eClassroom

Practice Paper

International GCSE Mathematics A (4MA1) · Higher Tier

Mixed topics

Name _____

Date _____

Time allowed: 65 minutes

Total marks: 50

Instructions

- Use black ink or ball-point pen.
- Answer all questions in the spaces provided.
- You must write down all the stages in your working.
- Calculators may be used.
- Diagrams are NOT accurately drawn, unless otherwise indicated.

Information

- The total mark for this paper is 50.
- The marks for each question are shown in brackets.
- This paper covers: Algebra, Probability, Ratio, Statistics, Geometry, Number, Trigonometry.
- Grades 4–9.
- All questions are original and written in the style of the specification.

1

(a) Find the value of 2^{-2}

Answer _____

(1)

(b) Find the value of $64^{2/3}$

Answer _____

(2)

(c) Find the value of $\left(\frac{9}{36}\right)^{-½}$

Answer _____

(2)

(Total for Question 1 is 5 marks)

2 A box contains 400 counters.
The ratio of red counters to blue counters is 2 : 3.
10% of the red counters are removed from the box.

(a) Work out the number of red counters that are removed.

Answer _____

(2)

(b) Work out the new ratio of red counters to blue counters.
Give your answer in its simplest form.

Answer _____

(3)

(Total for Question 2 is 5 marks)

3 The 3rd term of an arithmetic series is 26.
The 8th term of the same series is 61.

(a) Find the first term a and the common difference d.

Answer _____

(2)

(b) Find an expression, in terms of n, for the nth term.

Answer _____

(1)

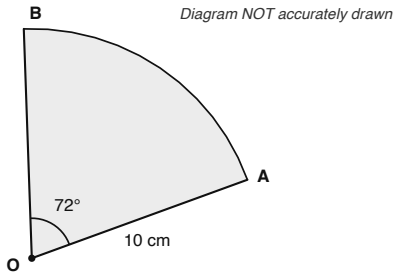
(c) Find the sum of the first 20 terms of the series.

Answer _____

(2)

(Total for Question 3 is 5 marks)

4 The diagram shows a sector OAB of a circle with centre O.
OA = 10 cm and angle AOB = 72°.



(a) Work out the length of the arc AB.
Give your answer correct to 3 significant figures.

Answer _____

(2)

(b) Work out the area of the sector.
Give your answer correct to 3 significant figures.

Answer _____

(2)

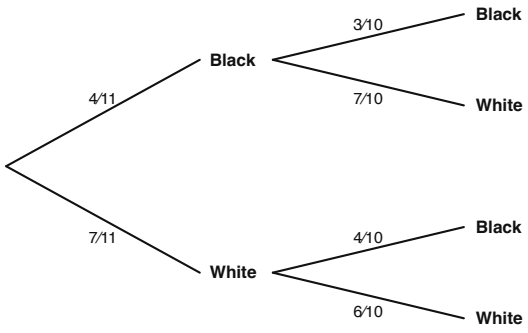
(c) Work out the perimeter of the sector.
Give your answer correct to 3 significant figures.

Answer _____

(1)

(Total for Question 4 is 5 marks)

5 A box contains 4 black pens and 7 white pens.
Ben takes at random two pens from the box, without replacement.
The tree diagram shows this information.



(a) Find the probability that both pens are the same colour.

Answer _____

(3)

(b) Find the probability that the two pens are different colours.

Answer _____

(2)

(Total for Question 5 is 5 marks)

6 375 and 100 are two whole numbers.

(a) Write 375 as a product of its prime factors.

Answer _____

(2)

(b) Find the highest common factor (HCF) of 375 and 100.

Answer _____

(2)

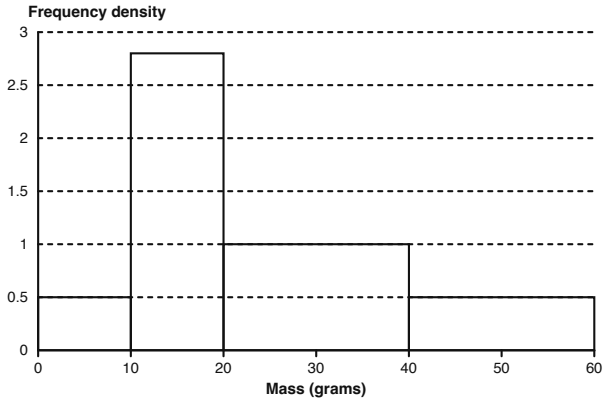
(c) Find the lowest common multiple (LCM) of 375 and 100.

Answer _____

(2)

(Total for Question 6 is 6 marks)

7 The histogram shows information about the mass, m grams, of some pebbles.



(a) Work out the number of pebbles with a mass between 10 g and 20 g.

Answer _____

(2)

(b) Work out an estimate for the number of pebbles with a mass between 25 g and 50 g.

Answer _____

(3)

(Total for Question 7 is 5 marks)

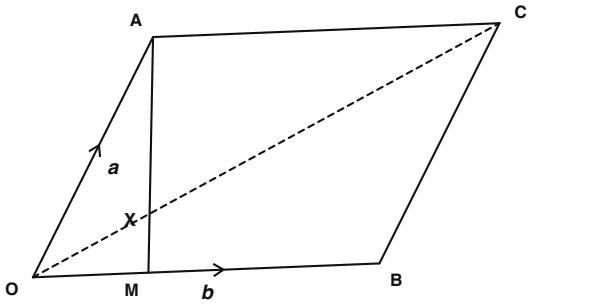
8 OACB is a parallelogram.

OA = **a** and OB = **b**.

M is the point on OB such that OM = $\frac{1}{3}$**b**.

The lines AM and OC cross at the point X.

Diagram NOT accurately drawn



(a) Find AM in terms of **a** and **b**.

Answer _____

(1)

(b) X lies on OC, so OX = λ(**a** + **b**) for some scalar λ.

Find the value of λ.

Answer _____

(4)

(c) Hence find OX in terms of **a** and **b**.

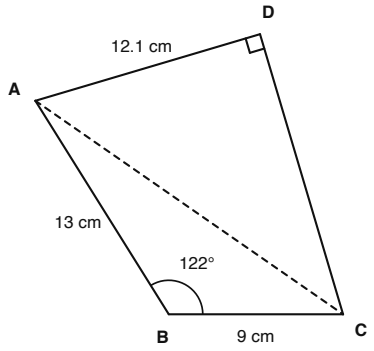
Answer _____

(1)

(Total for Question 8 is 6 marks)

9 The diagram shows the quadrilateral ABCD.
AB = 13 cm, BC = 9 cm and AD = 12.1 cm.
Angle ABC = 122° and angle ADC = 90°.

Diagram NOT accurately drawn



(a) Calculate the length of AC.
Give your answer correct to 3 significant figures.

Answer _____

(3)

(b) Calculate the length of DC.
Give your answer correct to 3 significant figures.

Answer _____

(2)

(c) Work out the total area of quadrilateral ABCD.
Give your answer correct to 3 significant figures.

Answer _____

(3)

(Total for Question 9 is 8 marks)

TOTAL FOR PAPER: 50 MARKS

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Mark Scheme & Worked Solutions

International GCSE Mathematics A (4MA1) · Higher Tier

M1 method mark · A1 accuracy mark · B1 independent mark
ft = follow through · oe = or equivalent · 3 s.f. unless stated

1 Number · 5 marks · band 2

(a) A negative index means the reciprocal.

$2^{-2} = \frac{1}{2^2} = \frac{1}{4}$

$\frac{1}{4}$

B1 $\frac{1}{4}$ oe

(b) The bottom of the index is the root, the top is the power.

$64^{2/3}$ = (cube root of 64)^2 = 4^2 = 16

16

M1 cube root of 64 = 4, oe $(64^{1/3})^2$

A1 16

(c) The negative index turns the fraction upside down, then the half is a square root.

$(\frac{9}{36})^{-½} = (\frac{36}{9})^{½} = \frac{\sqrt{36}}{\sqrt{9}} = \frac{6}{3} = \frac{2}{1}$

2

M1 inverting to $\frac{36}{9}$ or square rooting, oe

A1 $\frac{2}{1}$ oe

(a) There are 5 parts, so one part is 400 / 5 = 80.
The red counters number 2 × 80 = 160.
10% of 160 = 16

16

M1	400 / 5 × 2 = 160 oe
A1	16

(b) After the removal there are 160 − 16 = 144 red counters.
The blue counters are unchanged at 240.
144 : 240 = 3 : 5
Only one part changed, so the original ratio cannot simply be scaled.

3 : 5

M1	160 − 16 = 144 oe
M1	144 : 240 seen
A1	3 : 5

(a) The terms are 5 apart, so 5d = 61 − 26 = 35, giving d = 7.
Then a = 26 − 2(7) = 12.

a = 12, d = 7

M1	5d = 35 oe
A1	a = 12 and d = 7

(b) nth term = a + (n − 1)d = 12 + (n − 1)(7)
which simplifies to 7n + 5.

7n + 5

B1	7n + 5 (ft their a and d)

(c) Use the formula from the sheet: $S^n = n/2\,[2a + (n - 1)d]$.
S = ²0⁄₂ [2(12) + 19(7)] = 1570

1570

M1	S = ²0⁄₂ [2(12) + 19(7)] oe
A1	1570

(a) The sector is 72 / 360 of the whole circle.
72 / 360 × 2π × 10 = 12.6 cm

12.6 cm

M1	72 / 360 × 2π × 10 oe
A1	12.6

(b) 72 / 360 × π × 10^2 = 62.8 cm^2

62.8 cm^2

M1	72 / 360 × π × 10^2 oe
A1	62.8

(c) The perimeter is the arc PLUS the two radii.
12.6 + 10 + 10 = 32.6 cm

32.6 cm

B1	32.6 (ft)

(a) Nothing is replaced, so the second denominator is 10.

Both black: $^4/_{11} \times {}^3/_{10} = \frac{6}{55}$

Both white: $7/_{11} \times 6/_{10} = \frac{21}{55}$

Adding gives $\frac{27}{55}$

$$\mathbf{\frac{27}{55}}$$

M1 $^4/_{11} \times {}^3/_{10}$ or $7/_{11} \times 6/_{10}$

M1 adds both products

A1 $\frac{27}{55}$

(b) "Different colours" is everything else.

$1 - \frac{27}{55} = \frac{28}{55}$

$$\mathbf{\frac{28}{55}}$$

M1 1 - their answer to (a), oe

A1 $\frac{28}{55}$

(a) Divide repeatedly by the smallest prime that goes in.

$375 = 3 \times 5^3$

3×5^3

M1	factor tree or repeated division by primes
A1	3×5^3

(b) Write both numbers as products of primes.

$375 = 3 \times 5^3$ and $100 = 2^2 \times 5^2$

The HCF takes the lowest power of each shared prime: HCF = 25

25

M1	$100 = 2^2 \times 5^2$, or lists factors of both
A1	25

(c) The LCM takes the highest power of every prime that appears.

LCM = 1500

Check: $375 \times 100 \div 25 = 1500$

1500

M1	highest power of each prime, oe $375 \times 100 \div 25$
A1	1500

(a) Frequency = frequency density × class width, which is the area of the bar.

2.8 × 10 = 28

28

M1	2.8 × 10
A1	28

(b) The range 25 to 50 covers part of two bars.

From 25 to 40: 1 × 15 = 15

From 40 to 50: 0.5 × 10 = 5

Total = 20

This assumes the values are spread evenly within each class.

20

M1	1 × 15 oe
M1	adds 0.5 × 10 oe
A1	20

(a) AM = AO + OM = $-\mathbf{a} + \frac{1}{3}\mathbf{b}$

AM = $-\mathbf{a} + \frac{1}{3}\mathbf{b}$

$\mathbf{-a + \frac{1}{3}b}$

B1 $-\mathbf{a} + \frac{1}{3}\mathbf{b}$ oe

(b) X is on AM as well, so OX = OA + t AM for some scalar t.

OX = $\mathbf{a} + t(-\mathbf{a} + \frac{1}{3}\mathbf{b}) = (1 - t)\mathbf{a} + (t/3)\mathbf{b}$

X is also on OC, so OX = $\lambda\mathbf{a} + \lambda\mathbf{b}$.

Comparing **a**: $\lambda = 1 - t$. Comparing **b**: $\lambda = t/3$.

So $1 - t = t/3$, giving $t = \frac{3}{4}$ and $\lambda = \frac{1}{4}$.

$\boldsymbol{\lambda = \frac{1}{4}}$

M1 OX = OA + t AM for some scalar t oe

M1 a correct expression in **a** and **b**, e.g. $(1 - t)\mathbf{a} + (t/3)\mathbf{b}$

M1 compares the coefficients of **a** and **b** to form two equations

A1 $\lambda = \frac{1}{4}$

(c) OX = $\frac{1}{4}(\mathbf{a} + \mathbf{b}) = \frac{1}{4}\mathbf{a} + \frac{1}{4}\mathbf{b}$

$\mathbf{\frac{1}{4}a + \frac{1}{4}b}$

B1 $\frac{1}{4}\mathbf{a} + \frac{1}{4}\mathbf{b}$ (ft their λ)

(a) Triangle ABC is not right-angled, so use the cosine rule.

$AC^2 = 13^2 + 9^2 - 2 \times 13 \times 9 \times \cos 122°$

$AC = 19.3$ cm (3 s.f.)

19.3 cm

M1	$AC^2 = 13^2 + 9^2 - 2 \times 13 \times 9 \times \cos 122°$
M1	$AC^2 = 374$
A1	19.3 (cm)

(b) Angle ADC = 90°, so use Pythagoras in triangle ACD.

$DC^2 = AC^2 - AD^2 = 374 - 146$

DC = 15.1 cm (3 s.f.)

15.1 cm

M1ft	$DC^2 = 19.3^2 - 12.1^2$
A1ft	15.1 (cm)

(c) Area ABC = $\frac{1}{2} \times 13 \times 9 \times \sin 122° = 49.6$ cm²

Area ACD = $\frac{1}{2} \times 12.1 \times 15.1 = 91.3$ cm²

Total = 141 cm² (3 s.f.)

141 cm²

M1	$\frac{1}{2} \times 13 \times 9 \times \sin 122° = 49.6$
M1ft	$\frac{1}{2} \times 12.1 \times 15.1 = 91.3$
A1ft	141 (cm²)