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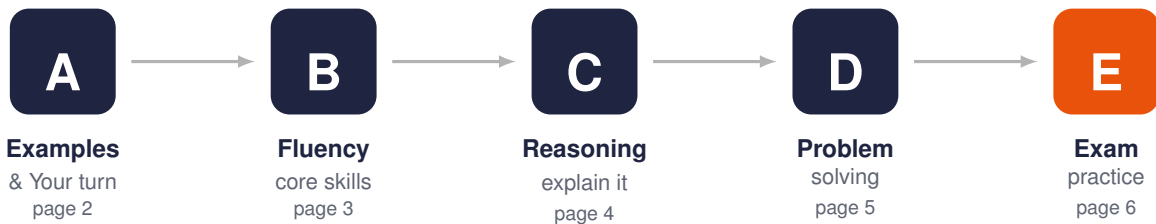
A-LEVEL MATHEMATICS • PURE 2

Sequences and Series

Worked Solutions

Pearson Edexcel International A Level (WMA12) & A Level (9MA0) • Calculator allowed: show every step

Full worked solutions for the free Sequences and Series worksheet, A-Level and International A-Level Maths



Tap a section to jump straight to it

Full worked solutions for every question on the Pure 2 Sequences and Series worksheet, in the same order and with the same question numbers.

How to mark your work. Tick each step you got right, not just the final answer. At A-level most marks are for method, so a wrong final answer can still earn most of the marks. If you lost marks, find the *first* line where your working is different from ours. That is the thing to fix.

In Section E: **M** = method mark, **dM** = method mark that depends on the previous M mark, **A** = accuracy mark (needs the M mark before it), **B** = independent mark.

Question sheet

Exam questions by topic

Sequences & Series revision engine

More worksheets



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A**Your turn answers**

Compare each line of your working with the solution.

CORE

A1 $a = 40, d = -3: u_{15} = 40 - 42 = \boxed{-2}; S_{15} = \frac{15}{2}(80 - 42) = \boxed{285}$

A2 $a = 64, r = -\frac{1}{2}: u_7 = 64 \times \frac{1}{64} = \boxed{1}; S_7 = \frac{64(1 + \frac{1}{128})}{\frac{3}{2}} = \boxed{43}$

A3 $u_2 = 10 - 10 = \boxed{0}, u_3 = 10 - 0 = \boxed{10}, u_4 = 10 - 20 = \boxed{-10}$

STRETCH

A4 $r = -\frac{20}{50} = -0.4, |r| < 1: \frac{50}{1.4} = \boxed{\frac{250}{7}}$

A5 26 terms, from 13 to 63: $\frac{26}{2}(13 + 63) = \boxed{988}$

A6 $S_n = 5(2^n - 1) > 20\,000$, so $2^n > 4001, n > 11.97 \dots: \boxed{n = 12}$

A7 $a + 4d = 23$ and $4(2a + 7d) = 196$, so $2a + 7d = 49$. With $a = 23 - 4d: 46 - d = 49: \boxed{d = -3}, \boxed{a = 35}$



B

Fluency solutions

One mark for each correct part. The notes in *italics* explain why each sequence was chosen.

CORE

B1 (a) $\boxed{43}$ (b) $\boxed{103}$ (c) $7 + 4(n - 1) = 199$: $\boxed{\text{the 49th}}$ (d) $7 + 4(n - 1) < 500, n < 124.25$: $\boxed{124}$

Every question about an AP becomes an equation or inequality in n

B2 (a) $\boxed{5050}$ (b) $\boxed{10100}$ (c) 50 terms: $\boxed{2500}$ (d) $15(14 + 116) = \boxed{1950}$

$S_n = \frac{n}{2}(\text{first} + \text{last})$ whenever the last term is known.

B3 (a) $2 \times 3^5 = \boxed{486}$ (b) $2 \times 3^9 = \boxed{39366}$ (c) $81 \times \left(\frac{1}{3}\right)^6 = \boxed{\frac{1}{9}}$ (d) $(-2)^8 = \boxed{256}$

The power is always $n - 1$

B4 (a) $\boxed{1023}$ (b) $\boxed{765}$ (c) $\frac{81 \left(1 - \frac{1}{729}\right)}{\frac{2}{3}} = \boxed{\frac{364}{3}}$ (d) $\frac{1 - 256}{3} = \boxed{-85}$

Use $\frac{a(1 - r^n)}{1 - r}$ when $r < 1$, so that the bottom stays positive.

B5 (a) $\boxed{20}$ (b) $\frac{12}{\frac{4}{3}} = \boxed{9}$ (c) $|r| > 1$: $\boxed{\text{no sum to infinity}}$

A sum to infinity exists only when $|r| < 1$

B6 (a) $\boxed{1, 3, 1}$ (b) $u_3 = u_1$: $\boxed{\text{order 2}}$ (c) 10 pairs of 3 + 1: $\boxed{40}$ (d) $40 + u_{21} = 40 + 3 = \boxed{43}$

Count whole cycles, then add the terms left over.

STRETCH

B7 (a) $3 \times 55 = \boxed{165}$ (b) $165 + 20 = \boxed{185}$ (c) $\sum_1^{20} - \sum_1^{10} = 670 - 185 = \boxed{485}$ (d) $2(2^8 - 1) = \boxed{510}$

A sum from 11 to 20 is the sum to 20 minus the sum to 10

B8 (a) $\frac{n}{2}(3n + 5) > 1000$; $S_{25} = 1000$ exactly: $\boxed{n = 26}$ (b) $3 \times 1.2^{n-1} > 100$: $\boxed{n = 21}$

(c) $500 \times 0.9^{n-1} < 10, n - 1 > 37.1$: $\boxed{n = 39}$ (d) $2^{n+1} - 2 > 10000$: $\boxed{n = 13}$

"Greater than" at an exact boundary needs the next integer; dividing by $\log 0.9 < 0$ reverses the inequality.



B9 (a) $r^3 = 8$: $r = 2, a = 6$ (b) $r^3 = -27$: $r = -3, a = 2$ (c) $1 - r = \frac{15}{60}$: $r = \frac{3}{4}$ (d) $a = 60 \times \frac{5}{4} = 75$

Dividing two terms removes a and leaves a power of r

B10 (a) $u_{n+1} - u_n = 3 > 0$: **increasing** (b) each term is half the last: **decreasing**

(c) $2, -2, 2$: **periodic, order 2** (d) $1, -\frac{1}{2}, -2, 1$: **periodic, order 3**

Periodic means a term repeats; the order is how many steps it takes.

C

Reasoning solutions

Your wording can be different. Check that your reason makes the same point.

CORE

C1 The n th term is ar^{n-1} , so the power is $10 - 1 = 9$: $3 \times 2^9 = 1536$

C2 $r = 1.5$ and $|r| > 1$, so the terms grow and the sum has no limit: **there is no sum to infinity**

C3 The correct formula is $\frac{n}{2}(2a + (n-1)d)$, with $2a$, not a

$10(10 + 57) = 670$

C4 $S_{22} = 968 < 1000$, so 22 is not enough: n must be the next whole number above 22.4 $n = 23$

STRETCH

C5 The ratio is $-\frac{1}{2}$, so $1 - r = 1 + \frac{1}{2}$: $\frac{64}{1.5} = \frac{128}{3}$

C6 $u_2 = 3, u_3 = 2 = u_1$, so the terms repeat every 2: periodic, order 2

25 pairs of $2 + 3$ and then $u_{51} = 2$: $125 + 2 = 127$

C7 After 1 year the population is 2000×1.03 , so after 6 years it is 2000×1.03^6 **2388**



C8 (a) $S_n = a + (a + d) + \dots + (a + (n - 1)d)$; reversed, $S_n = (a + (n - 1)d) + \dots + a$

Adding, each of the n pairs is $2a + (n - 1)d$: $2S_n = n(2a + (n - 1)d)$

as required

(b) $a = 1, d = 2$: $S_n = \frac{n}{2}(2 + 2n - 2) = n^2$

as required

D

Problem solving solutions

There is often more than one correct method. Any complete, correct method scores full marks.

CORE

D1 (a) $3 + 7(n - 1) = 360$, so $7(n - 1) = 357$: 52 terms

(b) $\frac{52}{2}(3 + 360) = 9438$

D2 (a) $a + 3d = 11$ and $6(2a + 11d) = 222$, so $2a + 11d = 37$; then $22 + 5d = 37$: $d = 3$, $a = 2$

(b) $\frac{n}{2}(3n + 1) > 1000$, so $3n^2 + n - 2000 > 0$, $n > 25.6 \dots$: $n = 26$

D3 (a) $24000 \times 0.88^5 \approx \text{£12 700}$

(b) $24000 \times 0.88^n < 5000$, so $n > \frac{\log(\frac{5}{24})}{\log 0.88} = 12.3 \dots$ (dividing by $\log 0.88 < 0$ reverses the inequality):

13 years

D4 (a) $a = 75(1 - 0.6) = 30$ (b) $75(1 - 0.6^5) = 69.168$

(c) 5.832; every term is positive, so the sum to infinity is more than any partial sum.

STRETCH

D5 (a) $u_2 = \frac{12}{5}$, $u_3 = 5 = u_1$, so the terms repeat: periodic of order 2

(b) $\sum_{n=1}^{40} 2n = 2 \times 820 = 1640$; $\sum_{n=1}^{40} u_n = 20 \left(5 + \frac{12}{5} \right) = 148$: 1788

D6 (a) $8 \times 0.75^3 = 3.375 \text{ m}$

(b) After the first fall, each rebound height is travelled up and then down. Rebounds: $6 + 4.5 + \dots = \frac{6}{0.25} = 24$

Total = $8 + 2 \times 24 = 56 \text{ m}$



D7 (a) $a(1+r) = 15$ and $\frac{a}{1-r} = 27$, so $27(1-r)(1+r) = 15$, $1-r^2 = \frac{5}{9}$: $9r^2 = 4$ as required

(b) $r = \frac{2}{3}$: $a = 9$; $r = -\frac{2}{3}$: $a = 45$; both have $|r| < 1$

D8 (a) The terms are a , $a+2d$ and $a+8d$; for a GP, $(a+2d)^2 = a(a+8d)$

$a^2 + 4ad + 4d^2 = a^2 + 8ad$, so $4d^2 = 4ad$, and $d \neq 0$, so $d = a$ as required

(b) The terms are a , $3a$, $9a$: $r = 3$ (c) $\frac{3(3^{10} - 1)}{2} =$ 88572

E**Exam practice solutions**

Mark-scheme notes are shown on the right of each line.

CORE

Exam questions for everyone

E1 (a) $S_3 = \frac{a(1-r^3)}{1-r} = S_\infty(1-r^3)$, so $81(1-r^3) = 78$ M1 A1

$r^3 = \frac{1}{27}$: $r = \frac{1}{3}$ A1

(b) $a = 81 \left(1 - \frac{1}{3}\right) =$ 54 B1

(c) $81 \left(1 - \frac{1}{729}\right) =$ $\frac{728}{9}$ B1

E2 (a) $2 + 19 \times 0.25 =$ 6.75 km B1

(b) $\frac{20}{2}(2 + 6.75)$ M1

87.5 km A1

(c) $\frac{N}{2}(4 + 0.25(N-1)) \geq 400$, so $N(15+N) \geq 3200$: $N^2 + 15N - 3200 \geq 0$ M1 A1

$N \geq 49.56 \dots$: $N = 50$ A1



STRETCH

E3 (a) $ar = 48$ and $ar^4 = \frac{3}{4}$, so $r^3 = \frac{1}{64}$

M1

$$r = \frac{1}{4}, \quad a = 192$$

A1 A1

(b) $|r| < 1$: $\frac{192}{\frac{3}{4}} = 256$

B1

(c) $192 \left(\frac{1}{4}\right)^{n-1} < 0.001$, so $4^{n-1} > 192\,000$

M1

$$n - 1 > \frac{\log 192\,000}{\log 4} = 8.77 \dots$$

dM1

$$n = 10$$

A1

(d) $S_n = \frac{192 \left(1 - \left(\frac{1}{4}\right)^n\right)}{\frac{3}{4}} = 256 (1 - 4^{-n})$ as required

B1

