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INTERNATIONAL A LEVEL MATHEMATICS • FURTHER PURE MATHEMATICS 2

Further Pure 2

Chapter 3: Complex Numbers

Exam Questions by Topic

Pearson Edexcel International Advanced Level (WFM02/01) • Calculator allowed

Multi-step exam questions at the difficulty of the real WFM02 paper, with a full mark scheme

Chapters (tap a chapter to jump to it)		Qs	Marks	Page	My score
1	**Inequalities** Rational and modulus inequalities, critical values, graphs	10	80	separate file	
2	**Series** Method of differences, partial fractions, telescoping sums	10	80	separate file	
3	**Complex Numbers** Exponential form, de Moivre's theorem, trigonometric identities, nth roots	10	80	2	
4	**Further Argand Diagrams** Loci, regions and transformations of the complex plane	10	80	separate file	
5	**First-Order Differential Equations** Integrating factors, substitutions, general and particular solutions	10	80	separate file	
6	**Second-Order Differential Equations** Auxiliary equations, complementary functions, particular integrals, substitutions	10	80	separate file	
7	**Maclaurin and Taylor Series** Higher derivatives, standard and compound series, Taylor series, series solutions	10	80	separate file	
8	**Polar Coordinates** Polar curves, areas, tangents parallel and perpendicular to the initial line	10	80	separate file	

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CHAPTER 3 • KEY SKILLS

3 Complex Numbers

Specification FP2: Further complex numbers • 10 questions • 80 marks • about 96 minutes

1 Exponential form

$z = re^{i\theta} = r(\cos\theta + i\sin\theta)$, with $-\pi < \theta \leqslant \pi$ for the principal argument.

2 Multiply and divide

$r_1e^{i\theta_1} \times r_2e^{i\theta_2} = r_1r_2e^{i(\theta_1+\theta_2)}$; for division divide the moduli and subtract the arguments.

3 de Moivre

$(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$, so $\left(re^{i\theta}\right)^n = r^ne^{in\theta}$

4 Multiple angles

Expand $(c+is)^n$ with binomial coefficients; real part gives $\cos n\theta$, imaginary part $\sin n\theta$; then use $s^2 = 1 - c^2$

5 Powers of cos and sin

With $z = e^{i\theta}$: $z^n + z^{-n} = 2\cos n\theta$ and $z^n - z^{-n} = 2i\sin n\theta$. Expand $(z+z^{-1})^n$ and pair the terms.

6 nth roots

$z^n = re^{i\theta}$ gives $z = r^{\frac{1}{n}}e^{i\frac{\theta+2k\pi}{n}}$, $k = 0, 1, \ldots, n-1$: n roots.

7 Geometry of roots

The roots are equally spaced on a circle of radius $r^{\frac{1}{n}}$: a regular polygon, and their sum is 0

8 Solving with identities

Turn a polynomial in x into $\cos n\theta = k$ with $x = \cos\theta$; list every solution, then choose the one asked for.

9 Hence and working

Use the result just proved. A calculator-only solution of the polynomial scores nothing.

Skills map every skill above is tested at least once; interleaved skills come from GCSE or other chapters

Skill	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Marks	6	6	6	6	6	10	10	10	10	10
1 Exponential form	•	•			•			•		•
2 Multiply and divide	•							•		
3 de Moivre	•	•	•			•	•			
4 Multiple angles			•			•	•			
5 Powers of cos and sin				•					•	
6 nth roots					•			•		•
7 Geometry of roots					•			•		•
8 Solving with identities			•			•	•			•
9 Hence and working			•	•		•	•		•	•
Interleaved: binomial expansion (P2)			•	•		•	•		•	
Interleaved: trigonometric equations and integration (P3, P4)			•	•		•	•		•	
Interleaved: complex numbers and Argand diagrams (FP1)	•	•			•			•		•

Exam difficulty. Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WFM02 paper, where one part leads into the next and skills from different topics are combined.

Must learn. Write out the full binomial expansion with its coefficients, then extract the real or imaginary terms. When solving, find all the angles first, then select the value the question asks for.

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1. The complex numbers $z = 2e^{\frac{\pi}{3}i}$ and $w = 3e^{-\frac{\pi}{4}i}$

(a) Find zw in the form $re^{i\theta}$

(1)

(b) Find $\frac{z}{w}$ in the form $re^{i\theta}$

(1)

(c) Write z in the form $a+bi$, giving exact values.

(1)

(d) Show that z^6 is real, and find its value.

(2)

(e) Find w^2 in the form $re^{i\theta}$, where $-\pi < \theta \leqslant \pi$

(1)

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Question 1 continued

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(Total for Question 1 is 6 marks)

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2. **(a)** Use de Moivre's theorem to find $\left(\sqrt{3}+\mathrm{i}\right)^8$ in the form $a+b\mathrm{i}$, where a and b are exact.

(4)

(b) Find the smallest positive integer n for which $\left(\sqrt{3}+\mathrm{i}\right)^n$ is real and positive.

(2)

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Question 2 continued

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(Total for Question 2 is 6 marks)

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3. **(a)** Use de Moivre's theorem to show that $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

(4)

(b) Hence find the three roots of $8x^3 - 6x - 1 = 0$, giving your answers to 3 decimal places.

(2)

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Question 3 continued

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4.

(a) By considering $\left(z+\frac{1}{z}\right)^4$, where $z=\cos\theta+\mathrm{i}\sin\theta$, show that

$$\cos^4\theta=\frac{1}{8}(\cos 4\theta+4\cos 2\theta+3)$$

(4)

(b) Hence find the exact value of $\int_0^{\frac{\pi}{2}}\cos^4\theta\,\mathrm{d}\theta$

(2)

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Question 4 continued

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(Total for Question 4 is 6 marks)

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5. **(a)** Solve the equation $z^3 = 8i$, giving your answers in the form $re^{i\theta}$, where $-\pi < \theta \leqslant \pi$

(4)

(b) Show the roots on an Argand diagram, and describe the shape formed by joining them.

(2)

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Question 5 continued

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6. **(a)** Use de Moivre's theorem to show that $\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$

(5)

(b) Hence find the smallest positive root of the equation $32x^5 - 40x^3 + 10x - 1 = 0$, giving your answer to 3 significant figures.

(4)

(c) Explain why the equation in part (b) has exactly five real roots.

(1)

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Question 6 continued

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Question 6 continued

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7. **(a)** Use de Moivre's theorem to show that $\sin 5\theta = 16\sin^5\theta - 20\sin^3\theta + 5\sin\theta$

(5)

(b) Hence solve, for $0 < \theta < \pi$, the equation $\sin 5\theta = \sin\theta$, giving your answers as exact multiples of π.

(5)

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Question 7 continued

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Question 7 continued

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(Total for Question 7 is 10 marks)

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8. **(a)** Solve the equation $z^5 = 16 + 16\sqrt{3}\,\mathrm{i}$, giving your answers in the form $re^{i\theta}$, where $-\pi < \theta \leqslant \pi$

(4)

(b) The points representing the roots are joined to form a pentagon. Find its exact area, and give its value to 3 significant figures.

(3)

(c) Show that the sum of the five roots is zero.

(2)

(d) Write down the product of the five roots.

(1)

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Question 8 continued

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Question 8 continued

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9. **(a)** Show that $32\cos^6\theta = \cos 6\theta + 6\cos 4\theta + 15\cos 2\theta + 10$

(5)

(b) Hence find the exact value of $\int_0^{\frac{\pi}{2}} \cos^6\theta \, d\theta$

(3)

(c) Find the exact value of $\int_0^{\frac{\pi}{4}} \cos^6\theta \, d\theta$

(2)

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Question 9 continued

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Question 9 continued

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10. **(a)** By writing $w = z^3$, solve $w^2 + w + 1 = 0$, giving the roots in the form $e^{i\theta}$

(2)

(b) Hence find the six roots of $z^6 + z^3 + 1 = 0$ in the form $e^{i\theta}$, where $-\pi < \theta \leqslant \pi$

(4)

(c) By pairing conjugate roots, express $z^6 + z^3 + 1$ as a product of three quadratic factors with real coefficients.

(3)

(d) Deduce that $\cos\frac{2\pi}{9} + \cos\frac{4\pi}{9} + \cos\frac{8\pi}{9} = 0$

(1)

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Question 10 continued

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Question 10 continued

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TOTAL FOR CHAPTER 3 IS 80 MARKS

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