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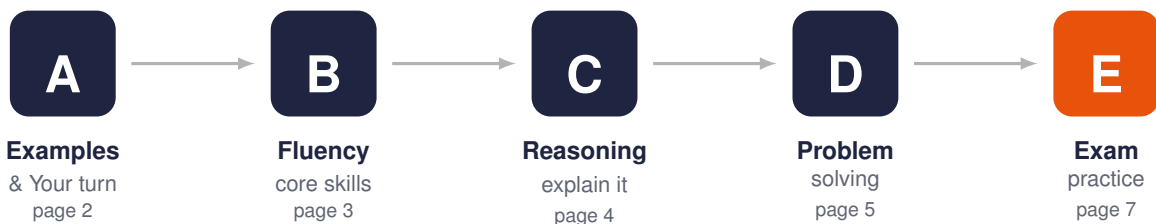
A-LEVEL MATHEMATICS • PURE 1

Graphs and Transformations

Worked Solutions

Pearson Edexcel International A Level (WMA11) & A Level (9MA0) • Calculator allowed: show every step

Full worked solutions for the free Graphs and Transformations worksheet, A-Level and International A-Level Maths



Tap a section to jump straight to it

Full worked solutions for every question on the Pure 1 Graphs and Transformations worksheet, in the same order and with the same question numbers.

How to mark your work. Tick each step you got right, not just the final answer. At A-level most marks are for method, so a wrong final answer can still earn most of the marks. If you lost marks, find the *first* line where your working is different from ours. That is the thing to fix.

In Section E: **M** = method mark, **dM** = method mark that depends on the previous M mark, **A** = accuracy mark (needs the M mark before it), **B** = independent mark.

Question sheet

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A**Your turn answers**

Compare each line of your working with the solution.

CORE**A1** $y = x(4 - x^2) = x(2 - x)(2 + x)$. Negative x^3 : from top left to bottom right, crossing three times.

$$(-2, 0), (0, 0), (2, 0)$$

A2 Branches in the second and fourth quadrants; asymptotes $x = 0$ and $y = 0$

$$-\frac{8}{x} = 2, \text{ so } x = -4: (-4, 2)$$

A3 $x(x+1)(x-3) - 5x = x(x^2 - 2x - 8) = x(x-4)(x+2)$, and $y = 5x$

$$(0, 0), (4, 20), (-2, -10)$$

STRETCH**A4** Negative x^4 : both ends go down. x^2 touches at O ; $(x-2)$ and $(x+3)$ cross.Touches $(0, 0)$; crosses $(2, 0)$ and $(-3, 0)$; the y intercept is the origin.**A5** $y = \frac{4}{x}$ moved 1 right and 2 up: asymptotes $x = 1$ and $y = 2$

$$x = 0: y = -4 + 2 = -2. \quad y = 0: \frac{4}{x-1} = -2, \text{ so } x - 1 = -2$$

$$\text{Using } y = \frac{4}{x-1} + 2: (0, -2), (-1, 0)$$

A6 $g\left(\frac{x}{3}\right)$: multiply x coordinates by 3; the y intercept is $g(0) = -5$

$$(-3, 0), (15, 0), (0, -5), \text{ minimum } (6, -9)$$

 $-2g(x)$: multiply y coordinates by -2 , so the minimum becomes a maximum: $(2, 18)$ **A7** $f(x+2) - 1$: 2 left, 1 down: $(-6, 5)$

$$-f\left(\frac{x}{2}\right): x \text{ doubled, } y \text{ negated: } (-8, -6)$$

$$3f(-x): x \text{ negated, } y \text{ trebled: } (4, 18)$$



B**Fluency solutions**

One mark for each correct part. The notes in *italics* explain why each sequence was chosen.

CORE

B1 (a) $(1, 0)$, $(2, 0)$, $(-3, 0)$, $(0, 6)$; it crosses at all three

(b) $(-3, 0)$, $(0, 3)$; it touches at $(1, 0)$

(c) $(1, 0)$, $(2, 0)$, $(-3, 0)$, $(0, -6)$; it crosses at all three

(d) $(0, 0)$; it touches at $(1, 0)$

A squared factor touches the axis. In (c), $(1 - x)$ makes the x^3 term negative, so the shape flips.

B2 (a) $x(x - 3)(x + 3)$: $0, 3, -3$ (b) $x(x + 1)^2$: $0, -1$

(c) $x(2x - 1)(x - 3)$: $0, \frac{1}{2}, 3$ (d) $2x(3 - x)(3 + x)$: $0, 3, -3$

Always take out the common factor x first; (b) is a perfect square, so that curve touches at $x = -1$

B3 (a) $x = 0, y = 0$; no axis points (b) $x = 0, y = 1$; $(-2, 0)$

(c) $x = -1, y = 0$; $(0, 2)$ (d) $x = -1, y = -4$; $(0, -2)$, $\left(-\frac{1}{2}, 0\right)$

Each move carries the asymptotes with it. In (d), $\frac{2}{x+1} = 4$ gives $x + 1 = \frac{1}{2}$

B4 (a) **1** (b) **3** (c) **1** (d) **2**

In (a) $y = x^2$ meets only the first-quadrant branch of $y = \frac{4}{x}$; in (b) the lower curve $y = x^2 - 4$ dips below the axis and meets the third-quadrant branch twice; in (d) both curves are symmetrical about the y -axis.

B5 (a) $(2, 8)$ (b) $(-1, 5)$ (c) $\left(\frac{2}{3}, 5\right)$

Outside the bracket changes y ; inside the bracket changes x the opposite way.

B6 (a) $x = 2, y = 2$ (b) $x = 5, y = -1$ (c) $x = 2, y = -2$ (d) $x = 4, y = -1$

A vertical change moves only the horizontal asymptote; a horizontal change moves only the vertical one.



STRETCH

B7 (a) $(\pm 1, 0)$, $(\pm 2, 0)$, $(0, 4)$; crosses four times

(b) crosses $(\pm 3, 0)$; touches $(0, 0)$

(c) touches $(1, 0)$ and $(-2, 0)$; $(0, 4)$

(d) $(\pm 2, 0)$, $(0, -4)$; $x^2 + 1$ is never zero

Four, three, two, then two roots: a squared factor touches, and a factor with no real roots never meets the axis.

B8 (a) $(x+2)^2 - 4(x+2) = x^2 - 4$ (b) $2x^2 - 8x$ (c) $(2x)^2 - 4(2x) = 4x^2 - 8x$ (d) $x^2 + 4x$

Replace every x inside the bracket; multiply the whole function outside it.

B9 (a) $(0, -4)$, minimum (b) $(4, 2)$, maximum (c) $(-4, -2)$, minimum (d) $(8, -6)$, minimum

Only a negative sign outside the bracket turns a minimum into a maximum.

B10 (a) 3 (b) 2 (c) 1 (d) 1

Below the maximum but above the minimum 0: three; at the maximum: the line touches, two; above the maximum or below the minimum: one.

C

Reasoning solutions

Your wording can be different. Check that your reason makes the same point.

CORE

C1 The factor $(x-2)$ is squared, so $(2, 0)$ is a repeated root: the curve touches the x -axis there and does not cross it.

y intercept: $(0-2)^2(0+1) = 4$: $(0, 4)$

C2 A change inside the bracket moves the graph horizontally: $f(x-2)$ is a translation 2 units to the right.

$(3, 3)$

C3 The vertical asymptote is where the denominator is zero, $x+3=0$; as x gets large, $\frac{5}{x+3}$ tends to 0, so the horizontal asymptote is $y=0$, not $y=5$

$x = -3$, $y = 0$; $x = 0$: $y = \frac{5}{3}$: $\left(0, \frac{5}{3}\right)$



C4 Dividing by x loses the solution $x = 0$.

$$x^3 - 9x = 0, \text{ so } x(x-3)(x+3) = 0, \text{ and } y = 5x: (0, 0), (3, 15), (-3, -15)$$

STRETCH

C5 In $f(3x)$ the x coordinates are *divided* by 3, and y stays the same. Asha multiplied by 3.

$$(-2, 4). \text{ The curve } y = f\left(\frac{x}{3}\right) \text{ has its maximum at } (-18, 4)$$

C6 $(x+1)(x^2 - x + 2) = x^3 - x^2 + 2x + x^2 - x + 2 = x^3 + x + 2$

$$x^2 + 1 = -\frac{2}{x}, \text{ so } x^3 + x + 2 = 0, \text{ so } (x+1)(x^2 - x + 2) = 0$$

$$x^2 - x + 2 = 0 \text{ has } b^2 - 4ac = 1 - 8 = -7 < 0: \text{ no real roots. So } x = -1 \text{ is the only solution, and } y = x^2 + 1:$$

$$(-1, 2)$$

C7 Tom's curve has y intercept $(0+1)(0-2)^2 = 4$, not 8. He has left out the constant.

$$y = k(x+1)(x-2)^2 \text{ and } 4k = 8, \text{ so } k = 2: y = 2(x+1)(x-2)^2$$

C8 (a) $x(x-6) = 16$, so $x^2 - 6x - 16 = 0$, so $(x-8)(x+2) = 0$: $x = 8$ and $x = -2$, which are 10 units apart.

(b) $f(x-7)$ is a translation 7 units to the right. Both points move 7 to the right, so the distance between them is unchanged.

(c) $f(ax)$ divides every x coordinate by a , so the distance becomes $\frac{10}{a} = 4$: $a = \frac{5}{2}$

D

Problem solving solutions

There is often more than one correct method. Any complete, correct method scores full marks.

CORE

D1 (a) $f(x) = k(x+2)(x-3)^2$, and $f(0) = 18k = -9$, so $k = -\frac{1}{2}$

$$f(x) = -\frac{1}{2}(x+2)(x-3)^2: k = -\frac{1}{2}, p = 2, q = -3$$

(b) The curve is above the x -axis only to the left of $(-2, 0)$: $x < -2$



D2 (a) 2 units left: $(-7, 0)$, $(-1, 0)$, maximum $(-5, 8)$

(b) x coordinates halved, and the y intercept is unchanged: $(-\frac{5}{2}, 0)$, $(\frac{1}{2}, 0)$, $(0, \frac{5}{4})$, maximum $(-\frac{3}{2}, 8)$

D3 (a) $x = 2$ and $y = 0$

(b) $\frac{3}{x-2} = x-4$, so $3 = (x-4)(x-2)$, so $x^2 - 6x + 5 = 0$, so $(x-1)(x-5) = 0$, and $y = x-4$

$A(1, -3)$, $B(5, 1)$

D4 (a) **3**: the line $y = 2$ lies between the minimum value 0 (at O) and the maximum value 4, so it crosses C three times.

(b) **2**: the line $y = 4$ touches C at the maximum and crosses it once more, to the left of O

(c) $(x+1)^2(2-x) = (x+1)^2(3-(x+1))$, so this is $f(x+1) = 2$: a translation 1 unit left does not change the number of intersections with $y = 2$: **3**

STRETCH

D5 (a) $x = -1$, $y = 3$

$x = 0$: $y = 2 + 3 = 5$: $(0, 5)$. $y = 0$: $\frac{2}{x+1} = -3$, so $x+1 = -\frac{2}{3}$: $(-\frac{5}{3}, 0)$

(b) $f(x-2) - 3 = \frac{2}{x-1} + 3 - 3$: $y = \frac{2}{x-1}$, with asymptotes $x = 1$ and $y = 0$

D6 (a) **4**: the line $y = \frac{1}{2}$ lies between the minimum value 0 and the maximum value 1, so it meets C four times.

(b) $x^2(x-2)^2 = (x^2 - 2x)^2 = 1$, so $x^2 - 2x = 1$ or $x^2 - 2x = -1$

$x^2 - 2x - 1 = 0$ gives $x = 1 \pm \sqrt{2}$; $(x-1)^2 = 0$ gives $x = 1$: $x = 1 - \sqrt{2}, 1, 1 + \sqrt{2}$

(c) Strictly between the minimum and the maximum: $0 < k < 1$

D7 (a) At $x = 0$, $y = f(b) = (b-1)(b-5) = 0$: $b = 1$ or $b = 5$

(b) $af(3) = -4a = 10$: $a = -\frac{5}{2}$. As $a < 0$ the curve is reflected, so $(3, 10)$ is a **maximum** point.



D8 (a) $B: y = (4)(1)(-1) = -4$. Gradient $\frac{-4 - 0}{3 - (-1)} = -1$: $y = -x - 1$

(b) $(x+1)(x-2)(x-4) = -(x+1)$

$(x+1)[(x-2)(x-4)+1] = 0$, so $(x+1)(x^2 - 6x + 9) = 0$, so $(x+1)(x-3)^2 = 0$

as required

(c) $x = 3$ is a repeated root, so l touches C at B : it is a tangent to C at B

E

Exam practice solutions

Mark-scheme notes are shown on the right of each line.

CORE

Exam questions for everyone

E1 (a) The same shape, translated 2 units to the right

B1

crosses at $(-2, 0)$, touches at $(4, 0)$

B1

maximum $(0, 8)$, which is also the y intercept

B1

(b) $(-2, 0)$, touches at $(1, 0)$, y intercept $(0, 4)$

B1

maximum $(-1, 8)$, in the second quadrant

B1

A curve whose maximum is on the y -axis or in the first quadrant in (b) has been stretched the wrong way.

E2 (a) The shape of $y = \frac{3}{x}$ moved up 2 units

B1

asymptotes $x = 0$ and $y = 2$

B1

$y = 0$: $\frac{3}{x} = -2$, so $\left(-\frac{3}{2}, 0\right)$

B1

(b) $2 + \frac{3}{x} = 5 - 2x$

M1: equates

$2x + 3 = 5x - 2x^2$, so $2x^2 - 3x + 3 = 0$

A1

$b^2 - 4ac = 9 - 24 = -15 < 0$, so there are no real roots

A1*

so l does not meet C

as required



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STRETCH

E3 (a) **a stretch parallel to the y -axis, scale factor 4** B1

or a stretch parallel to the x -axis, scale factor 2, since $\frac{4}{x^2} = \frac{1}{\left(\frac{x}{2}\right)^2}$ B1

(b) **a translation by the vector $\begin{pmatrix} -1 \\ 0 \end{pmatrix}$** B1

(c) $\frac{1}{x^2} = \frac{4}{(x+1)^2}$ M1: equates and cross-multiplies

$(x+1)^2 = 4x^2$, so $3x^2 - 2x - 1 = 0$, so $(3x+1)(x-1) = 0$ M1: solves

$x = 1$ or $x = -\frac{1}{3}$ A1

$y = \frac{1}{x^2}$ gives $y = 1$ and $y = 9$ M1

$P(1, 1)$, $Q\left(-\frac{1}{3}, 9\right)$ A1

