



Partial Fractions

Worked Solutions

Suitable for:

Pearson Edexcel International A-Level (iAL) — Pure Mathematics 4



Question 1 — Worked Solution

Two distinct linear factors

Write the partial fractions form and multiply through:

$$\frac{5x - 11}{(x + 1)(x - 3)} \equiv \frac{A}{x + 1} + \frac{B}{x - 3}$$

$$5x - 11 \equiv A(x - 3) + B(x + 1)$$

Cover-up: let $x = -1$: $-16 = -4A \Rightarrow A = 4$

Cover-up: let $x = 3$: $4 = 4B \Rightarrow B = 1$

$$\therefore \frac{5x - 11}{(x + 1)(x - 3)} = \frac{4}{x + 1} + \frac{1}{x - 3}$$



Question 2 — Worked Solution

Factorise denominator first

$$x^2 - 1 = (x + 1)(x - 1)$$

$$\frac{7x - 1}{(x + 1)(x - 1)} \equiv \frac{A}{x + 1} + \frac{B}{x - 1}$$

$$7x - 1 \equiv A(x - 1) + B(x + 1)$$

$$x = -1: -8 = -2A \Rightarrow A = 4$$

$$x = 1: 6 = 2B \Rightarrow B = 3$$

$$\therefore \frac{7x - 1}{x^2 - 1} = \frac{4}{x + 1} + \frac{3}{x - 1}$$





Question 3 — Worked Solution

Three distinct linear factors

$$\frac{4x^2 - 3x - 4}{x(x+2)(x-1)} \equiv \frac{A}{x} + \frac{B}{x+2} + \frac{C}{x-1}$$

$$4x^2 - 3x - 4 \equiv A(x+2)(x-1) + Bx(x-1) + Cx(x+2)$$

$$x = 0: -4 = A(2)(-1) = -2A \Rightarrow A = 2$$

$$x = -2: 16 + 6 - 4 = B(-2)(-3) = 6B \Rightarrow B = 3$$

$$x = 1: 4 - 3 - 4 = C(1)(3) = 3C \Rightarrow C = -1$$

$$\therefore \frac{4x^2 - 3x - 4}{x(x+2)(x-1)} = \frac{2}{x} + \frac{3}{x+2} - \frac{1}{x-1}$$





Question 4 — Worked Solution

Three distinct factors including $2x-1$

$$\frac{9x^2 - 12x - 3}{(x-2)(x+1)(2x-1)} \equiv \frac{A}{x-2} + \frac{B}{x+1} + \frac{C}{2x-1}$$

$$9x^2 - 12x - 3 \equiv A(x+1)(2x-1) + B(x-2)(2x-1) + C(x-2)(x+1)$$

$$x = 2: 36 - 24 - 3 = A(3)(3) = 9A \Rightarrow A = 1$$

$$x = -1: 9 + 12 - 3 = B(-3)(-3) = 9B \Rightarrow B = 2$$

$$x = 1/2: 9/4 - 6 - 3 = C(-3/2)(3/2) = -9C/4 \Rightarrow C = 3$$

$$\therefore \frac{9x^2 - 12x - 3}{(x-2)(x+1)(2x-1)} = \frac{1}{x-2} + \frac{2}{x+1} + \frac{3}{2x-1}$$





Question 5 — Worked Solution

Repeated linear factor

$$\frac{5x^2 + 3}{(x+1)(x-1)^2} \equiv \frac{A}{x+1} + \frac{B}{x-1} + \frac{C}{(x-1)^2}$$

$$5x^2 + 3 \equiv A(x-1)^2 + B(x+1)(x-1) + C(x+1)$$

$$x = -1: 5+3 = A(4) \Rightarrow A = 2$$

$$x = 1: 5+3 = C(2) \Rightarrow C = 4$$

$$\text{Compare } x^2 \text{ coefficients: } 5 = A + B \Rightarrow B = 3$$

$$\therefore \frac{5x^2 + 3}{(x+1)(x-1)^2} = \frac{2}{x+1} + \frac{3}{x-1} + \frac{4}{(x-1)^2}$$



Question 6 — Worked Solution

Repeated factor with x in denominator

$$\frac{2x^2 + 12x + 12}{x(x+2)^2} \equiv \frac{A}{x} + \frac{B}{x+2} + \frac{C}{(x+2)^2}$$

$$2x^2 + 12x + 12 \equiv A(x+2)^2 + Bx(x+2) + Cx$$

$$x = 0: 12 = 4A \Rightarrow A = 3$$

$$x = -2: 8 - 24 + 12 = C(-2) \Rightarrow -4 = -2C \Rightarrow C = 2$$

$$\text{Compare } x^2 \text{ coefficients: } 2 = A + B \Rightarrow B = -1$$

$$\therefore \frac{2x^2 + 12x + 12}{x(x+2)^2} = \frac{3}{x} - \frac{1}{x+2} + \frac{2}{(x+2)^2}$$





Question 7 — Worked Solution

Improper fraction — degrees equal, divide first

The numerator and denominator both have degree 2, so divide first. $(x+2)(x-1) = x^2+x-2$.

$$3x^2 + 4x - 10 = 3(x^2 + x - 2) + (x - 4)$$

Verify: $3(x^2+x-2)+(x-4) = 3x^2+3x-6+x-4 = 3x^2+4x-10$ ✓

$$\frac{3x^2 + 4x - 10}{(x + 2)(x - 1)} = 3 + \frac{x - 4}{(x + 2)(x - 1)}$$

Now decompose the proper fraction:

$$x - 4 \equiv B(x - 1) + C(x + 2)$$

$$x = -2: -6 = -3B \Rightarrow B = 2$$

$$x = 1: -3 = 3C \Rightarrow C = -1$$

$$\therefore \frac{3x^2 + 4x - 10}{(x + 2)(x - 1)} = 3 + \frac{2}{x + 2} - \frac{1}{x - 1}$$





Question 8 — Worked Solution

Improper fraction — cubic over quadratic, long division first

$(x-1)(x+2) = x^2+x-2$. Divide $2x^3+3x^2-2x+6$ by x^2+x-2 :

$$2x^3 + 3x^2 - 2x + 6 = (2x + 1)(x^2 + x - 2) + (x + 8)$$

Verify: $(2x+1)(x^2+x-2) = 2x^3+2x^2-4x+x^2+x-2 = 2x^3+3x^2-3x-2$. Add $(x+8)$: $2x^3+3x^2-2x+6$ ✓

$$\frac{2x^3 + 3x^2 - 2x + 6}{(x-1)(x+2)} = 2x + 1 + \frac{x + 8}{(x-1)(x+2)}$$

Now decompose the remainder:

$$x + 8 \equiv C(x + 2) + D(x - 1)$$

$$x = 1: 9 = 3C \Rightarrow C = 3$$

$$x = -2: 6 = -3D \Rightarrow D = -2$$

$$\therefore \frac{2x^3 + 3x^2 - 2x + 6}{(x-1)(x+2)} = 2x + 1 + \frac{3}{x-1} - \frac{2}{x+2}$$