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INTERNATIONAL A LEVEL MATHEMATICS • FURTHER PURE MATHEMATICS 1

Further Pure 1

Chapter 6: Transformations Using Matrices

Exam Questions by Topic

Pearson Edexcel International Advanced Level (WFM01/01) • Calculator allowed

Multi-step exam questions at the difficulty of the real WFM01 paper, with a full mark scheme

Chapters (tap a chapter to jump to it)		Qs	Marks	Page	My score
1	**Complex Numbers** Arithmetic, conjugates, Argand diagrams, modulus and argument, roots of polynomial equations	10	80	separate file	
2	**Roots of Quadratic Equations** Sums and products of roots, symmetric functions, equations with transformed roots	10	80	separate file	
3	**Numerical Solutions of Equations** Interval bisection, linear interpolation, Newton–Raphson	10	80	separate file	
4	**Coordinate Systems** Parametric equations, parabolas, rectangular hyperbolas, tangents and normals	10	80	separate file	
5	**Matrices** Matrix arithmetic, determinants, inverses of 2×2 matrices	10	80	separate file	
6	**Transformations Using Matrices** Linear transformations, combined transformations, area scale factor, inverses	10	80	2	
7	**Series** Standard sums of r, r^2 and r^3, sigma notation	10	80	separate file	
8	**Proof** Mathematical induction for sums, divisibility, matrices and sequences	10	80	separate file	

Full mark scheme Free courses More worksheets

Name ..... Class ..... Date .....

CHAPTER 6 • KEY SKILLS

6 Transformations Using Matrices

Specification FP1: Matrix transformations • 10 questions • 80 marks • about 96 minutes

1 Columns are images

The first column is the image of $\begin{pmatrix}1\\0\end{pmatrix}$, the second the image of $\begin{pmatrix}0\\1\end{pmatrix}$. The origin never moves.

2 Standard matrices

Rotation by θ anticlockwise about O: $\begin{pmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{pmatrix}$. Reflection in $y=x$: $\begin{pmatrix}0&1\\1&0\end{pmatrix}$; in $y=-x$: $\begin{pmatrix}0&-1\\-1&0\end{pmatrix}$

3 Full descriptions

Rotation: centre O, angle and direction. Reflection: the mirror line. Enlargement: centre O and scale factor.

4 Images

Multiply the matrix by each position vector; for a line, transform a general point $(t, mt+c)$

5 Combined transformations

$\mathbf{P}$ followed by $\mathbf{Q}$ is $\mathbf{QP}$: the first transformation is on the right.

6 Area scale factor

Image area = $|\det\mathbf{M}|\times$ original area. Do not divide by it or square it. A negative determinant includes a reflection.

7 Inverse transformation

$\mathbf{M}^{-1}$ maps images back to the original points.

8 Finding a matrix

From images of points, or from $\mathbf{CA}=\mathbf{B}$, giving $\mathbf{C}=\mathbf{BA}^{-1}$

9 Lines of points

Points with $\mathbf{M}\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}x\\y\end{pmatrix}$ do not move: for a reflection they form the mirror line.

Skills map every skill above is tested at least once; interleaved skills come from GCSE or other chapters

Skill	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Marks	6	6	6	6	6	10	10	10	10	10
1 Columns are images	•							•		
2 Standard matrices	•	•		•		•	•		•	
3 Full descriptions		•		•			•	•	•	
4 Images	•		•		•	•				•
5 Combined transformations				•		•	•		•	
6 Area scale factor			•		•	•	•	•	•	•
7 Inverse transformation					•		•			•
8 Finding a matrix						•	•	•		
9 Lines of points					•			•		•
Interleaved: matrix algebra and inverses (Chapter 5)			•	•	•	•	•	•	•	•
Interleaved: exact trigonometric values (P1)	•	•						•	•	
Interleaved: coordinate geometry and areas (P1)			•		•	•		•		•

Exam difficulty. Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WFM01 paper, where one part leads into the next and skills from different topics are combined.

Must learn. Check a matrix by finding the images of $\begin{pmatrix}1\\0\end{pmatrix}$ and $\begin{pmatrix}0\\1\end{pmatrix}$ on a quick sketch before describing it. Write the first transformation on the right, and use the determinant for areas.

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1. Write down the 2×2 matrix that represents

(a) a rotation through 90° anticlockwise about the origin,

(1)

(b) a reflection in the line $y = -x$,

(1)

(c) an enlargement with centre O and scale factor 3,

(1)

(d) a stretch parallel to the x-axis with scale factor 4,

(1)

(e) a rotation through 45° anticlockwise about the origin, giving the entries exactly.

(1)

(f) Find the image of the point (2, 5) under the transformation in part (b).

(1)

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Question 1 continued

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(Total for Question 1 is 6 marks)

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2. Describe fully the single geometrical transformation represented by each of these matrices.

(a) $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

(2)

(b) $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

(1)

(c) $\begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$

(2)

(d) $\begin{pmatrix} -3 & 0 \\ 0 & -3 \end{pmatrix}$

(1)

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Question 2 continued

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3.

$$\mathbf{M} = \begin{pmatrix} 3 & -5 \\ 2 & 4 \end{pmatrix}$$

(a) Find $\det \mathbf{M}$.

(1)

(b) A region R of area 6 is transformed by $\mathbf{M}$. Find the area of its image.

(1)

(c) The triangle with vertices $(0,0)$, $(2,0)$ and $(0,3)$ is transformed by $\mathbf{M}$. Find the coordinates of the images of the vertices.

(2)

(d) Use your answer to part (c) to verify your answer to part (a).

(2)

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Question 3 continued

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4. The transformation P is a reflection in the x-axis. The transformation Q is a rotation through 90° anticlockwise about the origin.

(a) Find the matrix representing the transformation P followed by Q.

(4)

(b) Describe fully this combined transformation as a single transformation.

(2)

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Question 4 continued

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5.

$$\mathbf{M} = \begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix}$$

(a) The point A is mapped by $\mathbf{M}$ to the point (5, 8). Find the coordinates of A.

(3)

(b) Explain why $\mathbf{M}$ does not change the area of any shape.

(1)

(c) Find the equation of the image of the line $y = x$ under $\mathbf{M}$.

(2)

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Question 5 continued

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6.

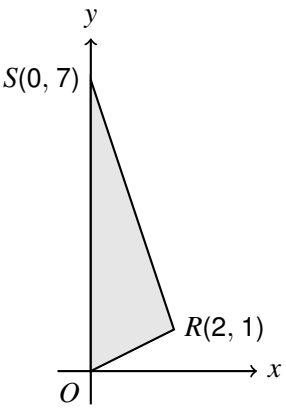

Figure 1

$$\mathbf{A} = \begin{pmatrix} p & 2 \\ -1 & 3 \end{pmatrix}, \quad \text{where } p \text{ is a constant.}$$

The triangle ORS has vertices $O(0, 0)$, $R(2, 1)$ and $S(0, 7)$, as shown in Figure 1. The transformation represented by $\mathbf{A}$ maps R to the point $R'(12, 1)$.

(a) Find the value of p.

(2)

(b) Find the area of triangle ORS.

(2)

(c) Hence, using your answer to part (b), find the area of the image of triangle ORS under $\mathbf{A}$.

(2)

(d) Find the coordinates of the image of S.

(1)

(e) Find the matrix representing the transformation $\mathbf{A}$ followed by a rotation through 90° anticlockwise about the origin.

(3)

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Question 6 continued

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Question 6 continued

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7.

$$\mathbf{A} = \begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix}, \qquad \mathbf{B} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

(a) A shape of area 4 is transformed by $\mathbf{A}$. Find the area of its image.

(2)

(b) Describe fully the single geometrical transformation represented by $\mathbf{B}$.

(2)

(c) Find the matrix $\mathbf{C}$ such that $\mathbf{CA} = \mathbf{B}$

(4)

(d) Describe, in terms of transformations, what the equation $\mathbf{CA} = \mathbf{B}$ means.

(2)

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Question 7 continued

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Question 7 continued

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8. A transformation T maps the point (1, 0) to (0.6, 0.8) and the point (0, 1) to (0.8, -0.6). T is represented by the matrix $\mathbf{M}$.

(a) Write down $\mathbf{M}$.

(1)

(b) Show that $\mathbf{M}^2 = \mathbf{I}$, and explain what this means geometrically.

(3)

(c) Find $\det \mathbf{M}$, and explain what its value tells you about T.

(2)

(d) T is a reflection. By finding the points that are not moved by T, find the equation of its mirror line.

(4)

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Question 8 continued

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Question 8 continued

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9. The matrix $\mathbf{R}$ represents a rotation through 60° anticlockwise about the origin, and $\mathbf{M} = 2\mathbf{R}$

(a) Write down $\mathbf{R}$, giving the entries exactly.

(1)

(b) Show that $\mathbf{M} = \begin{pmatrix} 1 & -\sqrt{3} \\ \sqrt{3} & 1 \end{pmatrix}$, and describe the transformation $\mathbf{M}$ as a combination of two transformations.

(3)

(c) Show that $\mathbf{M}^3 = -8\mathbf{I}$, and describe the transformation $\mathbf{M}^3$ geometrically.

(3)

(d) Write down the area scale factor of $\mathbf{M}^3$.

(1)

(e) Find the smallest positive integer n for which $\mathbf{M}^n = k\mathbf{I}$ with $k > 0$, and state the value of k.

(2)

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Question 9 continued

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Question 9 continued

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10.

$$\mathbf{T} = \begin{pmatrix} k & 2 \\ 1 & k \end{pmatrix}, \quad \text{where } k \text{ is a real constant.}$$

(a) Find the values of k for which $\mathbf{T}$ maps the whole plane onto a straight line.

(2)

(b) For the positive value of k found in part (a), find the equation of that line.

(3)

Given instead that $k = 3$

(c) find the area of the image of the triangle with vertices $(0, 0)$, $(1, 0)$ and $(0, 2)$,

(2)

(d) find the point whose image under $\mathbf{T}$ is $(8, 5)$.

(3)

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Question 10 continued

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TOTAL FOR CHAPTER 6 IS 80 MARKS

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