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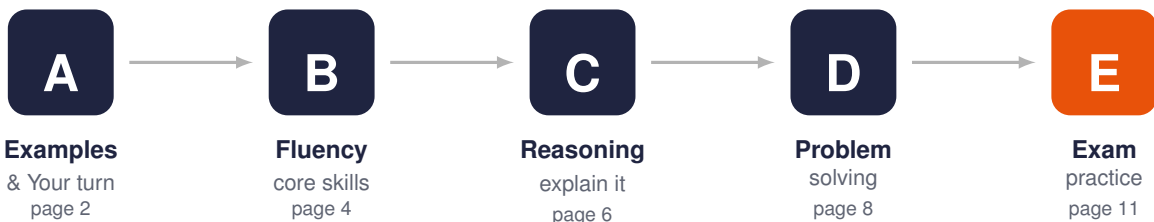
A-LEVEL MATHEMATICS • PURE 1

Integration

Worksheet

Pearson Edexcel International A Level (WMA11) & A Level (9MA0) • Calculator allowed: show every step

Free Integration worksheet for A-Level and International A-Level Maths, with full worked solutions



Tap a section to jump straight to it

By the end of this worksheet you can: integrate powers of x , including negative and fractional powers • write roots, brackets and fractions as powers before integrating • include the constant of integration every time • find the equation of a curve from its gradient function and a point • *Stretch:* integrate twice, use tangent information and two conditions to find unknown constants, and prove results.

Which questions are for me?

CORE The methods every A-level student must secure. Start here.

STRETCH Integrating twice, constants from tangents and from two conditions, and reasoning about families of curves.

Every section has both, clearly labelled. Questions get harder as you go, and every skill in Sections A to D is needed in Section E.

Track your progress

Section	B	C	D	E
Core marks	23	10	13	11
Stretch marks	16	14	17	8
My score				

Full worked solutions

Exam questions by topic

Integration revision engine

Formula sheet builder

Name

Class

Date



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A

Worked examples & Your turn

Read each example carefully. Cover it up, then answer the question beside it using the same method.

CORE

Secure the method

Worked example A1

Find $\int (6x^2 - 4x^3 + 5) \, dx$

Add 1 to the power, then divide by the new power.

Never forget +c

$$\frac{6x^3}{3} - \frac{4x^4}{4} + 5x + c$$

$$2x^3 - x^4 + 5x + c$$

Your turn A1

Find $\int (12x^3 + 6x - 7) \, dx$

Worked example A2

Find $\int (3x^{-2} + 4\sqrt{x}) \, dx$

$\sqrt{x} = x^{\frac{1}{2}}$. Dividing by $\frac{3}{2}$ means multiplying by $\frac{2}{3}$

$$\frac{3x^{-1}}{-1} + \frac{4x^{\frac{3}{2}}}{\frac{3}{2}} + c$$

$$-3x^{-1} + \frac{8}{3}x^{\frac{3}{2}} + c$$

Your turn A2

Find $\int (8x^{-3} - 6\sqrt{x}) \, dx$

Worked example A3

$f'(x) = 6x^2 - 2x$ and $f(1) = 5$. Find $f(x)$

$f(x) = 2x^3 - x^2 + c$

Use the point to find c : $2 - 1 + c = 5$, so $c = 4$. Then write $f(x)$ in full.

$$f(x) = 2x^3 - x^2 + 4$$

Your turn A3

$f'(x) = 4x^3 + 3$ and $f(-1) = 6$. Find $f(x)$



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STRETCH

Worked example A4: expand first

Find $\int (2x+3)^2 dx$

There is no rule for brackets: expand first.

$$\int (4x^2 + 12x + 9) dx$$

$$\frac{4}{3}x^3 + 6x^2 + 9x + c$$

Your turn A4

Find $\int (x-4)(3x+1) dx$

Worked example A5: split the fraction

Find $\int \frac{x^3+6}{x^2} dx$

Divide each term by x^2 : $x + 6x^{-2}$

$$\frac{1}{2}x^2 - 6x^{-1} + c$$

Your turn A5

Find $\int \frac{4x-3\sqrt{x}}{\sqrt{x}} dx$

Worked example A6: a curve through a point

A curve has $\frac{dy}{dx} = 3\sqrt{x} - \frac{2}{x^2}$ and passes through (1, 4). Find its equation.

$$y = 2x^{\frac{3}{2}} + 2x^{-1} + c$$

At (1, 4): $2 + 2 + c = 4$, so $c = 0$

$$y = 2x^{\frac{3}{2}} + \frac{2}{x}$$

Your turn A6

A curve has $\frac{dy}{dx} = 6\sqrt{x} + \frac{1}{x^2}$ and passes through (4, 20). Find its equation.

Worked example A7: integrating twice

$$f''(x) = 6x, f'(1) = 5 \text{ and } f(1) = 2. \text{ Find } f(x)$$

A constant at each integration: $f'(x) = 3x^2 + c_1$, so $c_1 = 2$

$$f(x) = x^3 + 2x + c_2, \text{ so } 1 + 2 + c_2 = 2$$

$$f(x) = x^3 + 2x - 1$$

Your turn A7

$$f''(x) = 12x^2 - 2, f'(0) = 3 \text{ and } f(0) = -1. \text{ Find } f(x)$$



B**Fluency**

Each question is a designed sequence. From one part to the next, ask yourself: what changed, and what did it do to the answer?

CORE*Secure the method***B1** Find

(a) $\int x^4 dx$

(b) $\int 5x^{-2} dx$

(c) $\int x^{\frac{2}{3}} dx$

(d) $\int 6x^{-\frac{1}{2}} dx$

[4 marks]

B2 Write each integrand as a power of x , then integrate.

(a) $\int \sqrt{x} dx$

(b) $\int \frac{1}{x^3} dx$

(c) $\int \frac{2}{\sqrt{x}} dx$

(d) $\int x\sqrt{x} dx$

[4 marks]

B3 Expand, then integrate.

(a) $\int x(x+2) dx$

(b) $\int (x+2)^2 dx$

(c) $\int (x+2)(x-2) dx$

(d) $\int (\sqrt{x}+2)^2 dx$

[4 marks]

B4 $f'(x) = 2x$. Find $f(x)$ when the curve $y = f(x)$ passes through

(a) $(0, 3)$

(b) $(1, 3)$

(c) $(2, 3)$

(d) $(-3, 3)$

[4 marks]



CORE continued: rewriting, and the two rules side by side**B5** Split each fraction into powers of x , then integrate.

(a) $\int \frac{x^2 + 1}{x^2} dx$

(b) $\int \frac{2x^3 - x}{x} dx$

(c) $\int \frac{x+1}{\sqrt{x}} dx$

[3 marks]

B6 For each expression, find (i) its derivative and (ii) its integral with respect to x

(a) $3x^2$

(b) $\frac{4}{x^2}$

(c) $\sqrt{x}$

(d) $x^{-\frac{3}{2}}$

[4 marks]

STRETCH**B7** Find the equation of each curve.

(a) $\frac{dy}{dx} = 3\sqrt{x}$, through (4, 10)

(b) $\frac{dy}{dx} = \frac{1}{x^2}$, through (1, 0)

(c) $\frac{dy}{dx} = \frac{1}{\sqrt{x}}$, through (9, 5)

(d) $\frac{dy}{dx} = 2x - 3x^2$, through (2, 0)

[4 marks]

B8 Find $f(x)$ in each case.

(a) $f''(x) = 6$, $f'(0) = 0$, $f(0) = 0$

(b) $f''(x) = 6x$, $f'(0) = 1$, $f(0) = 0$

(c) $f''(x) = 12x^2$, $f'(1) = 0$, $f(1) = 0$

(d) $f''(x) = 2x^{-3}$, $f'(1) = 0$, $f(1) = 0$

[4 marks]

B9 In each case, the curve $y = f(x)$ passes through both points. Find the value of the constant k

(a) $f'(x) = kx$, through (0, 0) and (2, 8)

(b) $f'(x) = k\sqrt{x}$, through (0, 0) and (4, 32)

(c) $f'(x) = 3x^2 + k$, through (0, 0) and (1, 5)

(d) $f'(x) = kx^{-2}$, through (1, 0) and (2, 3)

[4 marks]

B10 In each case $f(1) = 2$. Find $f(x)$, and hence the value of $f(4)$

(a) $f'(x) = 2x$

(b) $f'(x) = 3\sqrt{x}$

(c) $f'(x) = \frac{1}{x^2}$

(d) $f'(x) = \frac{6}{\sqrt{x}}$

[4 marks]



C**Reasoning**

Show your working and explain your thinking. A correct answer without a reason does not score full marks.

CORE*Secure the method*

C1 Sam writes $\int 4x^3 dx = x^4$

Explain what Sam has left out, and give the equations of two different curves that both have gradient function $4x^3$ [2 marks]

C2 Lily integrates $\frac{x^2 + 1}{x^2}$ by integrating the top and the bottom separately, and writes $\frac{\frac{1}{3}x^3 + x}{\frac{1}{3}x^3}$

Explain why this is wrong, and find the correct integral. [2 marks]

C3 A curve has gradient function $3x^2 - 4$ and passes through the point (2, 5). Kai writes $y = x^3 - 4x + c$, then uses $0 = 8 - 8 + c$ to get $c = 0$

Explain Kai's mistake, and find the correct equation of the curve. [3 marks]

C4 Jo writes $\int 6x^{-3} dx = \frac{6x^{-4}}{-4} = -\frac{3}{2}x^{-4} + c$

Explain Jo's mistake, find the correct integral, and show how differentiating your answer checks it.

[3 marks]



STRETCH

- C5** Max is told that $f''(x) = 6x$, $f'(1) = 4$ and $f(1) = 3$. He integrates twice, adding a single constant at the end: $f(x) = x^3 + c$, so $c = 2$

Explain Max's mistake, and find the correct $f(x)$

[3 marks]

- C6** Show that every curve with gradient function $4x - 3$ is a translation of the curve $y = 2x^2 - 3x$ parallel to the y -axis, and find the translation that gives the curve through $(1, 6)$

[3 marks]

- C7** Show, by differentiating, that $\int \frac{x^2 - 1}{x^2} dx = x + \frac{1}{x} + c$

Explain why differentiating is always a valid check of an integral.

[3 marks]

- C8** $f'(x) = 3x^2 - 6x + k$, where k is a constant. The tangent to the curve $y = f(x)$ at the point where $x = 1$ is horizontal, and the curve passes through $(0, 2)$

(a) Find the value of k .

(1)

(b) Find $f(x)$

(2)

(c) Show that $f(x) = (x - 1)^3 + 3$, and hence explain why the gradient of the curve is never negative.

(2)

[5 marks]



D**Problem solving**

You decide the method. Read the whole question first, and show every step.

CORE*Secure the method*

- D1** The curve C has gradient function $\frac{dy}{dx} = 4x^3 - 6x$

The curve passes through the point $(2, 7)$

(a) Find the equation of C .

(2)

(b) Find the coordinates of the point where C crosses the y -axis.

(1)

[3 marks]

- D2** *In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.*

$$f'(x) = \frac{3x^2 - 2}{\sqrt{x}} \quad x > 0$$

(a) Write $f'(x)$ in the form $Ax^p + Bx^q$, where A , B , p and q are constants.

(1)

Given that the curve $y = f(x)$ passes through the point $(4, 30)$,

(b) find $f(x)$, giving each term in its simplest form.

(3)

[4 marks]

- D3** The curve C has gradient function $\frac{dy}{dx} = 3x^2 + kx$, where k is a constant.

The point $P(1, 4)$ lies on C , and the gradient of C at P is 7

(a) Find the value of k .

(1)

(b) Find the equation of C .

(2)

[3 marks]



- D4** The height of a plant, h cm, t weeks after it is planted, is modelled by

$$\frac{dh}{dt} = 6\sqrt{t} \quad t \geq 1$$

One week after it is planted, the plant is 5 cm tall.

- (a) Find h in terms of t .

(2)

- (b) Find, according to the model, the height of the plant 9 weeks after it is planted.

(1)

[3 marks]

STRETCH

- D5** In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.

$$f''(x) = 6x - \frac{4}{x^3} \quad x > 0$$

Given that $f'(1) = 0$ and $f(1) = 4$,

- (a) find $f'(x)$

(2)

- (b) find $f(x)$

(2)

[4 marks]

- D6** The curve C has gradient function $\frac{dy}{dx} = kx - \frac{4}{x^2}$, $x \neq 0$, where k is a constant.

The line with equation $y = 5x - 3$ is the tangent to C at the point P , where $x = 2$

- (a) Find the value of k .

(2)

- (b) Find the equation of C .

(2)

[4 marks]



D7 In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.

$$f'(x) = 3x^2 - 12$$

The curve $y = f(x)$ passes through the point $(2, -16)$

(a) Find $f(x)$

(2)

(b) Find the exact x coordinates of the points where the curve meets the x -axis.

(2)

[4 marks]

D8 $f'(x) = ax^2 + bx$, where a and b are constants.

The curve $y = f(x)$ has gradient 1 at the point where $x = 1$, and gradient 12 at the point where $x = 2$

(a) Show that $a + b = 1$ and $2a + b = 6$

(2)

(b) Find the value of a and the value of b .

(1)

Given that the curve passes through the point $(1, 2)$,

(c) find $f(x)$

(2)

[5 marks]



E**Exam practice**

Exam-style questions in the style of WMA11. Write your answers in the spaces, and show every step.

CORE

Exam questions for everyone

- E1** In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.

The curve C has equation $y = f(x)$, $x > 0$, where

$$f'(x) = \frac{5x^2 - 8}{2\sqrt{x}}$$

- (a) Write $f'(x)$ in the form $Px^a + Qx^b$, where P , Q , a and b are constants.

.....
(2)

Given that the point $P(4, 10)$ lies on C ,

- (b) find $f(x)$, giving each term in its simplest form.

.....
(3)

(Total for Question E1 is 5 marks)



E2 The curve C has equation $y = f(x)$, where

$$f'(x) = 6x^2 - 2x + k$$

and k is a constant. The curve C passes through the points $(0, -3)$ and $(1, 3)$

(a) Find $f(x)$ in terms of k , and show that the constant of integration is -3

(1)

(b) Find the value of k .

(2)

(c) Find the equation of the normal to C at the point $(1, 3)$, giving your answer in the form $px + qy + r = 0$, where p , q and r are integers.

(3)

(Total for Question E2 is 6 marks)



STRETCH

E3 In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.

The curve C has equation $y = f(x)$, $x \geq 0$, where

$$f'(x) = k\sqrt{x} - 3$$

and k is a constant. The curve passes through the points $(1, 1)$ and $(4, 6)$

(a) Show that $2k + 3c = 12$ and $16k + 3c = 54$, where c is the constant of integration.

(3)

(b) Hence find the value of k and the value of c , and write down $f(x)$

(2)

(c) Find the coordinates of the points on C where $y = 2$, giving exact answers.

(3)

(Total for Question E3 is 8 marks)

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