eClassroom

A-LEVEL MATHEMATICS • FURTHER PURE 1

Roots of Quadratic Equations

Worked Solutions

Pearson Edexcel International A Level (WFM01) • Calculator allowed: show every step

Full worked solutions for the free Roots of Quadratic Equations worksheet, A-Level and International A-Level Maths

A **Examples** & Your turn page 2

B **Fluency** core skills page 2

C **Reasoning** explain it page 4

D **Problem** solving page 5

E **Exam** practice page 6

Tap a section to jump straight to it

Full worked solutions for every question on the Further Pure 1 Roots of Quadratic Equations worksheet, in the same order and with the same question numbers.

How to mark your work. Tick each step you got right, not just the final answer. At A-level most marks are for method, so a wrong final answer can still earn most of the marks. If you lost marks, find the *first* line where your working is different from ours. That is the thing to fix.

In Section E: **M** = method mark, **dM** = method mark that depends on the previous M mark, **A** = accuracy mark (needs the M mark before it), **B** = independent mark.

Question sheet | Exam questions by topic | Further Pure 1 exam questions | More worksheets

★★★★★

Join students already acing their exams.

Book your FREE lesson at www.eclassroom.org.uk

A Your turn answers

Compare each line of your working with the solution.

CORE

A1 $\alpha+\beta=-\frac{4}{5}$, $\alpha\beta=-\frac{3}{5}$: $\boxed{-\frac{4}{5},\ -\frac{3}{5}}$

A2 $\left(-\frac{3}{2}\right)^2-2(-2)=\frac{9}{4}+4$: $\boxed{\frac{25}{4}}$

A3 Sum: $-3-2=-5$; product: $\alpha\beta-(\alpha+\beta)+1=-5+3+1=-1$: $\boxed{x^2+5x-1=0}$

STRETCH

A4 $\alpha+\beta=\frac{1}{3}$, $\alpha\beta=\frac{2}{3}$; $\frac{1}{27}-3\left(\frac{2}{3}\right)\left(\frac{1}{3}\right)$: $\boxed{-\frac{17}{27}}$

A5 Sum: $\frac{3/4}{3/2}=\frac{1}{2}$; product: $\frac{1}{3/2}=\frac{2}{3}$; $x^2-\frac{1}{2}x+\frac{2}{3}=0$: $\boxed{6x^2-3x+4=0}$

A6 $(\alpha-\beta)^2=16-4\left(\frac{3}{2}\right)=10$: $\boxed{\sqrt{10}}$

A7 Roots a and $3a$: $3a^2=12$, so $a=\pm2$; sum $4a=-k$: $\boxed{k=\pm8}$

B Fluency solutions

One mark for each correct part. The notes in italics explain why each sequence was chosen.

CORE

B1 **(a)** $\boxed{3}$ **(b)** $\boxed{\frac{1}{2}}$ **(c)** $9-1$: $\boxed{8}$ **(d)** $\frac{3}{1/2}$: $\boxed{6}$

Every answer is built from the sum and product only.

B2 **(a)** $16+4$: $\boxed{20}$ **(b)** $-64-3(-2)(-4)$: $\boxed{-88}$ **(c)** $\alpha\beta(\alpha+\beta)$: $\boxed{8}$ **(d)** $\frac{20}{4}$: $\boxed{5}$

Factorise each expression into sums and products first.

B3 **(a)** $\boxed{x^2 - 15x + 18 = 0}$ **(b)** $\boxed{x^2 - 7x + 8 = 0}$ **(c)** $\boxed{x^2 - 21x + 4 = 0}$ **(d)** $\boxed{2x^2 - 5x + 1 = 0}$

New sum and new product, then $x^2 - (\text{sum})x + \text{product} = 0$

B4 **(a)** $\boxed{x^2 - 5x + 6 = 0}$ **(b)** $\boxed{4x^2 + 6x + 1 = 0}$ **(c)** $\boxed{x^2 - 7 = 0}$ **(d)** $\boxed{9x^2 - 6x - 5 = 0}$

Multiply through to clear fractions, and keep "= 0".

B5 **(a)** $\boxed{-\frac{1}{3}, -\frac{4}{3}}$ **(b)** $\frac{1}{9} + \frac{16}{3} : \boxed{\frac{49}{9}}$ **(c)** $(\alpha+\beta)(\alpha-\beta) = -\frac{1}{3} \times \frac{7}{3} : \boxed{-\frac{7}{9}}$

$(\alpha-\beta)^2 = (\alpha+\beta)^2 - 4\alpha\beta$

B6 **(a)** $\frac{21/4}{-5/2} : \boxed{-\frac{21}{10}}$ **(b)** $-\frac{5}{2} - \frac{1}{2} + 1 : \boxed{-2}$ **(c)** $\boxed{\frac{25}{4}}$ **(d)** $4\alpha\beta - 2(\alpha+\beta) + 1 : \boxed{-8}$

Expand each product into $\alpha\beta$ and $\alpha+\beta$

STRETCH

B7 **(a)** sum $-27 + 9 = -18$, product 1: $\boxed{x^2 + 18x + 1 = 0}$ **(b)** sum 7, product 1: $\boxed{x^2 - 7x + 1 = 0}$

(c) sum $3(\alpha+\beta) = -9$, product $5\alpha\beta + 2(\alpha^2+\beta^2) = 19$: $\boxed{x^2 + 9x + 19 = 0}$ **(d)** sum $\alpha\beta(\alpha+\beta) = -3$, product $(\alpha\beta)^3 = 1$: $\boxed{x^2 + 3x + 1 = 0}$

Part (d) returns the original equation, because $\alpha\beta = 1$

B8 **(a)** $36 - 4k = 16$: $\boxed{k = 5}$ **(b)** $3a = 6$, $k = 2a^2$: $\boxed{k = 8}$ **(c)** $36 - 2k = 30$: $\boxed{k = 3}$ **(d)** $\frac{6}{k} = 3$: $\boxed{k = 2}$

Each condition becomes an equation in the sum and product.

B9 **(a)** $\boxed{\frac{k}{2}}$ **(b)** $\boxed{\frac{3}{2}}$ **(c)** $\boxed{\frac{k^2}{4} - 3}$ **(d)** $\frac{k^2}{4} = 4$: $\boxed{k = 4}$

A parameter changes the sum, not the product.

B10 **(a)** sum $2(3) - 2 = 4$, product $4(-1) - 2(3) + 1 = -9$: $\boxed{x^2 - 4x - 9 = 0}$

(b) $\left(\frac{y+1}{2}\right)^2 - \frac{3(y+1)}{2} - 1 = 0$ gives $y^2 - 4y - 9 = 0$, which agrees

(c) sum $11 + 2 = 13$, product $1 + 11 + 1 = 13$: $\boxed{x^2 - 13x + 13 = 0}$ **(d)** sum 0, product $-(\alpha-\beta)^2 = -13$: $\boxed{x^2 - 13 = 0}$

Substitution is a check on the sum-and-product method.

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

C Reasoning solutions

Your wording can be different. Check that your reason makes the same point.

CORE

C1 The sum is $-\frac{b}{a}$, not $\frac{b}{a}$: $\boxed{\alpha+\beta=-2}$

C2 $(\alpha+\beta)^2$ also contains $2\alpha\beta$: $16-2(1)$: $\boxed{14}$

C3 $(\alpha+3)(\beta+3)=\alpha\beta+3(\alpha+\beta)+9=-4+3+9=8$; he left out $3(\alpha+\beta)$: $\boxed{x^2-7x+8=0}$

C4 It is not an equation (no "= 0") and the coefficients are not integers: $\boxed{2x^2-5x+1=0}$

STRETCH

C5 The roots are $\frac{1}{p}$ and $\frac{1}{q}$, so $\frac{1}{p}+\frac{1}{q}=2$ and $\frac{1}{pq}=\frac{2}{3}$

$pq=\frac{3}{2}$, $p+q=2pq$: $\boxed{p+q=3,\ pq=\frac{3}{2}}$

C6 The identity is $(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)$: $8-3(-3)(2)$: $\boxed{26}$

C7 The question says "without solving", so solving scores nothing and gives only an approximation. $\alpha+\beta=4$, $\alpha\beta=1$: $64-12$: $\boxed{52}$

C8 **(a)** $(\alpha-\beta)^2=(\alpha+\beta)^2-4\alpha\beta=\frac{b^2}{a^2}-\frac{4c}{a}=\frac{b^2-4ac}{a^2}$, as required

(b) Equal roots $\iff$ $(\alpha-\beta)^2=0$ $\iff$ $b^2=4ac$; when real, $|\alpha-\beta|=\sqrt{\frac{b^2-4ac}{a^2}}=\frac{\sqrt{b^2-4ac}}{|a|}$, as required

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

D Problem solving solutions

There is often more than one correct method. Any complete, correct method scores full marks.

CORE

D1 **(a)** $\alpha+\beta=\frac{3}{4}, \alpha\beta=-\frac{5}{4}$

(b) $\frac{9}{16}+\frac{5}{2}$: $\frac{49}{16}$

D2 **(a)** $\alpha+\beta=-\frac{k}{2}, \alpha\beta=-3$; $\frac{k^2}{4}+6=10$: $k=\pm4$

(b) $k=4$: $\frac{\alpha+\beta}{\alpha\beta}=\frac{-2}{-3}$: $\frac{2}{3}$

D3 $\frac{\alpha^3+\beta^3}{\alpha\beta}=\frac{(-k)^3-3(2)(-k)}{2}$: $\frac{6k-k^3}{2}$

D4 **(a)** $\frac{16}{9}-\frac{4}{3}$: $\frac{4}{9}$

(b) $\frac{64}{27}-3\left(\frac{2}{3}\right)\left(\frac{4}{3}\right)$: $-\frac{8}{27}$

STRETCH

D5 Sum: $\alpha+\beta=3$; product: $5\alpha\beta-2(\alpha^2+\beta^2)=25-2(9-10)=27$: $x^2-3x+27=0$

D6 **(a)** $\alpha+\beta=\frac{2}{A}$; sum of new roots $3(\alpha+\beta)=\frac{6}{A}=6$: $A=1$

(b) $\alpha\beta=-3$, $\alpha^2+\beta^2=4+6=10$; $B=5\alpha\beta+2(\alpha^2+\beta^2)=-15+20$: $B=5$

D7 $\alpha+\beta=-\frac{3}{2}, \alpha\beta=\frac{k}{2}$; sum $=\frac{\alpha^3+\beta^3}{(\alpha\beta)^2}=\frac{-27/8+9k/4}{k^2/4}=\frac{18k-27}{2k^2}$; product $=\frac{1}{\alpha\beta}=\frac{2}{k}$

$2k^2x^2-(18k-27)x+4k=0$

D8 **(a)** $\frac{1}{pq}=\frac{3}{4}$, so $pq=\frac{4}{3}$; $\frac{p+q}{pq}=\frac{5}{4}$: $p+q=\frac{5}{3}$

(b) $p^2+q^2=\frac{25}{9}-\frac{8}{3}=\frac{1}{9}$, $p^2q^2=\frac{16}{9}$: $9x^2-x+16=0$

E Exam practice solutions

Mark-scheme notes are shown on the right of each line.

CORE *Exam questions for everyone*

E1 **(a)** $\boxed{\alpha+\beta=\frac{5}{3},\ \alpha\beta=\frac{4}{3}}$ *B1*

(b) $(\alpha+\beta)^2-2\alpha\beta=\frac{25}{9}-\frac{8}{3}$ *M1*

$\boxed{\frac{1}{9}}$ *A1*

(c) Sum $\alpha\beta(\alpha+\beta)=\frac{20}{9}$ *B1*

Product $(\alpha\beta)^3=\frac{64}{27}$ *B1*

$x^2-\frac{20}{9}x+\frac{64}{27}=0$: $\boxed{27x^2-60x+64=0}$ *A1*

E2 **(a)** $\boxed{\alpha+\beta=4,\ \alpha\beta=7}$ *B1*

(b) $16-14$ *M1*

$\boxed{2}$ *A1*

(c) Sum $=4+\frac{\alpha^2+\beta^2}{(\alpha\beta)^2}=4+\frac{2}{49}=\frac{198}{49}$ *M1 A1*

$\alpha^3+\beta^3=64-84=-20$; product $=\alpha\beta+\frac{\alpha^3+\beta^3}{(\alpha\beta)^2}+\frac{1}{(\alpha\beta)^2}=7-\frac{20}{49}+\frac{1}{49}=\frac{324}{49}$ *M1*

$\boxed{49x^2-198x+324=0}$ *A1*

STRETCH

E3 **(a)** $(\alpha+\beta)^3 = \alpha^3+3\alpha^2\beta+3\alpha\beta^2+\beta^3 = \alpha^3+\beta^3+3\alpha\beta(\alpha+\beta)$ *M1*

so $\alpha^3+\beta^3 = (\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)$, as required *A1**

(b) $\alpha+\beta = -\frac{p}{2}$, $\alpha\beta = \frac{5}{2}$; $\frac{p^2}{4}-5 = -1$ *M1*

$\boxed{p = 4}$ *A1*

(c) If the roots were real, $\alpha^2+\beta^2$ could not be negative *B1*

(d) $\alpha^3+\beta^3 = (-2)^3-3\left(\frac{5}{2}\right)(-2) = 7$ *M1*

Sum $= 7+2 = 9$ *A1*

Product $= (\alpha\beta)^3+(\alpha^3+\beta^3)+1 = \frac{125}{8}+8 = \frac{189}{8}$ *M1*

$\boxed{8x^2-72x+189 = 0}$ *A1*