



eClassroom

A-LEVEL MATHEMATICS • PURE 3

Trigonometric Functions

Worksheet

Pearson Edexcel International A Level (WMA13) & A Level (9MA0) • Calculator allowed: show every step

Free Trigonometric Functions worksheet for A-Level and International A-Level Maths, with full worked solutions



Tap a section to jump straight to it

By the end of this worksheet you can: sec, cosec and cot • their graphs and identities • *Stretch:* inverse trig functions and proofs.

Which questions are for me?

CORE The methods every A-level student must secure. Start here.

STRETCH Quadratics in sec and cosec, reading graphs, inverse functions and proofs.

Every section has both, clearly labelled. Questions get harder as you go, and every skill in Sections A to D is needed in Section E.

Track your progress

Section	B	C	D	E
Core marks	23	10	13	11
Stretch marks	16	14	17	8
My score				

Full worked solutions

Exam questions by topic

Trigonometry revision engine

Formula sheet builder

Name

Class

Date



Join students already acing their exams.

Book your FREE lesson at www.eclassroom.org.uk

A

Worked examples & Your turn

Read each example carefully. Cover it up, then answer the question beside it using the same method.

CORE

Secure the method

Worked example A1

Find the exact values of $\sec 60^\circ$, $\operatorname{cosec} 210^\circ$ and $\cot 135^\circ$

$\sec = \frac{1}{\cos}$, $\operatorname{cosec} = \frac{1}{\sin}$, $\cot = \frac{1}{\tan}$; take the sign from the quadrant.

$$\frac{1}{\frac{1}{2}}, \frac{1}{-\frac{1}{2}}, \frac{1}{-1}$$

$$2, -2, -1$$

Your turn A1

Find the exact values of $\operatorname{cosec} 45^\circ$, $\sec 150^\circ$ and $\cot 300^\circ$

Worked example A2

Prove that $\sec x - \cos x \equiv \sin x \tan x$

Write in sin and cos, then use $1 - \cos^2 x = \sin^2 x$

$$\frac{1}{\cos x} - \cos x = \frac{1 - \cos^2 x}{\cos x} = \frac{\sin^2 x}{\cos x}$$

$$= \sin x \times \frac{\sin x}{\cos x}$$

Your turn A2

Prove that $\operatorname{cosec} x - \sin x \equiv \cos x \cot x$

Worked example A3

Solve $\sec x = 2.5$ for $0 \leq x < 360^\circ$

$\sec x = 2.5$ means $\cos x = 0.4$

66.4° and $360^\circ - 66.4^\circ$

$$x = 66.4^\circ, 293.6^\circ$$

Your turn A3

Solve $\cot x = -0.5$ for $0 \leq x < 360^\circ$



STRETCH

Worked example A4: a quadratic in secSolve $2 \tan^2 x + \sec x = 1$ for $0 \leq x < 360^\circ$

$$\tan^2 x = \sec^2 x - 1: 2 \sec^2 x + \sec x - 3 = 0$$

$$(2 \sec x + 3)(\sec x - 1) = 0, \text{ so } \cos x = -\frac{2}{3} \text{ or } \cos x = 1$$

$$0^\circ, 131.8^\circ, 228.2^\circ$$

Your turn A4Solve $\cot^2 x - \operatorname{cosec} x = 1$ for $0 \leq x < 360^\circ$ **Worked example A5: inverse functions**Find the exact values of $\arcsin \frac{1}{2}$, $\arccos \left(-\frac{1}{2}\right)$ and $\arctan(-1)$ *Ranges: arcsin, arctan in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$; arccos in $[0, \pi]$*

$$\frac{\pi}{6}, \frac{2\pi}{3}, -\frac{\pi}{4}$$

Your turn A5Find the exact values of $\arccos \frac{\sqrt{3}}{2}$, $\arcsin \left(-\frac{\sqrt{2}}{2}\right)$ and $\arctan \sqrt{3}$ **Worked example A6: a triangle for an inverse**Find the exact value of $\tan \left(\arcsin \frac{3}{5} \right)$ *Let $\theta = \arcsin \frac{3}{5}$: a right-angled triangle with opposite 3, hypotenuse 5, adjacent 4*

$$\frac{3}{4}$$

Your turn A6Find the exact value of $\cos(\arctan 2)$ **Worked example A7: reading a graph**State the asymptotes and the range of $y = 2 \sec x$, $0 \leq x \leq 2\pi$ *$\sec x$ is undefined where $\cos x = 0$, and $|\sec x| \geq 1$*

$$x = \frac{\pi}{2}, \frac{3\pi}{2}; y \leq -2 \text{ or } y \geq 2$$

Your turn A7For $y = \operatorname{cosec} 2x$, $0 < x < \pi$, state the asymptote and the turning points.

B**Fluency**

Each question is a designed sequence. From one part to the next, ask yourself: what changed, and what did it do to the answer?

CORE*Secure the method***B1** Find the exact value of

(a) $\sec 45^\circ$

(b) $\operatorname{cosec} 300^\circ$

(c) $\cot 240^\circ$

(d) $\sec 180^\circ$

[4 marks]**B2** Simplify

(a) $\tan x \operatorname{cosec} x$

(b) $\cot x \sec x$

(c) $\sin x \sec x \cot x$

(d) $\sec^2 x - 1$

[4 marks]**B3** Solve for $0 \leq x < 360^\circ$

(a) $\operatorname{cosec} x = 2$

(b) $\sec x = -2$

(c) $\cot x = \sqrt{3}$

(d) $\operatorname{cosec} x = 0.5$

[4 marks]**B4** $\tan \theta = \frac{3}{4}$. Find the exact value of

(a) $\sec \theta$, given θ is acute

(b) $\operatorname{cosec} \theta$, given θ is acute

(c) $\cot \theta$

(d) $\sec \theta$, given θ is reflex

[4 marks]

CORE*continued: graphs and inverses***B5** For $0 \leq x \leq 2\pi$, state**(a)** the asymptotes of $y = \sec x$ **(b)** the range of $y = \sec x$ **(c)** the period of $y = \cot x$ **[3 marks]****B6** Find the exact value of**(a)** $\arcsin \frac{\sqrt{3}}{2}$ **(b)** $\arccos 0$ **(c)** $\arctan \frac{1}{\sqrt{3}}$ **(d)** $\arcsin \left(\sin \frac{5\pi}{6} \right)$ **[4 marks]****STRETCH****B7** Solve for $0 \leq x < 360^\circ$ **(a)** $\tan^2 x = 2 \sec x - 1$ **(b)** $\cot^2 x = 3$ **(c)** $2 \cot^2 x + 3 \operatorname{cosec} x = 0$ **(d)** $\sec^2 x = 4$ **[4 marks]****B8** Prove that**(a)** $\operatorname{cosec}^2 x - \cot^2 x \equiv 1$ **(b)** $(\sec x + \tan x)(\sec x - \tan x) \equiv 1$ **(c)** $\cot x + \tan x \equiv \sec x \operatorname{cosec} x$ **(d)** $\frac{1 + \cot^2 x}{1 + \tan^2 x} \equiv \cot^2 x$ **[4 marks]****B9****(a)** State the domain and range of $y = \arcsin x$ **(b)** State the domain and range of $y = \arccos x$ **(c)** State the domain and range of $y = \arctan x$ **(d)** Solve $\arctan(2x - 1) = \frac{\pi}{4}$ **[4 marks]****B10** Find, in exact or simplest form,**(a)** $\sin \left(\arccos \frac{3}{5} \right)$ **(b)** $\tan \left(\arccos \frac{2}{3} \right)$ **(c)** $\cos(\arcsin x)$ **(d)** $\sec(\arctan x)$ **[4 marks]**

C**Reasoning**

Show your working and explain your thinking. A correct answer without a reason does not score full marks.

CORE

Secure the method

- C1** Sam says $\sec 30^\circ = 2$, because $\sin 30^\circ = \frac{1}{2}$
Explain Sam's mistake, and find $\sec 30^\circ$ exactly. [2 marks]
- C2** Lily says $\arcsin\left(\sin \frac{2\pi}{3}\right) = \frac{2\pi}{3}$
Explain why Lily is wrong, and give the correct value. [2 marks]
- C3** Kai solves $\sec x = 0.5$ for $0 \leq x < 360^\circ$ by writing $\cos x = 0.5$, and gives 60° and 300°
Explain Kai's mistakes. [3 marks]
- C4** Jo writes the identity $1 + \tan^2 x \equiv \operatorname{cosec}^2 x$
Explain Jo's mistake, and prove the correct identity that involves $\operatorname{cosec}^2 x$ [3 marks]



STRETCH

C5 Max solves $\sec x = -2$ for $-\pi \leq x \leq \pi$ and gives $x = \frac{2\pi}{3}$ and $x = \frac{4\pi}{3}$

Explain Max's mistake, and give the correct solutions.

[3 marks]

C6 Ali solves $\sin^2(2y) = \frac{1}{4}$ for $0 < y < \frac{\pi}{4}$, giving the exact answer $y = 15$

Explain why Ali's answer would lose marks, and correct it.

[3 marks]

C7 Ali writes $\operatorname{cosec}^2 2x = \frac{1}{(2 \sin x \cos x)^2} = \frac{1}{2 \sin^2 x \cos^2 x}$

Explain Ali's mistake, and write $\operatorname{cosec}^2 2x$ correctly in terms of $\sin x$ and $\cos x$

[3 marks]

C8 (a) By drawing a right-angled triangle, prove that $\arctan x + \arctan \frac{1}{x} = \frac{\pi}{2}$ for $x > 0$

(2)

(b) Hence solve the equation $\arctan x = 2 \arctan \frac{1}{x}$ for $x > 0$, giving the exact answer.

(3)

[5 marks]



D**Problem solving**

You decide the method. Read the whole question first, and show every step.

CORE

Secure the method

- D1** In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.

Using the identity $1 + \tan^2 x \equiv \sec^2 x$, solve, for $0 \leq x < 360^\circ$, the equation

$$2 \tan^2 x - 3 \sec x = 0$$

You must form a quadratic equation in $\sec x$, solve it, and explain clearly why one of the roots of your quadratic gives no solutions in the interval.

[3 marks]

- D2** In this question you should write every function in terms of $\sin x$ and $\cos x$ first.

(a) Show that the equation $4 \cot x = 3 \sec x$ can be written in the form $4 \sin^2 x + 3 \sin x - 4 = 0$, stating the identity that you use.

(2)

(b) Hence solve the equation $4 \cot x = 3 \sec x$ for $0 < x < 2\pi$, giving your answers in radians to 3 significant figures.

(2)

[4 marks]

- D3** Figure 1 shows the graph of $y = a \sec x + b$, $0 \leq x \leq 2\pi$, where a and b are constants. The graph has a minimum point at $A(0, 5)$ on the y -axis and a maximum point at $B(\pi, 1)$, and vertical asymptotes

at $x = \frac{\pi}{2}$ and $x = \frac{3\pi}{2}$

(a) Using the coordinates of A and B , find the values of a and b

(2)

(b) State the range of y for this domain, using inequalities.

(1)

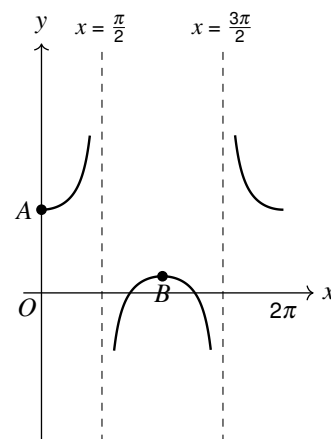


Figure 1

[3 marks]



- D4** In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.

The angle θ is defined by $\theta = \arcsin\left(-\frac{3}{5}\right)$

Using the range of the arcsin function to decide each sign, find the exact values of $\cos \theta$, $\tan \theta$ and $\sec \theta$. A sketch of a right-angled triangle may help, but you must explain how you choose each sign.

[3 marks]

STRETCH

- D5** In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.

(a) Prove that $\frac{1}{\operatorname{cosec} x - 1} + \frac{1}{\operatorname{cosec} x + 1} \equiv 2 \tan x \sec x$, for values of x where both sides are defined.

(2)

(b) Hence, factorising rather than dividing, solve $\frac{1}{\operatorname{cosec} x - 1} + \frac{1}{\operatorname{cosec} x + 1} = 4 \sin x$ for $0 < x < 360^\circ$, explaining why the solutions of $\sin x = 0$ must be rejected.

(2)

[4 marks]

- D6** Figure 2 shows the graphs of $y = \sec x$ and $y = 2 + \cos x$ for $0 < x < 2\pi$. The graphs meet at the points P and Q

(a) Show that the x coordinates of P and Q satisfy $\cos^2 x + 2 \cos x - 1 = 0$

(2)

(b) Find the x coordinates of P and Q , in radians to 3 significant figures, explaining why only one root of the quadratic can be used and how Figure 2 confirms that there are two points.

(2)

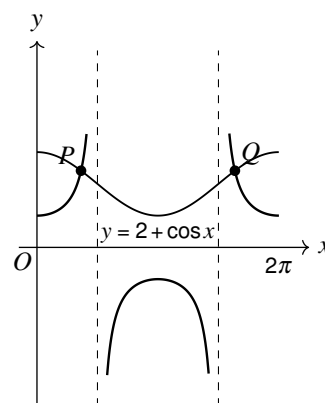


Figure 2

[4 marks]



- D7** *In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.*

Solve, for $0 \leq x < 360^\circ$, the equation

$$\sec(2x - 30^\circ) = 2$$

showing clearly the interval for $2x - 30^\circ$ that you use and finding every solution in the given interval.

[4 marks]

- D8** *In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.*

(a) Prove that $\sec^4 x - \tan^4 x \equiv \sec^2 x + \tan^2 x$

(2)

(b) Hence, using the identity $1 + \tan^2 x \equiv \sec^2 x$, solve $\sec^4 x - \tan^4 x = 3 \tan x$ for $0 \leq x < \frac{\pi}{2}$, giving one answer exactly and the other in radians to 3 significant figures.

(3)

[5 marks]



E**Exam practice**

Exam-style questions in the style of WMA13. Write your answers in the spaces, and show every step.

CORE

Exam questions for everyone

E1 *In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.*

(a) Using a suitable identity, show that the equation

$$2\cot^2 x + 5\operatorname{cosec} x + 4 = 0$$

can be written as $2\operatorname{cosec}^2 x + 5\operatorname{cosec} x + 2 = 0$

(2)

(b) Hence solve, for $0 \leq x < 360^\circ$, the equation $2\cot^2 x + 5\operatorname{cosec} x + 4 = 0$, giving your answers in degrees and explaining clearly why one root of your quadratic gives no solutions.

(3)

(Total for Question E1 is 5 marks)



E2 In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.

The functions $\arcsin$, $\arccos$ and $\arctan$ have their usual domains and ranges.

(a) Write down the exact value of $\arctan(\sqrt{3})$, and the exact value of $\arcsin\left(-\frac{1}{2}\right)$

.....
(2)

(b) Given that $\theta = \arccos\left(\frac{1}{4}\right)$, find the exact values of $\sin \theta$ and of $\cot \theta$, explaining how the range of $\arccos$ tells you the sign of $\sin \theta$

.....
(3)

(c) Explain why $\arccos\left(\cos \frac{5\pi}{3}\right) \neq \frac{5\pi}{3}$, and state its value.

.....
(1)

(Total for Question E2 is 6 marks)



STRETCH

- E3** Figure 3 shows the graph of the curve C with equation

$$y = 1 + \sec 2x \quad 0 \leq x \leq \pi$$

The point M is a maximum point of C , the curve meets the x -axis only at M , and C has two vertical asymptotes, shown as dashed lines in Figure 3.

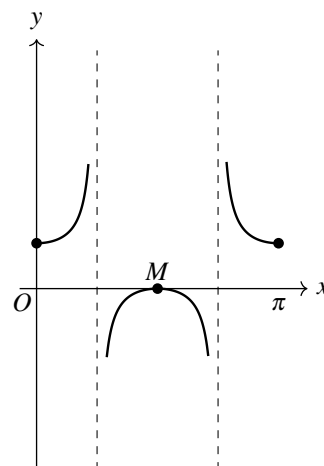


Figure 3

- (a) State the equations of the asymptotes of C

.....
(2)

- (b) State the coordinates of M

.....
(1)

- (c) Solve, for $0 \leq x \leq \pi$, the equation $1 + \sec 2x = 3$, giving your answers as exact multiples of π

.....
(3)

- (d) Using Figure 3, find the set of values of k for which the equation $1 + \sec 2x = k$ has exactly two solutions in $0 \leq x \leq \pi$

.....
(2)

(Total for Question E3 is 8 marks)

Want more exam practice? The **Exam Questions by Topic** booklets have 10 more full exam questions on Pure 3 Chapter 3, with a full mark scheme.

[Open the booklet](#)

Finished? Mark your work with the **full worked solutions**, fill in your scores on the front page, then keep the skill sharp with unlimited questions on the **Trigonometry revision engine**.



Join students already acing their exams.

Book your FREE lesson at www.eclassroom.org.uk