2x \right]_0^{\frac{\pi}{4}}$ Correct integral $\left(\frac{5\pi}{8} + \frac{3}{4} + 0 \right) - (0 + 0 + 1)$ $\frac{5\pi}{8} - \frac{1}{4}$	M1 A1 M1 A1 (4)
(c)	Replacing x by $2x$ halves the interval and halves the integral $\frac{5\pi}{16} - \frac{1}{8}$	M1 A1 (2)
Total for Question 7: 10 marks		



Question 8

10 marks

Part	Scheme	Marks
(a)	$\frac{\sec x \tan x + \sec^2 x}{\sec x + \tan x}$ (chain rule) Correct derivatives of $\sec x$ and $\tan x$ Numerator = $\sec x(\tan x + \sec x)$ Cancels to $\sec x$ *	M1 A1 M1 A1* (4)
(b)	$\ln(\sqrt{2} + 1) - \ln(1 + 0)$ $\ln(1 + \sqrt{2})$	M1 A1 (2)
(c)	$\int \tan x \, dx = \ln \sec x $ (or $-\ln \cos x $) $\ln 2 - \ln 1 = \ln 2$ *	M1 A1* (2)
(d)	$\sec x > \tan x$ on the interval: $\ln(1 + \sqrt{2}) - \ln \sqrt{2}$ $\ln\left(\frac{1 + \sqrt{2}}{\sqrt{2}}\right) \left(= \ln\left(1 + \frac{\sqrt{2}}{2}\right)\right)$	M1 A1 (2)
Total for Question 8: 10 marks		



Question 9

10 marks

Part	Scheme	Marks
(a)	$[2 \ln(2x+1)]_0^k = 2 \ln(2k+1)$ $(2k+1)^2 = 25$ $k = 2$	M1 M1 A1 (3)
(b)	$\left[\frac{1}{2}e^{2x} + e^x\right]_0^{\ln p}$ $\left(\frac{1}{2}p^2 + p\right) - \left(\frac{1}{2} + 1\right) = 10$ $p^2 + 2p - 23 = 0$ * $p = \frac{-2 \pm \sqrt{96}}{2}$ $p = -1 + 2\sqrt{6}$ (the negative root is rejected as $p > 1$)	M1 M1 A1* M1 A1 (5)
(c)	$[3 \ln(x-1)]_2^3 = 3 \ln 2 - 3 \ln 1$ $= \ln 2^3 = \ln 8$ *	M1 A1* (2)
Total for Question 9: 10 marks		



Question 10

11 marks

Part	Scheme	Marks
(a)	$y = 0: (e^x - 1)(e^x - 2) = 0$ $e^x = 1$ (the origin) or $e^x = 2$ $x = \ln 2$ *	M1 A1 A1* (3)
(b)	$e^{2x} \rightarrow 0$ and $e^x \rightarrow 0$: $y = 2$	B1 (1)
(c)	$\int (e^{2x} - 3e^x + 2) dx = \frac{1}{2}e^{2x} - 3e^x + 2x$ Correct integral $(2 - 6 + 2\ln 2) - (\frac{1}{2} - 3) = -\frac{3}{2} + 2\ln 2$ Region below the axis, so area = $\frac{3}{2} - 2\ln 2$ $\frac{3}{2} - \ln 4$	M1 A1 M1 M1 A1 (5)
(d)	$\frac{dy}{dx} = 2e^{2x} - 3e^x = 8 - 6 = 2$ at $x = \ln 2$ $y = 2(x - \ln 2)$	M1 A1 (2)
Total for Question 10: 11 marks		



8

Chapter 8: Numerical Methods

Mark scheme. Marks are shown on the right of each line.

Question 1

5 marks

Part	Scheme	Marks
(a)	$f(2) = -3$ and $f(3) = 13$ Sign change and f is continuous, so there is a root between 2 and 3	M1 A1 (2)
(b)	$x^3 = 3x + 5 \Rightarrow x = \sqrt[3]{3x + 5}$ *	B1* (1)
(c)	$x_2 = 2.2240$ $x_3 = 2.2684, x_4 = 2.2770$	M1 A1 (2)
Total for Question 1: 5 marks		

Question 2

5 marks

Part	Scheme	Marks
(a)	$f(1) = -2.28$ and $f(2) = 1.39$ Sign change, f continuous, so a root in $[1, 2]$	M1 A1 (2)
(b)	Interval $[1.7485, 1.7495]$ $f(1.7485) = -0.0025$ and $f(1.7495) = 0.0022$ Sign change, f continuous, so $\alpha = 1.749$ to 3 d.p. *	M1 A1 A1* (3)
Total for Question 2: 5 marks		



Question 3

5 marks

Part	Scheme	Marks
(a)	$f(1.9) = -7$ and $f(2.1) = 13$: opposite signs	B1 (1)
(b)	f is not continuous on the interval It has a vertical asymptote at $x = 2$, where the sign changes without passing through zero	B1 B1 (2)
(c)	$\frac{1}{x-2} = -3 \Rightarrow x-2 = -\frac{1}{3}$ $x = \frac{5}{3}$	M1 A1 (2)
Total for Question 3: 5 marks		

Question 4

5 marks

Part	Scheme	Marks
(a)	$x_2 = 0.8$ $x_3 = 0.728, x_4 = 0.706$ (0.7060)	M1 A1 (2)
(b)	$x_2 = 4.65, x_3 = 4.9245$ The values move away from β (they diverge), so the formula cannot find β	B1 B1 (2)
(c)	e.g. $x = 5 - \frac{3}{x}$ (or $x = \sqrt{5x-3}$)	B1 (1)
Total for Question 4: 5 marks		



Question 5

5 marks

Part	Scheme	Marks
(a)	α is a root of $g(x) = \ln x + x - 3 = 0$ $g(2) = -0.307$ and $g(3) = 1.099$ Sign change, g continuous for $x > 0$, so $2 < \alpha < 3$	B1 M1 A1 (3)
(b)	$x_2 = 2.3069$ $x_3 = 2.1641$	M1 A1 (2)
Total for Question 5: 5 marks		

Question 6

11 marks

Part	Scheme	Marks
(a)	$e^x + 2x$	B1 (1)
(b)	Tangent through the origin: $\frac{y_P}{x_P} = \text{gradient}$, so $e^\alpha + \alpha^2 = \alpha(e^\alpha + 2\alpha)$ $e^\alpha + \alpha^2 = \alpha e^\alpha + 2\alpha^2$ $e^\alpha(1 - \alpha) - \alpha^2 = 0$ *	M1 A1 A1* (3)
(c)	$h(0.5) = 0.574$ and $h(1) = -1$ Sign change, continuous, so $0.5 < \alpha < 1$	M1 A1 (2)
(d)	$1 - x = x^2 e^{-x} \Rightarrow x = 1 - x^2 e^{-x}$ *	B1* (1)
(e)	$x_2 = 0.7567$ $x_3 = 0.7313$	M1 A1 (2)
(f)	Interval $[0.7385, 0.7395]$: $h(0.7385) = 0.0019$, $h(0.7395) = -0.0011$ Sign change, continuous, so $\alpha = 0.739$ (3 d.p.)	M1 A1 (2)
Total for Question 6: 11 marks		



Question 7

10 marks

Part	Scheme	Marks
(a)	Product rule $f'(x) = 2x \cos x - x^2 \sin x$	M1 A1 (2)
(b)	$2x \cos x = x^2 \sin x \Rightarrow \tan x = \frac{2}{x}$ (as $x \neq 0$) $x = \arctan\left(\frac{2}{x}\right)$ (as $0 < x < \frac{\pi}{2}$) *	M1 A1* (2)
(c)	$x_2 = 1.1071$ $x_3 = 1.0652$ $x_4 = 1.0814$	M1 A1 A1 (3)
(d)	Interval $[1.0765, 1.0775]$ $f'(1.0765) = 0.0013$ and $f'(1.0775) = -0.0021$ Sign change, f' continuous, so $\beta = 1.077$ (3 d.p.)	M1 A1 A1 (3)
Total for Question 7: 10 marks		



Question 8

10 marks

Part	Scheme	Marks
(a)	$\frac{dC}{dt} = 8(-0.2e^{-0.2t} + e^{-t}) = 0$ $e^{0.8t} = 5$ $t = \frac{\ln 5}{0.8} = \frac{5}{4} \ln 5 \text{ hours}$	M1 A1 A1 (3)
(b)	$e^{-0.2T} - e^{-T} = 0.25 \Rightarrow e^{-0.2T} = 0.25 + e^{-T}$ $-0.2T = \ln(0.25 + e^{-T})$, so $T = -5 \ln(0.25 + e^{-T})$ *	M1 A1* (2)
(c)	$T_2 = 6.8821$ $T_3 = 6.9110$	M1 A1 (2)
(d)	$T \approx 6.912 \text{ hours}$ $6.912 - 0.394 = 6.518 \text{ hours}$ 6 hours 31 minutes	B1 M1 A1 (3)
Total for Question 8: 10 marks		



Question 9

10 marks

Part	Scheme	Marks
(a)	$f(1.2) = -1.03$ and $f(1.4) = 1.60$ Sign change, f continuous on $[1.2, 1.4]$, so a root	M1 A1 (2)
(b)	$f(1.5) = 9.60$ and $f(1.6) = -39.0$ $\tan x$ has an asymptote at $x = \frac{\pi}{2} = 1.5708$ in the interval: f is not continuous there	B1 B1 (2)
(c)	$x_2 = \arctan 3.9$ $x_2 = 1.3198$ $x_3 = 1.3234$	M1 A1 A1 (3)
(d)	Interval $[1.3235, 1.3245]$ $f(1.3235) = -0.0095$ and $f(1.3245) = 0.0042$ Sign change, continuous, so the root is 1.324 (3 d.p.)	M1 A1 A1 (3)
Total for Question 9: 10 marks		



Question 10

11 marks

Part	Scheme	Marks
(a)	$x = e^y - 2 \Rightarrow y = \ln(x+2)$ $f^{-1}(x) = \ln(x+2)$ Domain $x > -2$ (the range of f)	M1 A1 B1 (3)
(b)	The graphs are reflections in $y = x$, so they meet on it: $e^x - 2 = x$ $h(1) = -0.28$ and $h(2) = 3.39$ for $h(x) = e^x - x - 2$ Sign change, continuous, so $1 < \alpha < 2$	B1 M1 A1 (3)
(c)	$x_2 = 1.0986$ $x_3 = 1.1310$	M1 A1 (2)
(d)	$x_2 = -1.8347, x_3 = -1.8403$ $\ln(x+2)$ has gradient $\frac{1}{x+2} \approx 6$ near β , so that formula moves away from β $e^x - 2$ has gradient $e^x \approx 0.16$ near β (less than 1), so it converges to β	B1 B1 B1 (3)
Total for Question 10: 11 marks		

