eClassroom

A-LEVEL MATHEMATICS • DECISION 1

Algorithms

Worksheet

Pearson Edexcel International A Level (WDM11) • Calculator allowed: show every step

Free Algorithms worksheet for A-Level and International A-Level Maths, with full worked solutions

A **Examples** & Your turn page 2

B **Fluency** core skills page 4

C **Reasoning** explain it page 7

D **Problem** solving page 8

E **Exam** practice page 12

Tap a section to jump straight to it

By the end of this worksheet you can: follow algorithms and flow charts • bubble sort and quick sort • *Stretch:* bin packing with lower bounds and deductions, and binary search.

Which questions are for me?

CORE The methods every A-level student must secure. Start here.

STRETCH Sorting, searching and packing: the algorithms, the deductions they allow, and flow charts traced step by step.

Every section has both, clearly labelled. Questions get harder as you go, and every skill in Sections A to D is needed in Section E.

Track your progress

Section	B	C	D	E
Core marks	23	10	13	22
Stretch marks	16	14	17	14
My score				

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A Worked examples & Your turn

Read each example carefully. Cover it up, then answer the question beside it using the same method.

CORE *Secure the method*

Worked example A1: one bubble-sort pass

Carry out one pass of a bubble sort into ascending order on

8 3 9 1 6

Swap 8,3; keep 8,9; swap 9,1; swap 9,6

3 8 1 6 9

Your turn A1

Carry out one pass of a bubble sort into ascending order on

7 2 6 9 3

Worked example A2: one quick-sort pass

Carry out the first pass of a quick sort into ascending order on

14 7 22 9 18 3 11

Pivot: the middle (4th) item, 9

7 3 **9** 14 22 18 11

Your turn A2

Carry out the first pass of a quick sort into ascending order on

31 12 25 40 17 8 28

Worked example A3: first fit

Pack 5 7 3 4 6 2 into bins of size 10 using first fit.

Each item goes in the first bin it fits

[5, 3, 2] [7] [4, 6]

Your turn A3

Pack 6 4 8 5 3 7 into bins of size 12 using first fit.

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Worked example A4: a lower bound

Find a lower bound for the number of bins of size 10 needed for 5, 7, 3, 4, 6, 2

$\frac{27}{10} = 2.7$, *round up*

3

Your turn A4

Find a lower bound for the number of bins of size 15 needed for 9, 8, 7, 6, 5, 4

Worked example A5: binary search

Search for 26 in

3 8 12 15 21 26 30 34 41

Middle $\left[\frac{1+9}{2}\right] = 5$*: 21, reject 1–5; then 8: 34, reject 8–9; then 7: 30*

found at position 6

Your turn A5

Search for 9 in

2 5 9 13 17 20 24 28

Worked example A6: first-fit decreasing

Pack 5 7 3 4 6 2 into bins of size 10 using first-fit decreasing.

Sort first: 7, 6, 5, 4, 3, 2

[7, 3] [6, 4] [5, 2]

Your turn A6

Pack 6 4 8 5 3 7 into bins of size 12 using first-fit decreasing.

Worked example A7: tracing an algorithm

Step 1 Input n. Step 2 Let $s = 0$, $k = 1$. Step 3 Let $s = s + k$. Step 4 Let $k = k + 2$. Step 5 If $k < 2n$ go to Step 3. Step 6 Output s. Trace it with $n = 4$

s: 1, 4, 9, 16; k: 3, 5, 7, 9

16

Your turn A7

Step 1 Input n. Step 2 Let $p = 1$, $k = 1$. Step 3 Let $p = p \times k$. Step 4 Let $k = k + 1$. Step 5 If $k \leqslant n$ go to Step 3. Step 6 Output p. Trace it with $n = 5$

B Fluency

Each question is a designed sequence. From one part to the next, ask yourself: what changed, and what did it do to the answer?

CORE *Secure the method*

B1 Use a bubble sort into ascending order on

9	4	7	2	8	5

(a) Write down the list after pass 1

(b) Write down the list after pass 2

(c) Complete the sort.

(d) State how many passes were needed in total.

[4 marks]

B2 Use a quick sort into ascending order on

23	41	16	35	8	29	12	47

(a) State the first pivot.

(b) Write down the list after pass 1

(c) Write down the list after pass 2

(d) Complete the sort.

[4 marks]

B3 Use a quick sort into descending order on

3.4	1.8	5.2	2.7	4.6	0.9	3.9

(a) State the first pivot.

(b) Write down the list after pass 1

(c) State the pivots for pass 2

(d) Complete the sort.

[4 marks]

B4 The items

7	12	9	5	13	6	8

are packed into bins of size 20

(a) Find a lower bound.

(b) Use first fit.

(c) Use first-fit decreasing.

(d) State which packing is optimal, with a reason.

[4 marks]

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B5 Use a binary search on

4	9	13	18	22	27	31	36	40	44	50

(a) to find 36

(b) to try to find 15

(c) and state the most iterations ever needed for this list. **[3 marks]**

B6 The items

7	5	3	8	2	6	4	5

are packed into bins of size 10

(a) Find a lower bound.

(b) Use the full-bin method.

(c) Use first fit.

(d) Compare the two packings.

[4 marks]

STRETCH

B7 Use the flow chart in Figure 1.

(a) Trace it with $n = 6$, listing each value of n

(b) State the output.

(c) State the output when $n = 7$

(d) Explain what c counts.

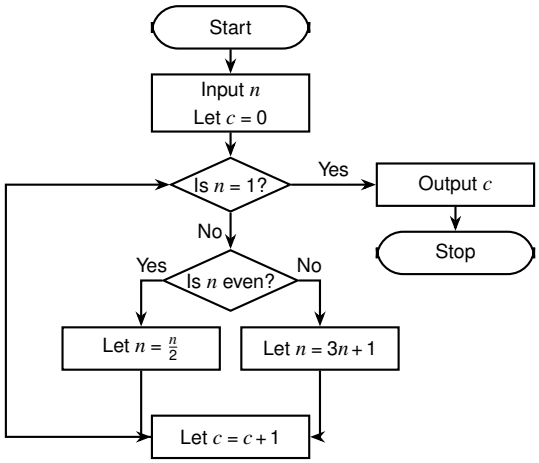


Figure 1

[4 marks]

B8 Use a bubble sort into ascending order on

5	1	4	2	8

(a) Give pass 1 with its comparisons and swaps.

(b) Give pass 2 with its comparisons and swaps.

(c) Explain why one more pass is needed.

(d) State the most comparisons a bubble sort of 5 items can need.

[4 marks]

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B9 First fit packs

23	18	31	12	27	9	16	21	14	8

into bins of size n as

Bin 1: 23 18 Bin 2: 31 9 Bin 3: 12 27 Bin 4: 16 21 Bin 5: 14 8

(a) Use Bin 1 to find a lower limit for n

(b) Use the 12 to find an upper limit for n

(c) State the possible values of n

(d) Check your values with the 8

[4 marks]

B10 After one pass of a quick sort into descending order, a list of ten numbers is

43	35	21	40	50	57	39	19	5	17

(a) Identify the pivot, with a reason.

(b) State its position in the original list of ten.

(c) Write down the sublists for pass 2

(d) State the pivots for pass 2

[4 marks]

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C Reasoning

Show your working and explain your thinking. A correct answer without a reason does not score full marks.

CORE *Secure the method*

C1 Packing 5, 7, 3, 4, 6, 2 into bins of size 10 using first fit, Amir puts the 2 into the bin holding 4 and 6 because "it was the last bin used". Explain his error. **[2 marks]**

C2 Asked to sort a list into descending order, Bea gives a correct sort into ascending order. Explain why she loses marks. **[2 marks]**

C3 In the first pass of a quick sort into ascending order on 14, 7, 22, 9, 18, 3, 11, Cal writes 3 7 **9** 11 14 18 22

Explain his error, and give the correct first pass. **[3 marks]**

C4 Dev's bubble sort reaches 2 4 5 7 8 9 after three passes and he stops. Explain why the algorithm needs another pass, and how many comparisons it makes. **[3 marks]**

STRETCH

C5 For a binary search for 9 in 2, 5, 9, 13, 17, 20, 24, 28, Eve takes the 4th item as the first middle and keeps it in the list she searches next. Explain her two errors. **[3 marks]**

C6 Fay says "first-fit decreasing always uses the fewest bins". Use the items 6, 5, 4, 3, 3, 3 in bins of size 12 to show that she is wrong. **[3 marks]**

C7 Gus says that a lower bound of 2 for the items 6, 6, 6 in bins of size 10 means two bins are enough. Explain why he is wrong. **[3 marks]**

C8 **(a)** Show that a full bubble sort of n items can need $\frac{n(n-1)}{2}$ comparisons.

(3)

(b) Hence find the most comparisons a bubble sort of 12 items can need, and give a list of 4 items that needs the most.

(2)

[5 marks]

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D Problem solving

You decide the method. Read the whole question first, and show every step.

CORE *Secure the method*

D1 The masses, in kg, of nine items that are to be moved by van are

220	340	180	410	260	150	300	270	190

Each van can carry at most 700 kg, and all the items will be moved at the same time.

(a) Calculate a lower bound for the number of vans required.

(1)

(b) Use the first-fit bin packing algorithm to allocate the items to the vans, and state whether your allocation is optimal.

(2)

[3 marks]

D2 A list of ten numbers is to be sorted into descending order. After one pass, the quick sort algorithm produces the list

43	35	21	40	50	57	39	19	5	17

(a) State, with a reason, which number was used as the pivot for the first pass.

(1)

(b) Continue the quick sort to obtain the fully sorted list. You must make your pivots clear.

(3)

[4 marks]

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D3 In this question INT(X) is the largest integer less than or equal to X. Consider the algorithm

Step 1 Input N. Step 2 Let $k = 1$, $c = 0$. Step 3 If INT$\left(\frac{N}{k}\right) = \frac{N}{k}$, let $c = c + 1$. Step 4 Let $k = k + 1$. Step 5 If $k \leqslant N$ go to Step 3. Step 6 Output c.

(a) Complete the table in the answer book with $N = 10$, showing the results of each step.

(2)

(b) Explain what the output tells you about N, and state what an output of 2 would mean.

(1)

Complete this table

k	$\frac{N}{k}$	INT$\left(\frac{N}{k}\right)$	c

Output _____

[3 marks]

D4 The ten numbers in the list below are packed into bins of size n, where n is a positive integer.

23 18 31 12 27 9 16 21 14 8

The first-fit bin packing algorithm gives Bin 1: 23 18, Bin 2: 31 9, Bin 3: 12 27, Bin 4: 16 21, Bin 5: 14 8

Based on this packing, determine the possible values of n. You must give reasons for your answer.

[3 marks]

STRETCH

D5 A list of seven distinct integers is to be sorted into descending order using a bubble sort.

12	x	5	18	9	15	3

The first two passes give

12 x 18 9 15 5 3 and 12 18 9 15 x 5 3

(a) Determine the range of possible values of x

(2)

(b) Given that $x = 7$, complete the bubble sort, writing down the list at the end of each pass.

(2)

[4 marks]

D6 The sorted list below shows the numbers of tickets sold for fifteen events.

3	7	11	14	19	23	26	30	34	38	41	45	49	52	58

(a) Use the binary search algorithm to try to locate 40 in the list. You must make your pivots clear and state which part of the list is rejected at each stage.

(3)

(b) State the maximum number of iterations the binary search could need for a list of fifteen items.

(1)

[4 marks]

D7 Eight boxes, with masses in kg

16	11	23	8	19	14	6	21

are to be loaded onto trolleys that each hold at most 40 kg.

(a) Calculate a lower bound for the number of trolleys needed.

(1)

(b) Use the first-fit decreasing bin packing algorithm to allocate the boxes to trolleys.

(2)

(c) Explain why your allocation is optimal.

(1)

[4 marks]

D8 Figure 2 shows a flow chart for an algorithm. The input is a positive integer.

(a) Complete the table in the answer book for $N = 30$, showing the results at each step.

(3)

(b) Explain what the output, c, tells you about the input.

(1)

(c) Without tracing the flow chart again, state the output when $N = 49$

(1)

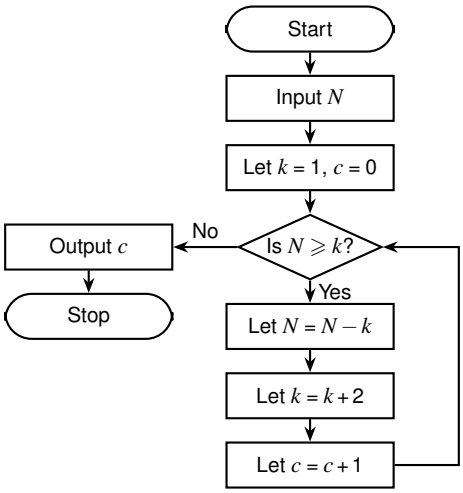


Figure 2

Complete this table

N	k	c

Output _____

[5 marks]

E Exam practice

Exam-style questions in the style of WDM11. Write your answers in the spaces, and show every step.

CORE *Exam questions for everyone*

E1 The list below shows the lengths, in minutes, of eleven video clips that are to be stored on discs. Each disc can hold at most 60 minutes of video, and a clip cannot be split between discs.

24	26	18	23	34	27	38	12	11	28	32

(a) Calculate a lower bound for the number of discs needed. You must show your working.

(2)

(b) Use the first-fit bin packing algorithm to allocate the clips to discs.

(2)

(c) Use a quick sort to obtain a list of the lengths in descending order. You should show the result of each pass and identify your pivots clearly.

(4)

(d) Apply the first-fit decreasing bin packing algorithm to your sorted list, and state whether the number of discs used is optimal.

(2)

(Total for Question E1 is 10 marks)

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Answer book: Question E1

Work in the space on this page, as you would in the WDM11 answer book. The list is printed again for you to work from.

24 26 18 23 34 27 38 12 11 28 32

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E2 A courier has ten parcels to deliver. The masses of the parcels, in kilograms, are listed below in the order in which they were collected.

4.8	2.3	6.1	1.7	3.9	5.4	2.9	0.8	4.2	3.3

(a) The list is to be sorted into ascending order. Use a quick sort to obtain the sorted list. You should show the result of each pass and identify your pivots clearly.

(4)

(b) Use the binary search algorithm on your sorted list to try to locate a parcel of mass 3.5 kg. Clearly indicate how you choose your pivots and which part of the list is rejected at each stage.

(3)

The parcels are to be packed into sacks for delivery. Each sack can hold a total mass of at most 10 kg, and a parcel cannot be split between sacks.

(c) Apply the first-fit decreasing bin packing algorithm to pack the parcels into sacks.

(3)

(d) Explain why the number of sacks used in part (c) is optimal.

(2)

(Total for Question E2 is 12 marks)

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Answer book: Question E2

Work in the space on this page, as you would in the WDM11 answer book. The list is printed again for you to work from.

4.8 2.3 6.1 1.7 3.9 5.4 2.9 0.8 4.2 3.3

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STRETCH

E3 The twelve numbers in the list below are to be packed into bins of size n, where n is a positive integer.

33	34	39	17	10	30	21	20	25	24	35	8

When the first-fit bin packing algorithm is applied to the list, the following allocation is obtained.

Bin 1: 33 34 10 Bin 2: 39 17 21 Bin 3: 30 20 25 Bin 4: 24 35 8

(a) Based on this packing, determine the possible values of n. You must give reasons for your answer.

(4)

When the first-fit decreasing bin packing algorithm is applied to the list, the following allocation is obtained.

Bin 1: 39 35 8 Bin 2: 34 33 10 Bin 3: 30 25 24 Bin 4: 21 20 17

(b) Determine the value of n. You must give a reason for your answer.

(2)

(c) Show that this number of bins is optimal.

(2)

The original list of twelve numbers, in the order given above, is now to be sorted into descending order using a bubble sort, starting at the left-hand end of the list.

(d) Perform the first pass, writing down the list at the end of the pass, and state the number of comparisons and the number of swaps made.

(3)

(e) Determine the maximum number of further comparisons that could be needed to complete the sort. You must explain your reasoning.

(3)

(Total for Question E3 is 14 marks)

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Answer book: Question E3

Work in the space on this page, as you would in the WDM11 answer book. The list is printed again for you to work from.

33 34 39 17 10 30 21 20 25 24 35 8

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