GCSE Mathematics

Algebraic Proof

Worked Solutions

Pearson Edexcel GCSE & iGCSE Mathematics

Section A — Foundation — Worked Solutions

[Fluency] Question 1

$(n+1)^2 - n^2 = n^2 + 2n + 1 - n^2 = 2n + 1$ ✓

∴ **Shown ✓**

[Fluency] Question 2

$(-3)^2 + (-3) - 6 = 9 - 3 - 6 = 0$ ✓

∴ **Verified ✓**

[Fluency] Question 3

$2n + (2n+2) = 4n + 2 = 2(2n+1)$

Hmm — 4n+2 = 2(2n+1) which is even but not necessarily div by 4.

Correction: the sum is 4n+2=2(2n+1). This is divisible by 2, not 4.

For divisibility by 4: use 2n and 2n+4 (consecutive even with gap 4):

2n+(2n+4)=4n+4=4(n+1) ✓ — divisible by 4.

Note: consecutive even numbers differ by 2 (not 4), so use 2n, 2n+2.

Correct statement: sum = 4n+2 = 2(2n+1) is always even (not div by 4).

∴ **Sum = 4n+2 = 2(2n+1) — always even ✓ (not div by 4 in general)**

[Reasoning] Question 4

$(2n+1)^2 - 1 = 4n^2 + 4n + 1 - 1 = 4n^2 + 4n = 4n(n+1)$ ✓

∴ **4n(n+1) is always a multiple of 4 ✓**

[Reasoning] Question 5

$n + (n+1) + (n+2) = 3n + 3 = 3(n+1)$ ✓

∴ **3(n+1) is always a multiple of 3 ✓**

[Reasoning] Question 6

$n = 4:\ 16 + 4 + 1 = 21 = 3 \times 7$ — not prime ✓

∴ **n=4 gives 21 = 3×7, not prime ✓**

[Reasoning] Question 7

2 and 3 are both prime. 2×3=6, which is even.

Counter-example: **2×3=6** (not odd) ✓

∴ **Counter-example: 2×3=6 ✓**

[Problem Solving] Question 8

$$(n+3)^2-(n+1)^2=n^2+6n+9-(n^2+2n+1)=4n+8=4(n+2)$$

This equals 4 only when n+2=1, i.e. n=–1. For all other n, it ≠4. ✓

∴ **= 4(n+2), which equals 4 only when n=–1, not for all n ✓**

[Problem Solving] Question 9

S=1+2+...+n. Also S=n+(n-1)+...+1.

Adding: 2S=n(n+1) → S=n(n+1)/2 ✓

∴ **S = n(n+1)/2 ✓**

[Problem Solving] Question 10

$$(x+y)^2=x^2+2xy+y^2$$

Equal to x^2+y^2 only when 2xy=0, i.e. x=0 or y=0.

∴ **Equality holds only when x=0 or y=0 ✓**

Section B — Higher — Worked Solutions

[Fluency] Question 1

$(2n+1)(2n+3)=4n^2+8n+3=2(2n^2+4n+1)+1$ ✓

∴ **Odd (of form 2k+1) ✓**

[Fluency] Question 2

$2n+(2n+2)+(2n+4)=6n+6=6(n+1)$ ✓

∴ **6(n+1) — divisible by 6 ✓**

[Fluency] Question 3

Let n=2k+1.

$n^2=(2k+1)^2=4k^2+4k+1=2(2k^2+2k)+1$ ✓

∴ **n^2 is odd ✓**

[Reasoning] Question 4

$(n+1)^2-n^2=n^2+2n+1-n^2=2n+1$ ✓

∴ **2n+1 is always odd ✓**

[Reasoning] Question 5

$(n+1)^2=n^2+2n+1,\ (n-1)^2=n^2-2n+1$

$\text{Difference}=4n$ ✓

∴ **4n ✓**

[Reasoning] Question 6

Let the odd numbers be (2n+1) and (2n+3).

$(2n+1)^2+(2n+3)^2=4n^2+4n+1+4n^2+12n+9=8n^2+16n+10$

$=2(4n^2+8n+5)=4(2n^2+4n+2)+2=4k+2$ ✓

Remainder 2 when divided by 4 ✓

∴ **Sum = 4k+2, remainder 2 when divided by 4 ✓**

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[Reasoning] Question 7

Among any 3 consecutive integers, one is divisible by 2 and one by 3.

Therefore product is divisible by 2×3=6. ✓

Formal: n(n+1)(n+2). If n even: 2|n(n+1)(n+2). If n odd: n+1 even. Always 3| one of them.

∴ **Divisible by 6 ✓**

[Problem Solving] Question 8

$$x^2 + 6x + 10 = (x + 3)^2 + 1 \geq 1 > 0 \checkmark$$

∴ **$(x+3)^2+1 \geq 1 > 0$ for all real x ✓**

[Problem Solving] Question 9

nth odd number = 2n–1. Sum = 1+3+5+...+(2n–1).

Pairs from ends: 1+(2n–1)=2n, 3+(2n–3)=2n, etc. n/2 pairs (or use formula).

$$S = \tfrac{n}{2}(1 + (2n - 1)) = \tfrac{n}{2}(2n) = n^2 \checkmark$$

∴ **Sum = n^2 ✓**

[Problem Solving] Question 10

$$a > b > 0 \Rightarrow a^2 - b^2 = (a - b)(a + b)$$

a–b > 0 (since a > b) and a+b > 0 (since both positive).

Product of two positives is positive: $a^2–b^2 > 0 \rightarrow a^2 > b^2$ ✓

∴ **$a^2 > b^2$ ✓**