



eClassroom

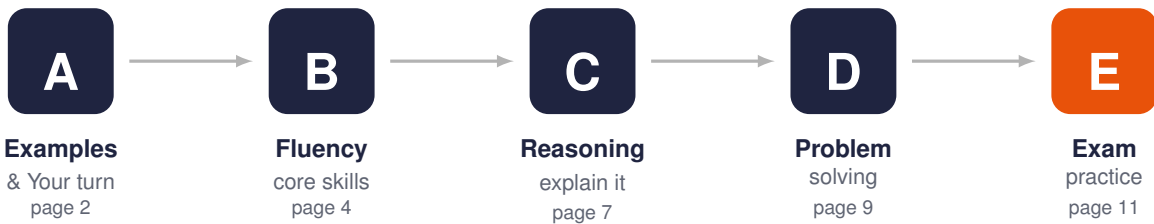
GCSE MATHEMATICS • ALGEBRA

Simultaneous Equations

Worksheet

Pearson Edexcel GCSE (1MA1) & International GCSE (4MA1) • No calculator

Free Simultaneous Equations worksheet for GCSE and iGCSE Maths, with full worked solutions



Tap a section to jump straight to it

By the end of this worksheet you can: solve simultaneous equations by elimination and by substitution • solve them from a graph • form simultaneous equations from a situation and solve them • *Higher:* a linear and a quadratic equation, and a line and a circle.

Which questions are for me?

FOUNDATION Everyone starts here. These skills appear on both tiers.

HIGHER Higher only: one linear and one quadratic equation, and lines meeting circles, plus harder elimination and reasoning. Grades 6 to 9.

Every section has both, clearly labelled. Questions get harder as you go, and every skill in Sections A to D is needed in Section E.

Track your progress

Section	B	C	D	E
Foundation marks	23	10	14	17
Higher marks	16	14	17	16
My score				

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Class

Date



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A

Worked examples & Your turn

Read each example carefully. Cover it up, then answer the question beside it using the same method.

FOUNDATION

Everyone starts here

Worked example A1

$$2x + y = 9$$

Solve

$$3x - y = 11$$

The y terms have opposite signs, so add the equations.

$$5x = 20, \text{ so } x = 4. \text{ Then } 8 + y = 9$$

$$x = 4, y = 1$$

Your turn A1

$$4x + 3y = 18$$

Solve

$$2x - 3y = 0$$

Worked example A2

$$3x + 2y = 16$$

Solve

$$x + 4y = 12$$

Double the first equation, then subtract.

$$6x + 4y = 32 \quad 5x = 20, \text{ so } x = 4. \text{ Then } 4 + 4y = 12$$

$$x = 4, y = 2$$

Your turn A2

$$5x + 2y = 11$$

Solve

$$3x - y = 11$$

Worked example A3

3 pens and 2 rulers cost £3.70. 1 pen and 4 rulers cost £2.40. Find the cost of a pen and of a ruler.

In pence: $3p + 2r = 370$, $p + 4r = 240$. Double the first and subtract: $5p = 500$

$$\text{pen } £1, \text{ ruler } 35p$$

Your turn A3

2 coffees and 3 teas cost £10.80. 3 coffees and 1 tea cost £9.90. Find the cost of a coffee and of a tea.

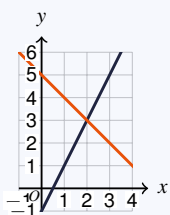
Worked example A4

Use the graph to solve

$$y = 2x - 1 \text{ and } y = -x + 5$$

The solution is where the lines cross.

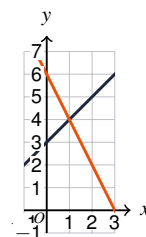
$$x = 2, y = 3$$



Your turn A4

Use the graph to solve

$$y = x + 3 \text{ and } y = -2x + 6$$



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HIGHER*Higher only: scaling both equations, quadratics and circles***Worked example A5: scaling both equations**

$$3x + 4y = 5$$

Solve

$$2x - 3y = 9$$

Multiply by 3 and by 4 so the y terms are $\pm 12y$, then add.

$$9x + 12y = 15 \quad 8x - 12y = 36 \quad 17x = 51$$

$$x = 3, y = -1$$

Your turn A5

$$5x + 3y = 21$$

Solve

$$2x - 7y = -8$$

Worked example A6: a line and a curve

$$y = x + 2$$

Solve

$$y = x^2$$

Substitute, then factorise: $(x - 2)(x + 1) = 0$

$x = 2$ or $x = -1$. Find each y from $y = x + 2$

$$(2, 4) \text{ and } (-1, 1)$$

Your turn A6

$$y = 3x - 2$$

Solve

$$y = x^2$$

Worked example A7: a line and a circle

$$x^2 + y^2 = 10$$

Solve

$$y = 2x + 1$$

Substitute: $5x^2 + 4x - 9 = 0$, so $(5x + 9)(x - 1) = 0$

$$x = 1 \text{ gives } y = 3; \quad x = -\frac{9}{5} \text{ gives } y = -\frac{13}{5}$$

$$(1, 3) \text{ and } \left(-\frac{9}{5}, -\frac{13}{5}\right)$$

Your turn A7

$$x^2 + y^2 = 25$$

Solve

$$y = x + 1$$



B**Fluency**

Each question is a designed sequence. From one part to the next, ask yourself: what changed, and what did it do to the answer?

FOUNDATION*Everyone starts here*

B1 Solve each pair of simultaneous equations.

(a) $x + y = 10$
 $x - y = 4$

(b) $x + y = 10$
 $x - y = -4$

(c) $2x + y = 10$
 $x + y = 7$

(d) $2x - y = 10$
 $x - y = 7$

[4 marks]

B2 Solve each pair of simultaneous equations.

(a) $x + 2y = 8$
 $3x + y = 9$

(b) $2x + 3y = 12$
 $x - y = 1$

(c) $4x - 3y = 1$
 $x + y = 9$

(d) $3x + 5y = 2$
 $x + 2y = 1$

[4 marks]

B3 Solve by substitution.

(a) $y = x + 3$
 $x + y = 11$

(b) $y = 2x$
 $3x + y = 20$

(c) $x = 2y - 1$
 $3x - y = 12$

(d) $y = 5 - x$
 $2x - 3y = 5$

[4 marks]

B4 Solve each pair of simultaneous equations.

(a) $x + y = 1$
 $x - y = 7$

(b) $2x + y = 0$
 $x - y = 6$

(c) $2x + 2y = 5$
 $x - y = 0$

(d) $3x + y = 2$
 $x + y = 4$

[4 marks]



FOUNDATION*continued: forming equations and checking solutions***B5** Form a pair of simultaneous equations and solve them.**(a)** Two numbers add up to 20 and have a difference of 6. Find the numbers.**(b)** 3 apples and 2 bananas cost 90p. 1 apple and 2 bananas cost 50p. Find the cost of each.**(c)** A rectangle is 3 cm longer than it is wide. Its perimeter is 26 cm. Find its length and width. **[3 marks]****B6** Is $x = 3$, $y = -2$ a solution of each pair? Show your working.

(a)
$$\begin{aligned}x + y &= 1 \\x - y &= 5\end{aligned}$$

(b)
$$\begin{aligned}2x + 3y &= 0 \\x - 2y &= 7\end{aligned}$$

(c)
$$\begin{aligned}4x + y &= 10 \\3x + 2y &= 6\end{aligned}$$

(d)
$$\begin{aligned}y &= 2x - 8 \\y &= x - 5\end{aligned}$$

[4 marks]**HIGHER***Higher tier only***B7** Solve each pair of simultaneous equations.

(a)
$$\begin{aligned}2x + 3y &= 7 \\3x + 2y &= 8\end{aligned}$$

(b)
$$\begin{aligned}3x - 4y &= 10 \\5x + 3y &= 7\end{aligned}$$

(c)
$$\begin{aligned}4x + 5y &= 23 \\3x + 2y &= 12\end{aligned}$$

(d)
$$\begin{aligned}6x + 5y &= 8 \\4x - 3y &= 18\end{aligned}$$

[4 marks]**B8** Solve each pair. Say if there is one solution or none.

(a)
$$\begin{aligned}y &= x^2 \\y &= 4\end{aligned}$$

(b)
$$\begin{aligned}y &= x^2 \\y &= x + 6\end{aligned}$$

(c)
$$\begin{aligned}y &= x^2 + 1 \\y &= 2x\end{aligned}$$

(d)
$$\begin{aligned}y &= x^2 \\y &= -1\end{aligned}$$

[4 marks]

HIGHER*continued: circles and other curves***B9** Solve each pair of simultaneous equations.

(a) $x^2 + y^2 = 25$
 $y = 3$

(b) $x^2 + y^2 = 25$
 $y = x - 1$

(c) $x^2 + y^2 = 20$
 $y = 2x$

(d) $x^2 + y^2 = 8$
 $x + y = 4$

[4 marks]**B10** Solve each pair of simultaneous equations.

(a) $xy = 12$
 $y = x + 1$

(b) $y = x^2 - 2x$
 $y = x$

(c) $y = x^2 - 4$
 $y = 3x$

(d) $y^2 = 4x$
 $y = x$

[4 marks]

C**Reasoning**

Show your working and explain your thinking. A correct answer without a reason does not score full marks.

FOUNDATION

Everyone starts here

- C1** Ben solves $2x + 3y = 12$ and $x + y = 5$. He multiplies the second equation by 2 and writes $2x + 2y = 5$
Explain Ben's mistake, and solve the equations. [2 marks]
- C2** Sam solves $3x + 2y = 13$ and $x - 2y = -1$. He writes $4x = 12$, so $x = 3$ and stops.
What has Sam not done? Complete the solution. [2 marks]
- C3** Explain why the equations $x + y = 5$ and $2x + 2y = 7$ have no solution. [3 marks]
- C4** Solve $2x + y = 7$ and $x - y = 2$ twice: once by elimination and once by substitution. Show that you get the same answer. [3 marks]



HIGHER*Higher tier only*

- C5** Mia solves $y = x^2$ and $y = x + 2$ and writes $x = 2$ or $x = -1$
Explain why her answer is not complete, and complete it. [3 marks]
- C6** Explain why a straight line and a circle can meet at 0, 1 or 2 points, but two different straight lines meet at 0 or 1 point. [3 marks]
- C7** Kai solves $x^2 + y^2 = 25$ and $y = x - 1$. He finds $x = 4$ or $x = -3$ and writes the solutions as $(4, -4)$ and $(-3, 3)$
Explain Kai's mistake, and write the correct solutions. [3 marks]
- C8** Show that the line $x + 2y = 10$ does not meet the circle $x^2 + y^2 = 16$
Hint: substitute $x = 10 - 2y$, then complete the square. [5 marks]



D**Problem solving**

You decide the method. Read the whole question first, and show every step.

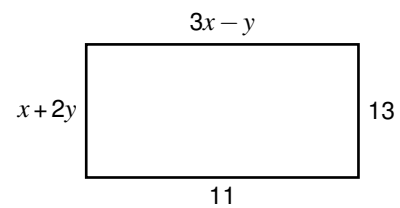
FOUNDATION

Everyone starts here

D1 Two numbers add up to 42. One number is 6 more than twice the other. Find the two numbers. [3 marks]

D2 Sam has 25 coins. They are all 10p or 20p coins. Altogether they are worth £3.60
How many of each coin does he have? [4 marks]

D3 The diagram shows a rectangle. The lengths are in centimetres.
Find x and y .



[4 marks]

D4 The line $y = mx + c$ passes through $(2, 7)$ and $(5, 16)$.
Form two equations in m and c , and solve them. [3 marks]



HIGHER*Higher tier only*

- D5** A ball is thrown upwards. Its height after t seconds is $h = 25t - 5t^2$ metres. A drone rises so that its height is $h = 10t + 10$ metres.
At what times are the ball and the drone at the same height, and what are the heights? [4 marks]
- D6** $x = 2$, $y = 3$ is the solution of the simultaneous equations $ax + by = 13$ and $bx - ay = -13$
Find the values of a and b . [4 marks]
- D7** The curve $y = x^2 + ax + b$ passes through $(1, 2)$ and $(3, 12)$.
Find a and b , and check your answer with a third point. [4 marks]
- D8** A line with gradient 2 has equation $y = 2x + k$. It touches the circle $x^2 + y^2 = 20$ at exactly one point.
(a) Show that $5x^2 + 4kx + k^2 - 20 = 0$
(b) The line touches the circle, so this equation has only one solution. Find the two possible values of k . [5 marks]



E**Exam practice**

Exam-style questions. Write your answers in the spaces. Aim for about one minute per mark.

FOUNDATION*Foundation and Higher crossover***E1** Solve the simultaneous equations

$$3x + y = 11$$

$$x - y = 1$$

$$x = \dots\dots\dots y = \dots\dots\dots$$

(Total for Question E1 is 3 marks)**E2** Solve the simultaneous equations

$$2x + 3y = 13$$

$$5x - 2y = 4$$

$$x = \dots\dots\dots y = \dots\dots\dots$$

(Total for Question E2 is 3 marks)**E3** 4 adult tickets and 3 child tickets cost £55. 2 adult tickets and 5 child tickets cost £45.

Work out the cost of an adult ticket and the cost of a child ticket.

$$\text{adult } \pounds \dots\dots\dots \text{ child } \pounds \dots\dots\dots$$

(Total for Question E3 is 4 marks)**E4** Solve the simultaneous equations

$$y = 2x - 3$$

$$3x + 2y = 22$$

$$x = \dots\dots\dots y = \dots\dots\dots$$

(Total for Question E4 is 3 marks)

- E5** Jo solves $x + y = 8$ and $x - y = 2$ by subtracting. She writes $2y = 10$, so $y = 5$
Explain Jo's mistake, and find the correct solution.

(Total for Question E5 is 2 marks)

- E6** Show that $x = 2$, $y = -1$ is a solution of both $3x + 2y = 4$ and $x - 4y = 6$

(Total for Question E6 is 2 marks)

HIGHER

Higher tier only

- E7** Solve the simultaneous equations
- $$y = x^2 - 3x + 2$$
- $$y = 2x - 4$$

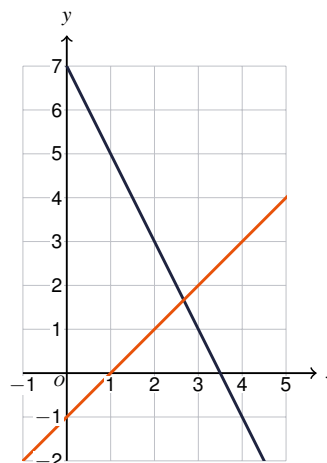
(Total for Question E7 is 4 marks)

- E8** The graphs of $2x + y = 7$ and $y = x - 1$ are drawn on the grid.

(a) Use the graph to estimate the solution of the simultaneous equations.

(1)

(b) Solve the equations algebraically to find the exact solution.



(2)

(Total for Question E8 is 3 marks)



- E9** A rectangle has a perimeter of 26 cm and an area of 40 cm^2 .
Form two equations and solve them to find the length and width of the rectangle.

.....
(Total for Question E9 is 4 marks)

- E10** The line $y = 3 - x$ meets the circle $x^2 + y^2 = 17$ at the points A and B .
Find the exact length of AB . Give your answer as a surd in its simplest form.

.....
(Total for Question E10 is 5 marks)

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