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A-LEVEL MATHEMATICS • STATISTICS 2

Poisson Distributions

Worksheet

Pearson Edexcel International A Level (WST02) • Calculator and statistical tables allowed: show every step

Free Poisson Distributions worksheet for A-Level and International A-Level Maths, with full worked solutions

A **Examples** & Your turn page 2

B **Fluency** core skills page 4

C **Reasoning** explain it page 6

D **Problem** solving page 7

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Tap a section to jump straight to it

By the end of this worksheet you can: state a Poisson model with its conditions in context • find probabilities from the formula and the tables • scale the rate to a new interval • use the mean and variance • *Stretch:* unknown rates, sums of independent Poisson variables and binomial models built on Poisson probabilities.

Which questions are for me?

CORE The methods every A-level student must secure. Start here.

STRETCH Unknown rates, sums of Poisson variables, conditional splits and binomial models built on Poisson probabilities.

Every section has both, clearly labelled. Questions get harder as you go, and every skill in Sections A to D is needed in Section E.

Track your progress

Section	B	C	D	E
Core marks	23	10	13	22
Stretch marks	16	14	17	14
My score				

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A Worked examples & Your turn

Read each example carefully. Cover it up, then answer the question beside it using the same method.

CORE *Secure the method*

Worked example A1: the formula

$X \sim \text{Po}(3.5)$. Find $\text{P}(X = 2)$

$$\text{P}(X = r) = \frac{\text{e}^{-\lambda}\lambda^{r}}{r!}$$

$$\frac{\text{e}^{-3.5}(3.5)^{2}}{2!}$$

0.1850

Your turn A1

$X \sim \text{Po}(2.5)$. Find $\text{P}(X = 4)$

Worked example A2: the tables

$X \sim \text{Po}(6)$. Find $\text{P}(X \geqslant 4)$ and $\text{P}(3 < X \leqslant 8)$

The tables give $\text{P}(X \leqslant x)$

$1 - \text{P}(X \leqslant 3) = 1 - 0.1512;\ 0.8472 - 0.1512$

0.8488, 0.6960

Your turn A2

$X \sim \text{Po}(4.5)$. Find $\text{P}(X > 6)$ and $\text{P}(2 \leqslant X < 7)$

Worked example A3: scaling the rate

Calls arrive at a mean rate of 12 per hour. Find the probability of no calls in a 15-minute period.

15 minutes is a quarter of an hour: Po(3)

$\text{P}(X = 0) = \text{e}^{-3}$

0.0498

Your turn A3

Calls arrive at a mean rate of 8 per hour. Find the probability of at most 2 calls in a 45-minute period.

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STRETCH

Worked example A4: adding Poisson variables

$X \sim \text{Po}(2)$ and $Y \sim \text{Po}(1.5)$ are independent. Find $\text{P}(X+Y=3)$

$X+Y \sim \text{Po}(2+1.5)$

$$\frac{\text{e}^{-3.5}(3.5)^3}{3!}$$

0.2158

Your turn A4

$X \sim \text{Po}(2.5)$ and $Y \sim \text{Po}(3.5)$ are independent. Find $\text{P}(X+Y \leqslant 4)$

Worked example A5: λ from a relation

$X \sim \text{Po}(\lambda)$ and $\text{P}(X=1) = \text{P}(X=2)$. Find λ

$$\text{e}^{-\lambda}\lambda = \frac{\text{e}^{-\lambda}\lambda^2}{2}, \text{ so } \lambda^2 = 2\lambda$$

$\lambda = 2$

Your turn A5

$X \sim \text{Po}(\lambda)$ and $\text{P}(X=2) = 2\text{P}(X=1)$. Find λ

Worked example A6: a conditional probability

$X \sim \text{Po}(5)$. Find $\text{P}(X=6 \mid X>4)$

$$\frac{\text{P}(X=6)}{1-\text{P}(X\leqslant 4)} = \frac{0.1462}{1-0.4405}$$

0.2613

Your turn A6

$X \sim \text{Po}(3)$. Find $\text{P}(X \leqslant 1 \mid X \leqslant 3)$

Worked example A7: a binomial built on a Poisson

Faults in a cable occur at 1.5 per metre. Ten separate 1-metre lengths are tested. Find the probability that exactly 2 lengths have no faults.

$p = \text{P}(\textit{no faults}) = \text{e}^{-1.5} = 0.2231$, *then* $\text{B}(10, p)$

$$\binom{10}{2}(0.2231)^2(0.7769)^8$$

0.2972

Your turn A7

Errors on a web page occur at 0.8 per page. Twelve pages are checked. Find the probability that at least one page has 2 or more errors.

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B Fluency

Each question is a designed sequence. From one part to the next, ask yourself: what changed, and what did it do to the answer?

CORE *Secure the method*

B1 $X \sim \text{Po}(4.5)$. Use the tables to find

(a) $\text{P}(X \leqslant 3)$

(b) $\text{P}(X < 3)$

(c) $\text{P}(X \geqslant 3)$

(d) $\text{P}(X > 3)$

[4 marks]

B2 Use the formula to find $\text{P}(X = 3)$ when

(a) $X \sim \text{Po}(1)$

(b) $X \sim \text{Po}(2)$

(c) $X \sim \text{Po}(3)$

(d) $X \sim \text{Po}(4)$

[4 marks]

B3 Buses pass a stop at a mean rate of 6 per hour. Find the probability that

(a) exactly 2 pass in 20 minutes

(b) none pass in 10 minutes

(c) more than 8 pass in 90 minutes

(d) at most 1 passes in 5 minutes

[4 marks]

B4 In each part μ and σ are the mean and the standard deviation of X

(a) $X \sim \text{Po}(9)$: find the standard deviation of X

(b) $X \sim \text{Po}(9)$: find $\text{P}(\mu - \sigma \leqslant X \leqslant \mu + \sigma)$

(c) X has variance 2.25: find its mean and standard deviation

(d) $X \sim \text{Po}(4)$: find $\text{P}(X > \mu + 2\sigma)$

[4 marks]

B5 State, with a reason, whether a Poisson distribution is likely to be a suitable model for

(a) the number of cars passing a checkpoint on a quiet road in 10 minutes.

(b) the number of cracked eggs in a box of 6 eggs.

(c) the number of customers entering a cafe in an hour, counted over a whole day including the lunchtime rush.

[3 marks]

B6 $X \sim \text{Po}(1.5)$ and $Y \sim \text{Po}(2.5)$ are independent. Find

(a) $\text{P}(X + Y = 4)$

(b) $\text{P}(X + Y \leqslant 2)$

(c) $\text{P}(X = 0 \text{ and } Y = 0)$

(d) $\text{P}(X = 1 \mid X + Y = 4)$

[4 marks]

STRETCH

B7 $X \sim \text{Po}(\lambda)$. Find λ when

(a) $\text{P}(X = 0) = 0.2$

(b) $\text{P}(X \geqslant 1) = 0.95$

(c) $\text{P}(X = 1) = 3\text{P}(X = 0)$

(d) $\text{P}(X = 3) = \text{P}(X = 4)$

[4 marks]

B8 Emails arrive at a mean rate of 3 per hour. An hour is *busy* if more than 4 emails arrive.

(a) Find the probability that an hour is busy.

(b) Find the probability that exactly 2 of 8 hours are busy.

(c) Find the expected number of busy hours in 40 hours.

(d) Find the probability that the first busy hour is the third hour.

[4 marks]

B9 Cars pass a point at a mean rate of 2 per minute. In an interval of t minutes,

(a) write down the probability that no cars pass

(b) find the longest t for which $\text{P(no cars)} \geqslant 0.5$

(c) find the least t for which $\text{P(at least one car)} > 0.99$

(d) find the least t for which $\text{P(at least two cars)} > 0.9$

[4 marks]

B10 $X \sim \text{Po}(4)$ and $Y = 2X + 3$

(a) Find $\text{E}(Y)$

(b) Find $\text{Var}(Y)$

(c) Find $\text{P}(Y > \text{E}(Y))$

(d) Find $\text{P}(Y \leqslant 7)$

[4 marks]

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C Reasoning

Show your working and explain your thinking. A correct answer without a reason does not score full marks.

CORE *Secure the method*

C1 Customers enter a shop at a mean rate of 7 every 10 minutes. Amir gives, as a condition for a Poisson model, "each customer has a constant probability of entering". Explain why this does not score, and give a suitable condition. **[2 marks]**

C2 $X \sim \text{Po}(6)$. Asked for $\text{P}(X = 5)$, Bea reads 0.4457 from the tables. Explain her error, and find $\text{P}(X = 5)$ **[2 marks]**

C3 Litter is found on a beach at a mean rate of 6 items per square metre, and 25% of the items are plastic. Cal models the number of plastic items in a 4 m^2 area as B(4, 0.25)

Explain his error, and find the probability that there are more than 4 plastic items in the area. **[3 marks]**

C4 Calls reach a helpline at a mean rate of 9 per hour. For the probability of at most 2 calls in 20 minutes, Dev uses Po(9) and gets 0.0062

Explain his error, and find the probability. **[3 marks]**

STRETCH

C5 $X \sim \text{Po}(2)$ and $Y \sim \text{Po}(3)$ are independent. For $\text{P}(X + Y = 1)$, Eve writes $\text{P}(X = 1) \times \text{P}(Y = 1) = 0.0404$

Explain her error, and find $\text{P}(X + Y = 1)$ **[3 marks]**

C6 The numbers of calls per minute in a sample have mean 4 and standard deviation 2. Fay says a Poisson model is unsuitable because the mean and the standard deviation are not equal.

Explain her error, and state what the data suggest. **[3 marks]**

C7 $X \sim \text{Po}(4.5)$. To find $\text{P}(X \leqslant 3)$, Gus uses a normal distribution with mean 4.5 and variance 4.5

Explain why this is not appropriate, and find $\text{P}(X \leqslant 3)$ **[3 marks]**

C8 $X \sim \text{Po}(\lambda)$ and $Y \sim \text{Po}(\mu)$ are independent.

(a) By considering the ways in which $X + Y = 1$ can occur, show that $\text{P}(X + Y = 1) = (\lambda + \mu)\text{e}^{-(\lambda+\mu)}$

(2)

(b) Show that $\text{P}(X = 1 \mid X + Y = 2) = \dfrac{2\lambda\mu}{(\lambda + \mu)^2}$

(3)

[5 marks]

D Problem solving

You decide the method. Read the whole question first, and show every step.

CORE *Secure the method*

D1 Flaws occur in a sheet of glass at a mean rate of 0.8 per square metre. A piece of glass of area 4 square metres is chosen at random.

(a) Find the probability that the piece has exactly 3 flaws.

(2)

(b) Find the probability that the piece has at least 3 flaws.

(1)

[3 marks]

D2 Calls to a helpline arrive at a mean rate of 2.5 every 10 minutes during the morning.

(a) State a suitable distribution to model the number of calls in a 30-minute period during the morning, and give two assumptions, in context, needed for the model to be valid.

(3)

(b) Find the probability that there are fewer than 5 calls in a randomly chosen 30-minute period during the morning.

(1)

[4 marks]

D3 A cafe sells cakes at a mean rate of 1.5 every 10 minutes and, independently, muffins at a mean rate of 1 every 10 minutes. Find the probability that in a randomly chosen 10-minute period the cafe sells exactly 2 of these items, both of them cakes. **[3 marks]**

D4 Accidents at a factory occur at a mean rate of 0.6 per week. Find the probability that, in a period of 10 weeks, there is at most 1 week in which an accident occurs. **[3 marks]**

STRETCH

D5 The random variable $Y \sim \text{Po}(\lambda)$. Given that $\text{P}(Y = 3) = 0.1687$ and $\text{P}(Y = 4) = 0.1898$, correct to 4 decimal places,

(a) by forming and solving 2 equations, find the value of λ to 2 significant figures

(2)

(b) find $\text{P}(Y > 5)$

(2)

[4 marks]

D6 Patients arrive at a hospital department at a mean rate of λ per hour. The probability that no patients arrive in a 30-minute period is 0.0498

(a) Show that $\lambda = 6$

(2)

(b) Find the probability that fewer than 4 patients arrive in a 40-minute period.

(2)

[4 marks]

D7 Emails arrive at an office at a mean rate of 4 per hour. An hour is described as *busy* if 6 or more emails arrive in it. Find the probability that, in a working day of 8 hours, at least 2 of the hours are busy.

[4 marks]

D8 A shop sells a particular television at a mean rate of 2.5 per week. At the start of each week the shop has k of these televisions in stock, and no more are delivered during the week.

(a) For $k = 4$, find the probability that the shop cannot meet the demand in a week.

(1)

(b) Find the least value of k for which the probability that the shop cannot meet the demand in a week is less than 0.02

(2)

(c) Using your value of k from part (b), find the probability that the shop meets the demand in each of 4 weeks.

(2)

[5 marks]

E Exam practice

Full-size exam questions in the style of WST02. Write your answers in the spaces, and show every step.

CORE *Exam questions for everyone*

E1 An online shop sells garden furniture through its website. During the afternoon, the website receives orders at a mean rate of 3.5 every 10 minutes.

(a) State a suitable distribution to model the number of orders received in a 20-minute period during the afternoon, and give two assumptions, in context, needed for this distribution to be a suitable model.

(3)

(b) Find the probability that exactly 5 orders are received in a 20-minute period.

(2)

(c) Find the probability that more than 8 orders are received in a 20-minute period.

(2)

(d) Find the probability that fewer than 2 orders are received in a 5-minute period.

(2)

Six separate 5-minute periods are chosen at random during the afternoon.

(e) Find the probability that fewer than 2 orders are received in exactly 4 of these 6 periods.

(3)

(Total for Question E1 is 12 marks)

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E2 A typist is checking the accuracy of her work. The numbers of typing errors on 120 randomly chosen pages of a long report that she typed are recorded in the table below.

Number of errors	0	1	2	3	4	5	6
Number of pages	27	40	30	15	6	2	0

(a) Calculate the mean and the variance of these data.

(3)

(b) Explain how your answers to part (a) support the use of a Poisson distribution to model the number of errors on a page.

(1)

The number of errors on a page is modelled by a Poisson distribution with mean 1.5

(c) Find the probability that a randomly chosen page has at least 3 errors.

(2)

(d) Find the probability that a randomly chosen 4-page letter typed by the same typist has fewer than 5 errors in total.

(2)

(e) A 4-page letter has exactly 4 errors in total. Show that the probability that there is exactly one error on each page is $\frac{3}{32}$

(2)

(Total for Question E2 is 10 marks)

STRETCH

E3 The number of breakdowns of a lift in a large office block is modelled by a Poisson distribution with a mean rate of λ breakdowns per month. The probability that the lift has no breakdowns in a 2-month period is 0.0907

(a) Show that $\lambda = 1.2$, to 2 significant figures.

(3)

(b) Find the probability that the first lift has more than 3 breakdowns in a randomly chosen 3-month period.

(3)

A second lift in the same office block breaks down, independently of the first lift, at a mean rate of 0.8 breakdowns per month.

(c) Find the probability that there is at most one breakdown altogether in a month.

(2)

(d) In a particular month there are exactly 3 breakdowns altogether. Find the probability that all 3 were breakdowns of the first lift.

(3)

(e) Find the least number of whole months, n, for which the probability that the first lift has at least one breakdown in n months is greater than 0.999

(3)

(Total for Question E3 is 14 marks)

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