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INTERNATIONAL A LEVEL MATHEMATICS • FURTHER PURE 2 (WFM02)

Further Pure 2 Practice Paper 2

Edexcel IAL Further Pure Mathematics 2

AIMING FOR A

Written in the style of the Pearson Edexcel International Advanced Level paper (WFM02/01) • Calculator allowed

A full 75-mark paper, harder than a typical series, with one printed result and chapters combined, and a separate mark scheme

Time	Total	You must have
1 hour 30 minutes	**75 marks**	**Mathematical Formulae and Statistical Tables, calculator**

Instructions

- Use black ink or ball-point pen. Pencil may be used for diagrams and sketches.
- Answer **all** questions, in the spaces provided.
- Show enough working to make your method clear. Answers without working may not gain full credit.
- Give inexact answers to **three significant figures** unless the question says otherwise.

Information

- There are 8 questions in this paper. The total mark is 75.
- The marks for each question are shown in brackets.

Advice

- Sit the paper in one go, in 1 hour 30 minutes, with the mark scheme closed.
- Read each question carefully and try every question. Check your answers if you have time.
- Then mark it line by line with the mark scheme and fill in your score.

Track your score

Q	Topic	Marks	Score
1	A modulus inequality and a sketch	7	
2	A differential equation by substitution	8	
3	Differences with three partial fractions	9	
4	A line mapped to a circle	9	
5	A Taylor series from a differential equation	10	
6	Fourth roots and the square they make	10	
7	A particular integral inside the complementary function	11	
8	The area inside two polar curves	11	
	Total	**75**	

Rough grade guide (out of 75): A* **64** • A **58** • B **51** • C **44**

Based on recent WFM02 grade boundaries. Real boundaries change every series, so treat this as a guide, not a grade.

Mark scheme | Exam questions by topic | More practice papers

Name ..... Date .....

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1. **(a)** On the same axes, sketch the graphs of $y = |x^2 - 4|$ and $y = 3x$, showing the coordinates of any points where the graph of $y = |x^2 - 4|$ meets the coordinate axes.

(2)

(b) Find the values of x for which $|x^2 - 4| = 3x$

(2)

(c) Hence, or otherwise, find the set of values of x for which

$$|x^2 - 4| > 3x$$

(3)

(Total for Question 1 is 7 marks)

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2.

$$x\frac{dy}{dx} + y = x^3y^2, \qquad x > 0$$

(a) Show that the substitution $z = \frac{1}{y}$ transforms this differential equation into the equation

$$\frac{dz}{dx} - \frac{z}{x} = -x^2$$

(3)

(b) By solving the equation in part (a), find the particular solution of the original differential equation for which $y = 1$ when $x = 1$, giving your answer in the form $y = f(x)$

(5)

(Total for Question 2 is 8 marks)

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3. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

(a) Express $\dfrac{4}{r(r+2)(r+4)}$ in partial fractions.

(3)

(b) Hence, using the method of differences, find

$$\sum_{r=1}^{n} \frac{4}{r(r+2)(r+4)}$$

giving your answer in the form $a - \dfrac{1}{2}\left(\dfrac{1}{n+1} + \dfrac{1}{n+2} - \dfrac{1}{n+3} - \dfrac{1}{n+4}\right)$, where a is a rational number to be found.

(4)

(c) Hence find the limit of $\displaystyle\sum_{r=1}^{n} \frac{4}{r(r+2)(r+4)}$ as $n \to \infty$

(2)

(Total for Question 3 is 9 marks)

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4. The transformation T from the z-plane, where $z = x + \mathrm{i}y$, to the w-plane, where $w = u + \mathrm{i}v$, is given by

$$w = \frac{z + 2\mathrm{i}}{z - \mathrm{i}}, \qquad z \neq \mathrm{i}$$

(a) Show that the real axis in the z-plane is mapped by T onto a circle in the w-plane, and find the centre and radius of this circle.

(6)

(b) On an Argand diagram, shade the region of the w-plane onto which T maps the region of the z-plane for which $y > 0$

(3)

(Total for Question 4 is 9 marks)

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5.

$$\frac{dy}{dx} = y^2 - x$$

Given that $y = 2$ when $x = 1$

(a) find the value of $\frac{d^2y}{dx^2}$ and the value of $\frac{d^3y}{dx^3}$ when $x = 1$

(3)

(b) find a series solution for y in ascending powers of $(x - 1)$, up to and including the term in $(x - 1)^3$

(4)

(c) use your series to estimate the value of y when $x = 1.1$, giving your answer to 3 decimal places.

(2)

(d) Explain why your series is unlikely to give a reliable estimate of the value of y when $x = 2$

(1)

(Total for Question 5 is 10 marks)

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6. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

$$w = -8 - 8\sqrt{3}\mathrm{i}$$

(a) Express w in the form $r(\cos\theta + \mathrm{i}\sin\theta)$, where $r > 0$ and $-\pi < \theta \leqslant \pi$

(2)

(b) Solve the equation $z^4 = w$, giving your answers in the form $a + b\mathrm{i}$, where a and b are exact real numbers.

(5)

(c) The points representing the roots of $z^4 = w$ are the vertices of a quadrilateral on an Argand diagram. Show these points on an Argand diagram and find the exact area of the quadrilateral.

(3)

(Total for Question 6 is 10 marks)

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7.

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 6y = 10e^{3x}$$

(a) Explain why a particular integral of the form $y = \lambda e^{3x}$ cannot be used, and find a suitable particular integral.

(5)

(b) Find the general solution of the differential equation.

(2)

(c) Find the particular solution for which $y = 0$ and $\frac{dy}{dx} = 1$ when $x = 0$

(4)

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Question 7 continued

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(Total for Question 7 is 11 marks)

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8.

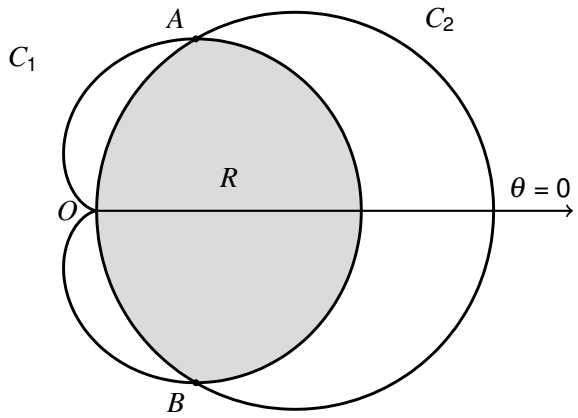

Figure 1

Figure 1 shows the curves C_1 and C_2 with polar equations

$$C_1 : r = 1 + \cos\theta, \quad -\pi < \theta \leqslant \pi \qquad C_2 : r = 3\cos\theta, \quad -\frac{\pi}{2} \leqslant \theta \leqslant \frac{\pi}{2}$$

The curves meet at the pole and at the points A and B

(a) Find the polar coordinates of A and the polar coordinates of B

(3)

The region R, shown shaded in Figure 1, lies inside both C_1 and C_2

(b) Use calculus to find the exact area of R

(8)

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Question 8 continued

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(Total for Question 8 is 11 marks)

TOTAL FOR PAPER IS 75 MARKS

END

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